

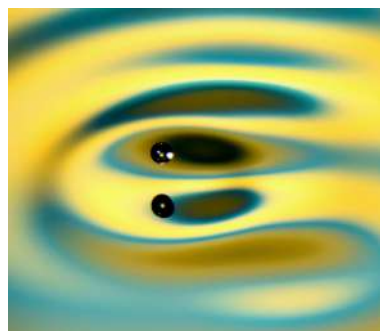
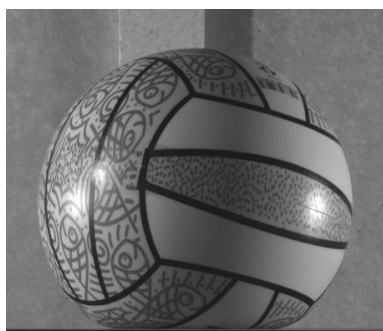
..... Épistémologie du modèle physique

HABILITATION À DIRIGER DES RECHERCHES

Discipline : Sciences pour l'ingénieur

Le modèle physique :
un outil pour appréhender les systèmes complexes ?

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Résumé

Le modèle est indissociable de la pratique scientifique : le physicien synthétise ses observations en une théorie mathématique, le biologiste résume sa vision d'un processus biochimique sur un schéma, l'ingénieur bâti des maquettes aux dimensions réduites pour étudier le comportement mécanique d'une structure.

Le modèle, entendu dans le sens de système modèle, considère une idéalisation matérielle du phénomène étudié en épargnant les aspects contraignants du système réel. Pour l'ingénieur, une maquette réduite permet d'éviter de bâtir le système à l'échelle 1, le biologiste peut, quant à lui, mettre de côté la variabilité biologique. Le physicien, lui, s'engage à étudier des systèmes épurés et la notion de système modèle apparaît comme l'essence de son activité.

Le premier chapitre de ce mémoire concerne un rare exemple de système modèle pour la physique : les gouttes marcheuses comme un analogue à la mécanique quantique. Ce système permet de tester les différences entre les résultats de la mécanique quantique et la dualité onde-corpuscule à l'échelle macroscopique. Le second chapitre concerne les interactions fluide-solide au niveau des feuilles d'arbre. Des feuilles artificielles sont utilisées comme systèmes mécaniques idéalisés pour étudier les interactions avec les gouttes de pluie, le vent, et les mouvements induits par changements osmotiques au sein des tissus mous. La troisième partie de ce mémoire présente des exemples d'actionneurs mécaniques élémentaires inspirés de ces systèmes modèles idéalisés.

Les systèmes modèles, à l'intersection de la mécanique, la physique et la biologie offrent un terrain de jeu fantastique pour le scientifique, aussi bien pour des questions fondamentales que pour des applications concrètes.

Mots clés : Interactions fluide-solide, Biomécanique des plantes, actionneurs bioinspirés.

Abstract

Modelling is at the core of scientific practice: the physicist summarizes his observations into a theory, the biologist describes a biochemical process on a diagram, the engineer builds small-scale models to study the mechanical behavior of a structure.

The model system is a material idealization of the phenomenon under study, avoiding some restrictive aspects of the real system. For the engineer, a reduced mock-up avoids the cost of building the real system, while for the biologist, biological variability can be set aside with a mechanical model. Physicists, on the other hand, aims at studying minimal systems, which make the model system the essence of their practice.

The first chapter of this manuscript concerns a rare example of a model system for physics: walking drops as an hydrodynamic quantum analogue. This system enables to test the differences between quantum mechanics and wave-corpuscle duality at the macroscopic scale. The second chapter deals with fluid-solid interactions involving tree leaves. Artificial leaves are used as idealized mechanical systems to study interactions with raindrops, wind and osmosis-induced motions within soft tissues. The third part of this manuscript presents examples of elementary mechanical actuators emerging from such idealized model systems.

Model systems, at the intersection of mechanics, physics and biology, offer a fantastic scientific playground, both for fundamental questions and direct applications.

Keywords: Fluid-solid interaction, plant biomechanics, bioinspired actuators

Remerciements

Mes remerciements vont tout d'abord aux membres du jury, Jean-Marc Allain, Jacques Dumais, Benoît Roman, Pierre Lambert et Jean-Marc Linares qui m'ont fait l'amitié d'examiner mon travail. Ces grands chercheurs sont une source d'inspiration pour une science excitante et de qualité.

Je tenais aussi à remercier tous les chercheurs, ingénieurs, techniciens et étudiants que j'ai pu côtoyer et qui m'ont chacun aidé à former mon esprit et à prendre de la hauteur de vue.

J'ai une pensée reconnaissante pour ma famille : mes fils, ma femme, mes parents et mon frère qui m'ont soutenu, aidé, conforté et parfois réconforté. Il y a toujours une partie immergée à l'iceberg qu'est un chercheur.

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Avant-propos

La science foisonne d'une littérature débordante, à l'écriture froide et mécanique. La fantaisie n'est pas de mise pour convaincre (Chester et al., 1967). De ce fait, il est rare d'avoir une tribune personnelle pour exposer comment les idées se forment, quelles questions motivent le scientifique et quelles sont ses vues pour le futur. Ce rapport me donne l'opportunité d'apporter cet éclairage et d'exposer les travaux postérieurs à ma thèse dont les points de convergence sont (i) l'investigation d'un système complexe, biologique ou quantique, via un système mécanique modèle et (ii) la mécanique-physique non-linéaire.

Je tiens à préciser que ce document n'a pas pour vocation d'être un traité de philosophie exhaustif mais témoigne d'un point de vue personnel sur les systèmes modèles. J'insiste notamment sur l'utilité de ces modèles en biomécanique (Abourachid et al., 2023) et pour la bioinspiration.

Un parcours non-linéaire

J'ai une formation de physicien des fluides et de la matière molle. J'ai effectué ma thèse dans un laboratoire de mécanique des fluides. J'aime les plantes et j'ai un goût particulier pour les problèmes d'élasticité.

La première partie de ce rapport présente les travaux de recherche que j'ai menés pendant 5 ans à l'université de Liège, d'abord comme post-doc, puis comme chargé de recherche au FNRS. Ces travaux portent majoritairement sur la possibilité de considérer le système des gouttes marcheuses découvert par Yves Couder; Protiere et al. (2005) comme un analogue à la mécanique quantique.

En 2020, pendant la pandémie, j'ai été recruté comme maître de conférences à Aix-Marseille Université, à l'Institut des Sciences du Mouvement. Depuis, j'ai recentré mes recherches autour des systèmes fluides et solides et de leurs interactions, avec des systèmes biologiques pour inspiration. Ma position d'enseignant à l'Institut Universitaire de Technologie (IUT) m'a permis de tisser des liens avec plusieurs industriels aéronautiques.

Liste des principales contributions scientifiques postérieures à la thèse

ANALOGUE HYDRODYNAMIQUE À LA MÉCANIQUE QUANTIQUE

Les gouttes marcheuses sont un exemple de dualité onde-corpuscule macroscopique. Quelles sont les similarités et les différences entre ce système et la théorie quantique?

1. Les gouttes marcheuses sont-elles non-locales? Nous avons montré que le champ d'onde autour du marcheur est amorti exponentiellement et quantifié l'extension de ce champ d'onde. Nous avons montré que l'extension du champ d'onde ne peut pas être infinie même si l'on travaille à des nombres de mémoire infinie (aussi près que possible du seuil de Faraday).
2. Nous avons proposé une théorie visqueuse pour déterminer précisément le seuil de l'instabilité Faraday.
3. Nous avons montré que les gouttes marcheuses ont une dynamique verticale perturbée lorsqu'elles interagissent avec d'autres ondes. Cette dynamique verticale est une source de chaos.
4. Nous avons montré que la diffraction d'une goutte marcheuse crée un pattern de diffraction à 3 pics. Nous avons comparé de manière quantitative les patterns de diffraction en fonction de l'amplitude de l'onde avec la prédiction quantique. Les données expérimentales ne vont pas dans le sens d'une analogie entre les deux systèmes dans cette configuration.

2 PRINCIPALES PUBLICATIONS DU DOMAINE

1. **Tadrist, L.**, Shim, J. B., Gilet, T., & Schlagheck, P. (2018). Faraday instability and subthreshold Faraday waves : surface waves emitted by walkers. *Journal of Fluid Mechanics*, 848, 906-945.
2. **Tadrist, L.**, Sampara, N., Schlagheck, P., & Gilet, T. (2018). Interaction of two walkers : Perturbed vertical dynamics as a source of chaos. *Chaos : An Interdisciplinary Journal of Nonlinear Science*, 28(9).

BIOMÉCANIQUE DES FEUILLES

Peut-on comprendre les relations forme-fonction chez les feuilles avec des outils de mécanique?

1. Nous avons montré que l'optimisation des tailles de feuilles vis-à-vis de l'auto-support suit des lois d'échelles d'allocation de biomasse entre les parties portantes et les parties fonctionnelles.
2. Nous avons montré que le raidissement des tissus mous des plantes avec la turgescence ne peut pas être expliqué par une théorie linéaire. Au contraire, le raidissement provient d'une loi de comportement raidissante des parois cellulaires.

3. Nous avons montré que pour une cellule, la raideur effective est impactée par le comportement fluide (du plus souple au plus raide : isobare, isotherme/osmotique, adiabatique et isochore). Le coefficient de Laplace peut être utilisé comme un proxy pour déterminer l'effet raidissant du fluide sur la membrane.
4. COMPLÉMENT AUX MEMBRANES PRESSURISÉES : Nous avons montré que, lors d'un impact, le contact d'une membrane pressurisée au gaz est non linéaire avec un temps de contact d'autant plus faible que la vitesse et la pression initiales sont élevées. La transformation du gaz est adiabatique. L'énergie est principalement dissipée par des vibrations consécutives à l'impact.
5. Nous avons montré que la flexibilité des feuilles permet l'éjection de gouttes d'eau vers le haut et quantifié cet effet pour la propagation de maladies.

2 PRINCIPALES PUBLICATIONS DU DOMAINE

1. **Tadrist, L.**, & Darbois-Textier, B. (2016). Are leaves optimally designed for self-support? An investigation on giant monocots. *Journal of Theoretical Biology*, 396, 125-131.
2. Lacorre, P., Fiore, L., Linares, J. M., & **Tadrist, L.** (2024). Pressurized membranes between walls : Thermodynamic process changes force and stiffness. *Journal of the Mechanics and Physics of Solids*, 192, 105798.

ROBOTIQUE MOLLE ET BIO-INSPIRÉE

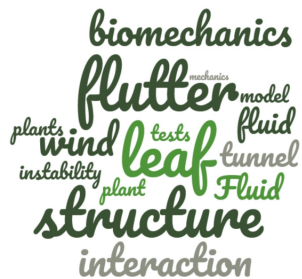
1. Nous avons construit un gripper capillaire nano-structuré pour la manipulation de micro-objets, inspiré de l'adhésion capillaire des insectes.
2. Nous avons construit une maquette de muscle artificielle composée d'actionneurs élémentaires. Nous avons montré que la redondance à la méso-échelle permet de conserver des performances élevées malgré les dommages.
3. Nous avons construit un pulvinus artificiel et montré que l'effet du gonflage aux petites pressions est assouplissant.
4. Nous avons construit un robot flexible minimal alimenté et contrôlé par des vibrations. Le déplacement est réalisé sur l'effet non réciproque de la force de friction et le déphasage des forces tangentes et normales.

2 PRINCIPALES PUBLICATIONS DU DOMAINE

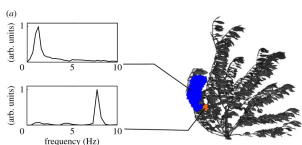
1. Barois, T., Boucherie, A., Tadrist, L., & Kellay, H. (2024). Controlled Locomotion of a Minimal Soft Structure by Stick-Slip Nonlinearity. *Physical Review Letters*, 133(23), 238301.
2. Iazzolino, A., Tourtit, Y., Chafaï, A., Gilet, T., Lambert, P., & **Tadrist, L.** (2020). Pick up and release of micro-objects : A motion-free method to change the conformity of a capillary contact. *Soft Matter*, 16(3), 754-763.



Test de Moulia



Expériences en soufflerie



Decomposition bi-orthogonale

Cheminement scientifique

Au début de mes études de doctorat, j'ai construit des feuilles biomimétiques pour étudier leurs déformations statiques sous gravité et en vent. J'ai étudié leur mécanique, modélisé finement leurs déformées, résolu de belles équations différentielles. Au bout de quelques mois, je racontais que les feuilles étaient courbées à cause de la gravité. Je me fourvoyais. C'était une des plus belles bêtises scientifiques de ma vie^a... dans mon esprit, le système modèle avait pris le pas sur le système réel. Il a fallu toute la sagacité didactique de Bruno Moulia pour me faire comprendre en douceur qu'un système modèle n'est qu'une idéalisation de la réalité. Il m'a fait cueillir une feuille, elle était bien courbée comme fléchie par gravité. Puis, il m'a demandé de la retourner. Elle était courbée vers le haut! Cette courbure n'était pas le fait de la gravité mais le fait de la croissance^b! Les faits sont têtus. Un système modèle biomimétique dont les idéalizations sont perdues de vue est un grand danger.

Néanmoins, ces feuilles biomimétiques m'ont permis de comprendre comment le vent les faisaient flotter (critère de Den Hartog). En combinant un modèle de turbulence pour le mouvement des branches, nous avons pu montrer que le mouvement du feuillage et la superposition du mouvement des feuilles prédominant à basse vitesse de vent et de celui des branches qui prend le dessus à haute vitesse. Finalement, nous avons rationalisé un arbre en vent avec des outils classiques d'interaction fluide-structure.

D'un point de vue expérimental, j'ai développé des compétences dans la mise au point et l'instrumentation de feuilles modèles en petite soufflerie. Le but était de détecter le seuil d'apparition du flottement.

D'une manière plus ambitieuse, nous avons placé un arbre complet dans la très grande soufflerie climatique

^a. Une part de l'intégrité scientifique est de reconnaître ses a priori et erreurs.

^b. J'appelle cette preuve le « test de Moulia ». Depuis ce jour, je ne me suis encore jamais aventuré sur le domaine des feuilles en croissance et prends bien soin de vérifier comment les échelles de temps mécanique et biologique sont dissociées.



Caractérisation vibratoire de végétaux.

Optimalité de la répartition de biomasse pour l'autosupport



Etirement Vs élongation.
Longueur de déploiement d'une
feuille courbée



Etirement Vs élongation.
Apparition de D-cônes



Jules Verne du Centre Scientifique et Technique du Bâtiment (CSTB) de Nantes. L'instrumentation était très riche avec une balance d'efforts, plusieurs anémomètres et 3 caméras haute fréquence (>200 Hz). J'ai aussi développé des compétences en analyse d'images en pratiquant la décomposition bi-orthogonale (en temps et en espace) d'un signal vidéo.

Simultanément à ma thèse, je me suis intéressé à l'optimalité de la répartition de biomasse des feuilles pour l'auto-support. Les validations expérimentales ont été effectuées sur des grandes feuilles de palmier^a, avec des expériences de "coin de jardin". Ces feuilles avaient une belle veine centrale et correspondaient bien aux feuilles idéalisées considérées dans le modèle.

Pour en comprendre d'avantage, nous nous sommes intéressés aux stratégies de rigidification géométrique par courbure de Gauss. Cette stratégie arrive régulièrement chez les plantes à feuilles sessiles qui grandissent enroulées autour de la tige centrale. J'ai pu développer des techniques de mesure de courbure par Moiré (mesure optique) et de modélisation analytique en considérant uniquement les énergies de courbure (bending) et d'étirement (stretching).

En expérimentant, j'avais été gêné par l'apparition de défauts où l'énergie d'étirement se concentrait, allant jusqu'à la plastification locale des feuilles élastiques. Nous avons donc prolongé ce travail en déterminant la condition d'apparition des d-cônes (zones de stretching concentré) permettant de relâcher les contraintes de manière globale et réduisant la longueur de déploiement des feuilles.

Après ces premières interactions avec les feuilles, j'avais le sentiment que les questions profondes se trouvaient à l'intérieur des tissus souples des plantes, sans réussir à formaliser clairement ces questions. J'avais observé, par exemple, que les feuilles de cerisier s'assouplissaient en période de manque d'eau pour limiter leur interception solaire au moment les plus chauds de la journée; des modifications de rigidité où jouent fluides, solides et thermodynamique. Cette observation est le point de départ des actionneurs biomimétiques que je développe aujourd'hui.

^a. Jardin botanique du Rayol, France.

Expérience professionnelle à l'étranger

2015-2017 : Postdoc, Microfluidics lab, Université de Liège.

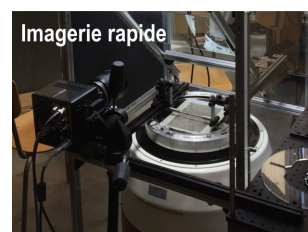
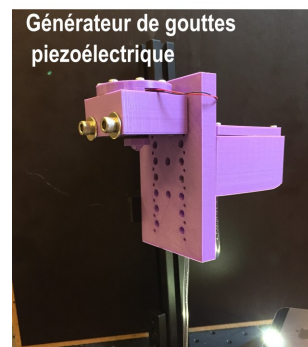
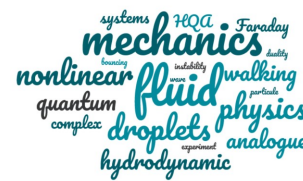
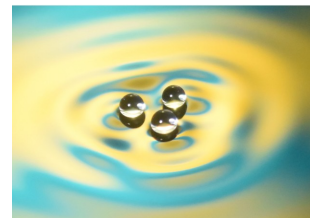
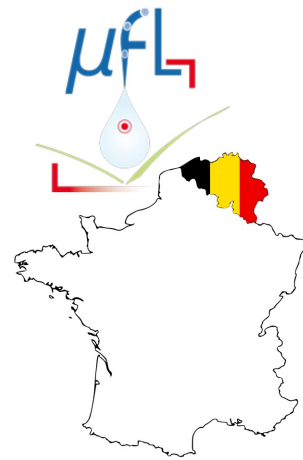
2017-2020 : FNRS Research fellow, Microfluidics lab.

2018 : Visiting scholar, MIT.

Je suis parti en post-doctorat chez Tristan Gilet (Université de Liège), que j'avais rencontré à la conférence plant biomechanics en 2012 (Clermont-Ferrand), sur un sujet complètement différent : les gouttes marcheuses comme analogue à la mécanique quantique. J'avais découvert cette expérience lors de mon master de science (2009/2010) en visite au laboratoire Matière et Systèmes Complexes. Cette expérience était toute récente à l'époque, et pour un physicien réaliste comme moi, représentait un énorme espoir de pouvoir enfin appréhender la mécanique quantique sans parler d'onde de probabilité. Je suis parti à Liège avec une ouverture d'esprit maximale : je n'avais pas pour objectif de montrer l'analogie entre goutte marcheuse et mécanique quantique mais de tester les limites de l'analogie entre les deux systèmes. Les faisceaux de preuves s'accumulaient, prouvant la similarité des deux systèmes : quantification orbitale, quantification dans un puits de potentiel, densité de probabilité de présence dans une cavité. Seule l'expérience de diffraction à deux fentes manquait à l'appel.

J'ai entrepris de mieux comprendre ce système en déterminant une expression de l'extension spatiale du champ d'onde, une expression approchée du seuil de Faraday en tenant compte de la viscosité et en étudiant l'effet des perturbations de phase de rebond des gouttes sur la prédictibilité du mouvement (chaos par sensibilité aux conditions initiales).

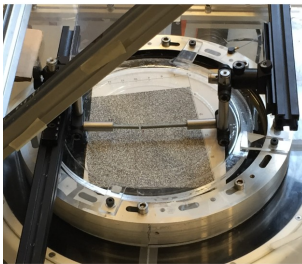
Pour vérifier les prédictions théoriques sur l'extension du champ d'onde, j'ai mis en place des techniques expérimentales nouvelles au laboratoire : un générateur de goutte piézoélectrique imprimé en 3D, de la mesure de champ d'onde par corrélation d'image 2D avec acquisition rapide. Ces montages expérimentaux et les traitements d'image subséquents m'ont permis de développer des compétences expérimentales en mesure optique, traitement d'image avancé et automatisation expérimentale. En effet, pour la plupart des expériences,



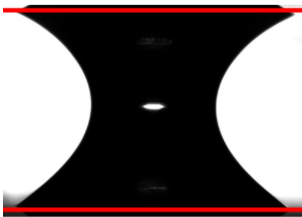
Expérience automatisée
Labview



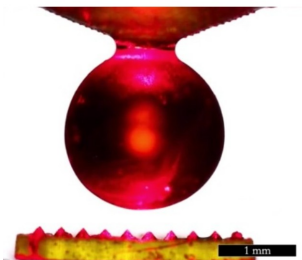
Mesure de champ d'onde
Corrélation d'images 2D



Rupture de ponts liquides



Evaporation sous laser IR



Aérodynamique des balles
1/2 lisse et 1/2 rugueuses



le banc expérimental était asservi en vibration à l'aide du logiciel Labview; l'acquisition, le formatage et le pré-traitement des données étaient sans aucune intervention humaine. L'éloignement de l'intervention humaine sur l'acquisition des données était une limitation du biais lié à l'opérateur.

Avec ces arguments, nous avons démontré que les conditions expérimentales utilisées dans la communauté ne permettaient pas de pouvoir observer des interférences. Pour se placer dans une configuration favorable pour observer des interférences, nous avons proposé d'utiliser une huile moins visqueuse pour le bain dans le but d'augmenter la non-localité des marcheurs^a.

En 2018, j'ai été reçu comme visiting scholar dans le groupe de John Bush au Massachusetts Institute of Technology (MIT). John Bush s'est positionné comme chercheur leader sur les gouttes marcheuses après la mise en retrait d'Yves Couder qui avait découvert le phénomène en 2005. J'ai pu approfondir mes compétences en imagerie rapide et en expérience automatisée longue durée (jusqu'à 24h consécutives). L'institution MIT possède des ressources mutualisées de grande qualité comme le Edgerton Center pour l'imagerie rapide qui permet de diffuser la connaissance technique de manière très efficace.

Ma pratique de l'expérience des gouttes marcheuses m'a convaincu d'un écueil important : l'interaction de l'onde capillaire avec les fentes submergées. L'onde est très amortie au passage des fentes ce qui empêche l'apparition des interférences dans l'espace postérieur aux fentes. Pour éviter cet écueil nous avons réalisé une expérience de diffraction sur onde stationnaire : la diffraction de Kapitza-Dirac. Les résultats finaux ne confirment pas l'analogie entre les deux systèmes. Cette expérience est unique car l'asservissement de l'amplitude du pot vibrant, permettant de gérer la lévitation et le déplacement des gouttes marcheuses, n'était pas réalisé sur des mesures inertiels (accéléromètres) mais sur l'amplitude de l'onde stationnaire au sein d'une rainure plus profonde. La mesure de l'onde, quelques microns, était réalisée en temps réel par corrélation d'image 1D.

^a. Je vous renvoie au chapitre 2 pour une histoire complète sur les marcheurs.

En modélisant les gouttes marcheuses, j’ai pu développer des compétences en simulation numérique de systèmes chaotiques et leur analyse.

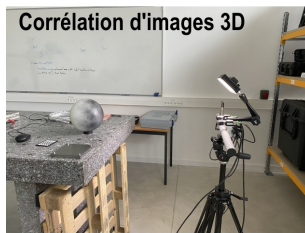
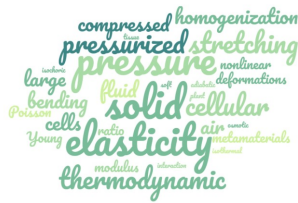
Pendant mon passage en Belgique, j’ai continué à m’intéresser à des systèmes bioinspirés et biomimétiques au niveau microfluidique. Les impacts de gouttes sur feuilles d’arbre modèles pour comprendre la dissémination de pathogènes et l’utilisation de surfaces texturées (Impression 3D à résolution sub-micrométrique) à la manière des insectes pour la préhension d’objets millimétriques et sub-millimétriques. Ce gripper bioinspiré fut le départ d’une étude théorique sur la rupture des ponts capillaires menée pendant le confinement de la dernière pandémie.

Au cours de mon passage en Belgique, je suis resté en contact avec la communauté d’interaction fluide-structure et ingénierie du vent. Nous avons mis au point une expérience en soufflerie pour mesurer l’effet de la rugosité dissymétrique sur des balles de cricket. Nous avons montré que le déclenchement de la crise de traînée n’arrive pas en même temps sur le côté lisse que sur le côté usé. Cette rupture de symétrie crée une force latérale nette qui dévie la balle de cricket. Cette déviation est prévisible mais très peu de lanceurs arrivent à lancer suffisamment fort pour entrer dans ce régime, elle est donc difficile à anticiper pour les batteurs.

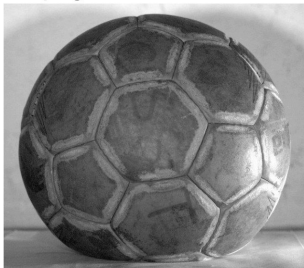
Enfin, pendant mon séjour en Belgique, j’ai commencé à poser les jalons de mon travail futur sur la mécanique des membranes pressurisées en condition d’impact avec une approche expérimentale et théorique. Au niveau expérimental, je me suis principalement appuyé sur les techniques expérimentales du laboratoire : l’imagerie rapide et les techniques de traitement d’image.

Maître de conférence

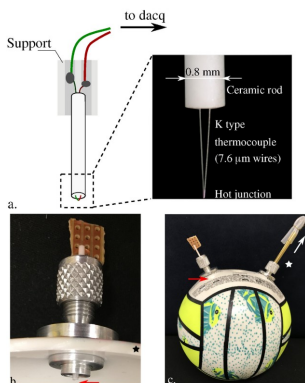
Institut des sciences du mouvement, Aix-Marseille Université.



**Membranes pressurisées:
couplage fluide-solide**



Mesure thermique ultra-rapide



En 2020, j'ai eu l'opportunité d'intégrer le groupe de recherche en conception bioinspirée à l'Institut des Sciences du Mouvement à l'université d'Aix-Marseille. Dans cet environnement nouveau, j'ai travaillé sur 3 axes de recherche différents : (1) Les actionneurs multi-degré de liberté bioinspirés, (2) La biomécanique des feuilles et les membranes pressurisées et (3) les vis-à-rouleaux à planétaire.

Ces trois thèmes sont complémentaires : les actionneurs multi-degrés de liberté se retrouvent sur l'axe 2 matériau cellulaire pilotable / biomécanique des plantes. Enfin l'axe 3 m'a permis d'être en contact avec l'industrie et de pouvoir orienter ma recherche fondamentale vers les applications, notamment pour les actionneurs.

(2) La biomécanique des feuilles et les membranes pressurisées. À mes débuts à l'institut des sciences du mouvement, j'avais prévu de travailler sur les impacts de membranes pressurisées avec en arrière pensée la volonté de comprendre comment ces objets souples interagissent avec l'humain. J'avais en tête deux applications : (1) l'utilisation des objets souples gonflés pour la protection aux chocs (ex : casque de vélo gonflable) et (2) comprendre la réglementation de la pression de gonflage des balles de sport. Nous avons déterminé le temps de contact en fonction de la pression et de la vitesse d'impact avec une transformation adiabatique du gaz. Ce temps de contact est primordial pour la protection aux chocs car il sert d'échelle aux accélérations échangées entre les solides. Le deuxième point que j'ai interrogé est l'aspect dissipatif du choc. Nous avons montré que pour une membrane pressurisée, la dissipation avait lieu par transfert non linéaire vers des modes de vibrations hautes fréquences. La source de non linéarité principale étant le contact. J'ai poursuivi ces travaux sur les membranes pressurisées mais dans le but de comprendre l'assouplissement du tissu des plantes lors de la perte de la turgescence. Les expériences étaient d'ailleurs toujours réalisées sur des balles de sport gonflées que l'on a choisi comme objet de référence. Nous

Expérience automatisée
Node-red

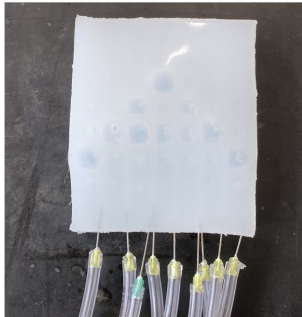


Couplage
fluide-solide-thermodynamique

Rigidification osmotique
du parenchyme de patate



Cell piloted metamaterials



Cell piloted actuator



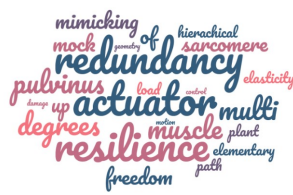
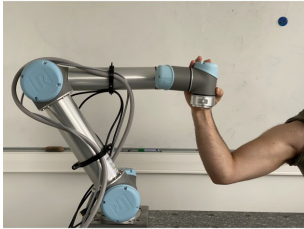
Inspiré de Mimosa Pudica

avons montré que la rigidité cellulaire est non linéaire et dépend fortement de (1) la transformation fluide considérée (isochore, adiabatique, isotherme, isobare) et (2) la géométrie de la cellule et des contacts.

Les études sur les membranes pressurisées m'ont permis de mettre en place des techniques expérimentales et numériques originales :

1. la mesure de l'élévation de température du gaz contenu dans un ballon lors d'un choc a fait appel à la fabrication artisanale de micro-thermocouples ayant des temps de réponse très brefs (inférieurs à la seconde).
2. la résolution par éléments finis homogénéisés de problèmes de solides cellulaires pressurisés.
3. la mise en place d'expériences automatisées de traction de parenchyme de pomme de terre en solution tampon de mannitol. Le banc de traction a une résolution à 10^{-4} N avec une capacité de force maximale de 3 N.
4. la fabrication de solides cellulaires en silicone (ecoflex) pouvant être considérés comme un métamatériau souple lorsque les cellules sont en matrices ou bien comme actionneur lorsque la forme est plus allongée.

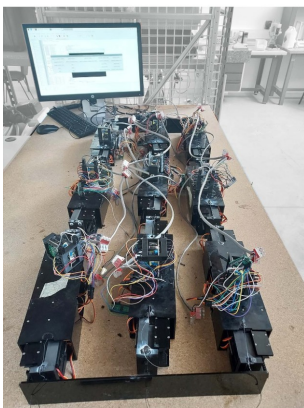
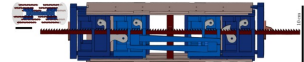
En complément de mon approche expérimentale et numérique, j'essaye toujours de simplifier le problème avec un modèle analytique. Cette approche a échoué sur la modélisation des membranes sphériques entre deux plans – le problème est complexe, fortement non linéaire et je n'ai pas su trouver d'hypothèse simplificatrice suffisante pour mener plus loin ce sujet que ce qui existe déjà dans la littérature. Cependant, quand une solution est possible, elle nous permet de comprendre la physique du phénomène. Par exemple, l'assouplissement des solides cellulaires à cases cubiques lors du gonflement observée numériquement et expérimentalement. Les modèles analytiques s'éloignent souvent plus de la réalité que les modèles numériques mais ont la capacité de mieux faire comprendre / résumer un phénomène physique. Dans ma pratique scientifique, j'essaye autant que possible d'écrire un modèle simple d'un phénomène : cette approche m'aide à comprendre.



Actionneurs aéronautiques
Fatigue d'une vis-à-rouleaux



Maquette de muscle



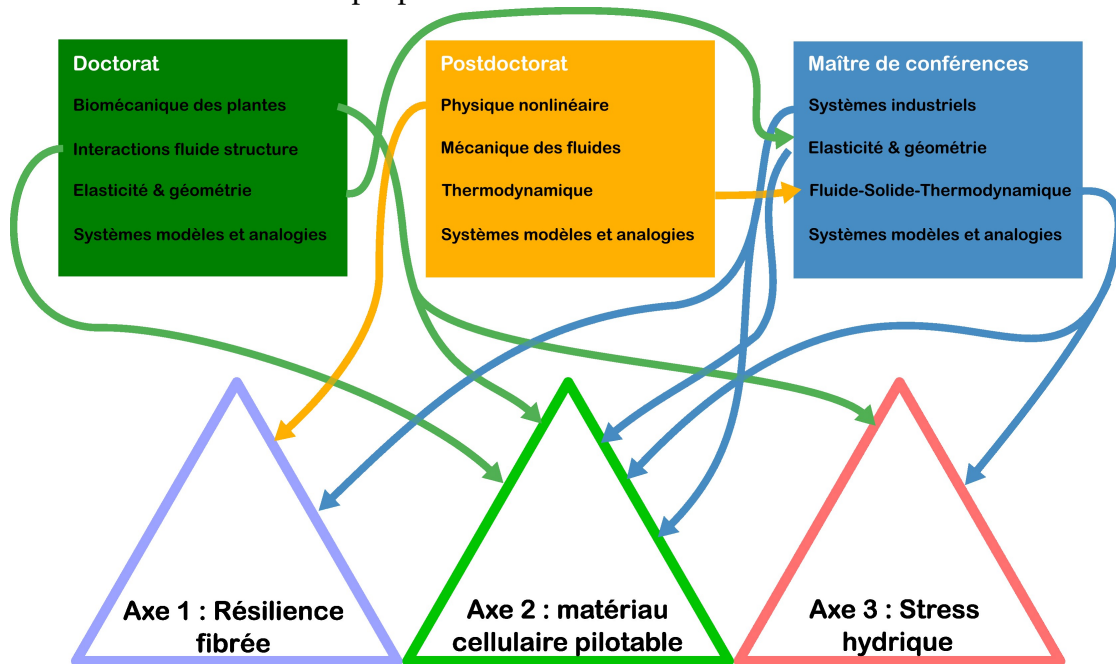
Vers un actionneur résilient

(3) **les vis-à-rouleaux à planétaire.** J'avais toujours fait de la recherche fondamentale assez éloignée des applications et ce travail relevait pour moi du défi. Les vis-à-rouleaux sont un élément critique : le chemin d'effort passe entièrement par cet élément qui ne peut pas être dupliqué. Pour le monter sur une machine volante, les constructeurs aéronautiques doivent prouver qu'une défaillance est hautement improbable (10^{-9} défaillance par heure de vol). Airbus Helicopters souhaitait être capable de maîtriser la conception de la vis à rouleaux en fatigue pour spécifier les dimensions de ses futures commandes. Nous avons mis au point un code numérique pour calculer la durée de vie d'une vis à rouleaux en fonction des propriétés mécaniques des aciers, de la géométrie de la vis et des chargements, éventuellement alternés.

L'industrie aéronautique s'intéresse aux vis à rouleaux à planétaire pour leur faible encombrement et faible masse par rapport aux charges qu'elles peuvent supporter. Dans cette industrie où chaque gramme compte, on ne néglige pas sur la sécurité : tous les actionneurs sont dupliqués voire tripliqués. Cette redondance a un coût massique important, préserver la sécurité sans duplication serait un avantage concurrentiel majeur. Dans la nature la plupart des mouvement sont critiques mais les actionneurs ne sont pas dupliqués : les actionneurs animaux (muscles) ou végétaux peuvent continuer à fonctionner malgré des dégâts. Si l'on parvenait à créer un actionneur industriel pouvant continuer à fonctionner malgré des dégâts, le pilote d'avion ou d'hélicoptère pourrait se poser et préserver l'intégrité de l'appareil et les vies de l'équipage.

(1) **Les actionneurs multi-degré de liberté bioinspirés.** Ce thème était nouveau pour moi mais il émergeait d'une volonté de développement stratégique de l'équipe de recherche. Une spécialité du laboratoire étant le muscle strié squelettique, je me suis lancé sur cet actionneur comme inspiration pour un actionneur résilient. De part mon expérience sur les végétaux, je me suis aussi lancé vers les pulvinus, les moteurs végétaux. Nous avons d'abord essayé de comprendre en quoi ces actionneurs pouvaient amener des concepts intéressant pour l'industrie. Nous avons mis en évidence que leur

structure multi-échelle permettait d'avoir des propriétés intéressantes. Notamment en cas de dégâts, les performances des actionneurs biologiques ne s'effondrent pas mais suivent une décroissance quasi linéaire. Nous avons nommé cet effet résilience aux dégâts. Cette propriété structurelle permet d'envisager de remplacer la duplication des actionneurs générant les mouvements critiques. Nous avons construit une maquette de muscle bio-inspirée qui a confirmé ce concept. Mon arrivée à Aix-Marseille Université m'a mis en contact avec des problèmes industriels capables de nourrir une activité de recherche à part entière. Mon projet de recherche s'appuie sur le développement de systèmes modèles capables de répondre à des questions de science fondamentale et de proposer des innovations industrielles.



Thème du mémoire

Ce mémoire reprend un thème que j’ai exploré lors de mes études d’histoire des sciences : la construction d’une théorie. En 2010/2011, j’avais essayé de comprendre comment les travaux de Léonard de Vinci (1452-1519) sur les tourbillons d’eau et d’air avaient pu influencer l’avènement de la théorie de la turbulence au XIX^{ème} siècle. Notamment, j’avais montré que les dessins de tourbillons de Léonard de Vinci portaient en eux des prémices d’un changement de paradigme¹. Les dessins de Léonard de Vinci sur les tourbillons proposaient (1) un vocabulaire nouveau et précis, (2) une argumentation basée sur l’expérimentation et (3) une idéalisation de la représentation des tourbillons.

Près de 15 ans plus tard, vieilli de quelques aventures scientifiques en biomécanique des plantes, en mécanique des fluides, en physique non-linéaire et en ingénierie bioinspirée, j’essaye dans ce mémoire d’analyser la qualité d’un système modèle. Le système modèle est similaire aux points (2) et (3) de l’approche de Léonard de Vinci : un système modèle mécanique est une idéalisation d’un système complexe qui doit permettre d’apporter des preuves expérimentales pour la compréhension du système original. J’analyse mes expériences vécues pour des systèmes analogues (gouttes marcheuses) et des systèmes modèles mécaniques idéalisant des systèmes biologiques pour essayer de déterminer le champ d’application des systèmes modèles et préciser l’étendue de leurs prédictions.

1. Les qualifier de révolutionnaire au sens de T.S. Kuhn (*de la structure des révolutions scientifiques*) serait trop fort : le paradigme nouveau proposé par Léonard de Vinci était incomplet, le fait d’un ingénieur assez isolé et il n’a pas diffusé au sein de la communauté scientifique à cette période.

Introduction

Le modèle : définitions

Le mot « modèle » est issu du latin *modus* dont le sens premier signifie mesure / outil de mesure. D'un point de vue étymologique, faire un modèle, c'est donc donner une mesure / un outil de mesure. Son acception scientifique est assez moderne avec une utilisation rare avant les années 1950 et beaucoup plus utilisée aujourd'hui, voir figure 1.

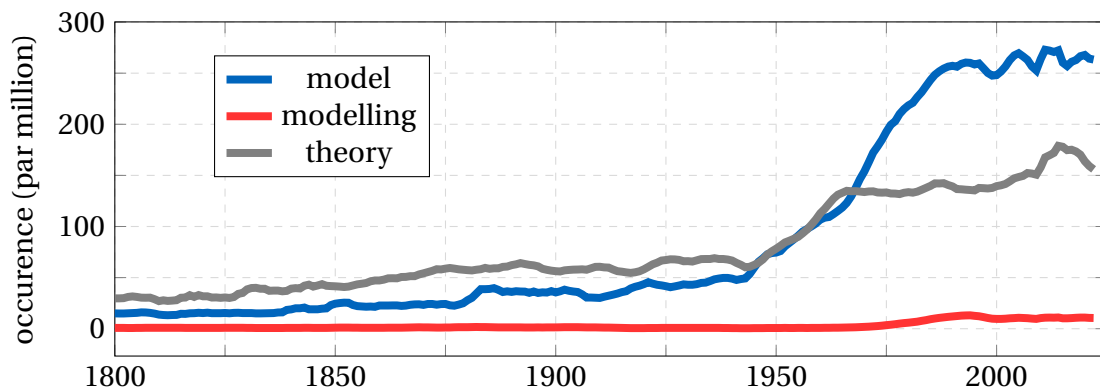


FIGURE 1. – Occurrence du mot modèle dans les production écrites en anglais (Source : google Ngram). L'utilisation du mot *model*, *theory* et *modelling* décolle à partir des années 1950.

En science, le mot modèle a deux significations principales :

1. Le modèle dans le sens de théorie.
2. Le système modèle comme idéalisation du système réel. 4 types de systèmes modèles peuvent être utilisés : (i) Le modèle physique, (ii) Le modèle réduit, (iii) Le modèle numérique, (iv) Le modèle vivant.

Le modèle théorique.

Ce modèle cherche à regrouper des observations dans une théorie unifiée, logique et synthétique. Un modèle théorique en physique se résume à une équation mathématique, par exemple l'équation de Schrödinger pour décrire une particule quantique. Dans d'autres disciplines, un modèle théorique peut être moins mathématisé et peut prendre des formes diverses comme un schéma en biologie, par exemple le cycle de

Krebs, pour expliquer la récupération d'énergie par dégradation moléculaire chez les organismes aérobies. En psychologie, un modèle théorique est un ensemble descriptif et explicatif d'un fonctionnement psychique (exemple : la théorie du développement cognitif de l'enfant par Jean Piaget). Les caractéristiques communes aux modèles théoriques indépendamment de la science considérée sont : (i) un vocabulaire précis, (ii) un contenu logique en accord avec les observations réalisées et les connaissances scientifiques.

Le système modèle.

La démarche scientifique consiste à créer un artefact idéalisant un phénomène physique. Cet artefact doit posséder des éléments représentant le système réel tout en évitant certains aspects difficiles à maîtriser ou à explorer. Cet artefact peut être un code numérique, un modèle physique (parfois appelé maquette physique), un modèle réduit ou bien même un être vivant modèle.

Prenons un exemple pour fixer les idées. Si l'on se pose la question de l'interaction entre le vent et une feuille d'arbre à l'origine de leur mouvement, on pourra étudier le flottement d'une feuille idéalisée en soufflerie : il sera facile de contrôler les dimensions et les rigidités de l'artefact ; de faire varier son orientation vis-à-vis du vent, la vitesse du vent et la turbulence de l'écoulement. Ces paramètres mécaniques sont difficiles à mesurer *in situ* et presque impossibles à maîtriser. De plus la feuille d'arbre est un objet biologique dont les propriétés mécaniques sont variables, entre espèces, entre individus. Même, au sein du feuillage d'un seul arbre, des feuilles peuvent avoir des propriétés mécaniques différentes selon leur exposition au soleil.

Les systèmes modèles peuvent se rencontrer dans des domaines scientifiques variés :

1. **En neurosciences** : le neurone artificiel, implémenté numériquement pour comprendre le rôle de fonction d'activation et de la connectivité synaptique ;
2. **En biomécanique** : le robot escargot permet de comprendre comment les ondes du pied de l'escargot lui permet d'avancer ;
3. **En ingénierie** : le modèle réduit² permet d'investiguer le comportement dynamique d'une structure avant son implantation sur site réel. Souvent utilisé pour les interactions complexes fluide-solide en complément de simulations numériques ;
4. **En biologie** : Avec le choix d'un être vivant comme modèle de référence, par exemple l'arabette des dames (*arabidopsis thaliana*) chez les plantes dont le génome est entièrement séquencé (Meinke et al., 1998).

2. L'étude des modèles réduits, mais aussi des lois allométriques chez les animaux et les végétaux, fait appel au formalisme très puissant de l'analyse dimensionnelle. Le théorème de Vashy-Buckingham stipule que le nombre de paramètres indépendant régissant un problème est le nombre de variable diminué du rang de la matrice aux dimensions. En mécanique classique, il n'y a que 3 dimensions élémentaires : masse M, longueur L et temps T.

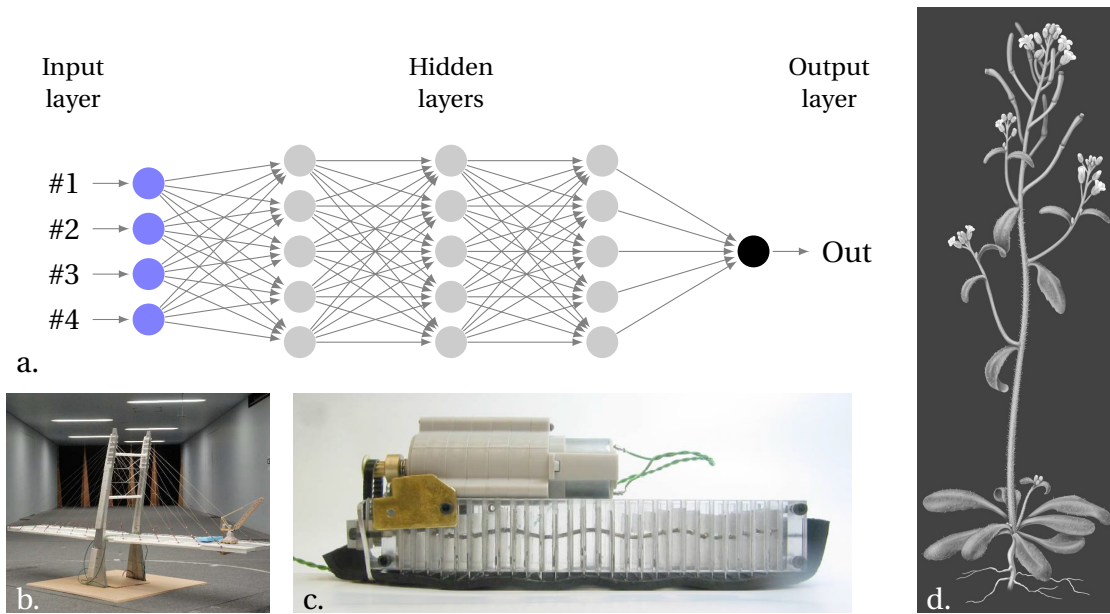


FIGURE 2. – Exemple des 4 grands ensembles de systèmes modèles. **a.** Modèle numérique : un réseau de neurones pour étudier l’intelligence. **b.** Modèle réduit : Pont de Korabelny ©CSTB, (Théodore, 2020). **c.** Modèle physique : un robot escargot pour étudier le rôle des ondes de déformation du pied pour la locomotion (Chan et al., 2005). **d.** Arabidopsis thaliana (Meinke et al., 1998), dessin par K. Sutliff.

Ces types de modèles permettent d’effectuer des expériences répétables en gagnant sur plusieurs tableaux : le coût et le temps des campagnes expérimentales, la répétabilité des résultats (signifiante statistique) avec un travail en laboratoire sur des systèmes bien contrôlés, le choix des échelles de temps et de dimensions, parfois, l’accès à des données extrêmement difficiles à mesurer. Le modèle numérique, par exemple, permet de simuler les déformations matérielles dans le cœur du matériau où l’instrumentation est difficile voire impossible. Ces modèles peuvent être complémentaires. En mécanique par exemple, on accompagne presque toujours un code numérique d’une maquette physique pour le valider.

Ce mémoire se concentre sur le rôle des modèles physiques comme méthode d’investigation d’un phénomène complexe avec un exemple en mécanique quantique et quelques exemples en biomécanique/mécanique bioinspirée.

Avant de poursuivre, il est nécessaire de préciser le rôle du système modèle en biomécanique. La confluence de deux sciences apportant son lot d’opportunités mais aussi d’incompréhension au niveau des questions posées, des moyens d’y répondre et même du vocabulaire³.

3. L’acceptation de l’existence d’une barrière sémantique entre disciplines scientifiques est un point fondamental d’une démarche interdisciplinaire (Abourachid et al., 2023).

La bioinspiration : un eldorado scientifique ou bien le mirage du physicien-mécanicien ?

Ingénierie bio-inspirée

L'ingénierie bio-inspirée est un moyen poursuivant 2 buts distincts :

1. **Biologie** ➡ **Ingénierie**

L'ingénieur peut trouver par l'observation de la nature de nouvelles idées pour répondre à un problème technique posé. Un exemple emblématique est la découverte et la mise au point du système de fixation Velcro (velours-crochets), que l'on appelle communément par l'onomatopée « scratch », par Georges de Mestral (1961). Les fruits de bardane s'accrochaient à ses vêtements et aux poils de son chien. Il observa leurs formes au microscope et trouva que les poils flexibles en forme de crochet de la bardane s'enchevêtraient dans les fibres textiles. Il reproduisit ce phénomène d'adhésion par enchevêtrement avec des fibres en nylon.

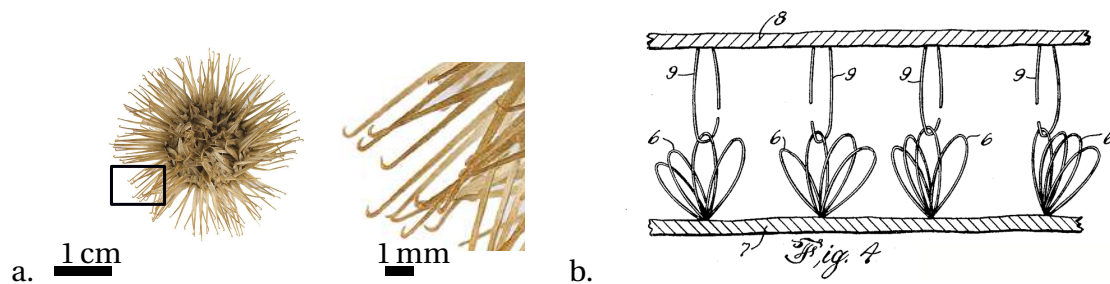


FIGURE 3. – La bioinspiration du Velcro (velours-crochets). Un exemple d'invention d'une solution technique par l'observation de la nature. **a.** Akènes de la bardane (*Arctium lappa*) et grossissement au niveau des bractées (crochets). Crédits : Roger Culos. **b.** Brevet de la fixation crochet et boucles textiles – ou scratch – par de Mestral (1961).

2. **Ingénierie** ➡ **Biologie**

À travers l'étude de systèmes idéalisés, le scientifique peut appréhender les lois gouvernant la Nature.

(i) **Les temps très courts : le système biologique est inerte.** Les systèmes idéalisés permettent de s'affranchir de la variabilité biologique pour déterminer des lois de fonctionnement mécanique. Prenons un exemple végétal : la dissémination vers le haut de pathogènes lors d'une pluie. Les pathogènes (septoriose du blé par exemple) sont présents au sol ou bien sur les feuilles basses du couvert végétal. Lors d'une pluie, les gouttes impactant le bas du couvert végétal contaminé sont ré-éjectées de manière inertielle, en partie vers le haut, et permettent de contaminer les feuilles du dessus. On peut faire cette observation facilement

un jour de pluie dans un champ de blé mais l’investigation directe est presque impossible.

Difficile de travailler directement avec des feuilles de plantes : une feuille sessile de blé est très différente d’une feuille à pétiole de pommier, elle même très différente d’une feuille de trèfle ou d’une palme de dattier. Les variations de taille, de forme et de rigidité dépendent de l’espèce, des conditions de croissance, de la maturité des feuilles, etc. Les feuilles végétales sont aussi des objets très fragiles, si on les coupe ou si elles sèchent, leurs propriétés mécanique peuvent varier sur un ordre de grandeur. Avec un artefact de feuille, le scientifique peut étudier l’éjection de gouttes d’eau par un substrat flexible de manière contrôlée. Il peut mettre en évidence les principaux modes d’éjection de gouttes d’eau et essayer d’en déterminer les lois mécaniques (Gilet & Tadrist, 2025).

(ii) **Les temps longs : Le système biologique est soumis à la sélection naturelle.** On peut s’interroger sur la notion d’optimalité en biologie. La solution mécanique retenue lors de l’évolution est-elle optimale? (Et que veut dire « être optimale »?) Quels sont les facteurs prépondérants sur la sélection d’un trait mécanique? Ici, le biomimétisme permet d’enrichir et de préciser la notion d’optimalité en biologie. Prenons de nouveau un exemple végétal : il existe une très grande variété de tailles de feuilles, de moins d’un millimètre pour les cyprès à près de 26m pour les feuilles de *Raphia Regalis* (Hallé, 2015). Pourquoi une telle diversité? À première vue, plus une feuille est grande, plus elle nécessite de biomatériau pour être maintenue en l’air et orientée pour capter la lumière. Grâce à un modèle d’optimisation de l’allocation de biomasse sous contrainte d’auto-support, on peut montrer que la surface utile pour l’interception lumineuse est indépendante de la taille (Tadrist & Darbois-Textier, 2016). L’auto-support n’est donc pas un critère de sélection de la taille de la feuille. D’autres phénomènes sont plus contraignants sur les tailles de feuille comme les flux de sève (Kaare H. Jensen et al., 2013), les conditions environnementales (Wright et al., 2017) ou encore l’illumination, directe ou diffuse.

L’optimalité en physique, en ingénierie et en biologie

En physique ou en mécanique, l’optimisation est toujours réalisée par rapport à une grandeur unique (l’énergie, le temps, la distance, etc). En mécanique analytique par exemple, la trajectoire unidimensionnelle d’un corps, $x(t)$, relié par un ressort est celle minimisant l’action,

$$S(x) = \int_0^t \left(\frac{1}{2} m \dot{x}^2 - V(x) \right) dt$$

où $V(x) = kx^2/2$ est le potentiel élastique et $m\dot{x}^2/2$ l’énergie cinétique. Cette minimisation est réalisée dans le cadre mathématique de la dérivée de Fréchet : on considère

une perturbation infinitésimale $\delta(t)$,

$$\begin{aligned} S(x(t) + \delta(t)) &= \int_0^t \left(\frac{1}{2} m(\dot{x}(t) + \dot{\delta}(t))^2 - \frac{1}{2} kx(t)^2 + \delta(t)^2 \right) dt \\ &= S(x(t)) + \int_0^t (m\dot{x}(t)\dot{\delta}(t) - kx(t)\delta(t)) dt + o(\delta(t)^2) \end{aligned}$$

On effectue une intégration par partie sur le premier membre de l'intégrale,

$$S(x(t) + \delta(t)) - S(x(t)) = \left[m\dot{x}(t)\delta(t) \right]_0^t + \int_0^t (-m\ddot{x}(t) - kx(t))\delta(t) dt + o(\delta(t)^2)$$

L'action est minimale lorsque $S(x(t) + \delta(t)) - S(x(t)) = 0$ quelle que soit la fonction perturbatrice infinitésimale $\delta(t)$. Le terme intégré correspond à des conditions aux limites. Le terme intégral par contre, est nul pour n'importe quelle fonction infinitésimale $\delta(t)$ si et seulement si le terme en bleu est nul. On trouve ainsi l'équation classique d'un oscillateur masse-ressort,

$$m\ddot{x}(t) + kx(t) = 0$$

Le mot optimisation en physique/mécanique a un sens bien défini et mathématisé.

En ingénierie, la notion d'optimalité devient multifactorielle. Considérons, par exemple, la conception d'un réservoir de pression léger : comment choisir le bon matériau ? L'ingénieur fait face à un compromis à déterminer.

Le problème physique est simple : le réservoir de pression doit supporter une pression P pour un rayon R . L'épaisseur e du réservoir s'obtient par la formule du chaudronnier,

$$e = s \frac{PR}{2\sigma_e}$$

où σ_e est la limite élastique du matériau et s un coefficient de sécurité. La masse M s'écrit

$$M = 4\pi\rho R^2 e = 2\pi s PR^3 \frac{\rho}{\sigma_e}$$

où ρ est la masse volumique du matériau.

La méthode de sélection des matériaux d'Ashby et al. (2018), permet d'optimiser rapidement le choix d'un matériau de manière graphique en plaçant sur un diagramme loglog les propriétés matériaux (en bleu dans les expressions de la masse ci-dessus). En considérant une base de donnée de matériaux, on trace σ_e en fonction de ρ , figure 4. Les iso-lignes de pente 1 déterminent les matériaux ayant les mêmes indices de performance. Le meilleur matériau est celui placé sur l'iso-ligne de pente 1 la plus haute, ici les composites et céramiques techniques.

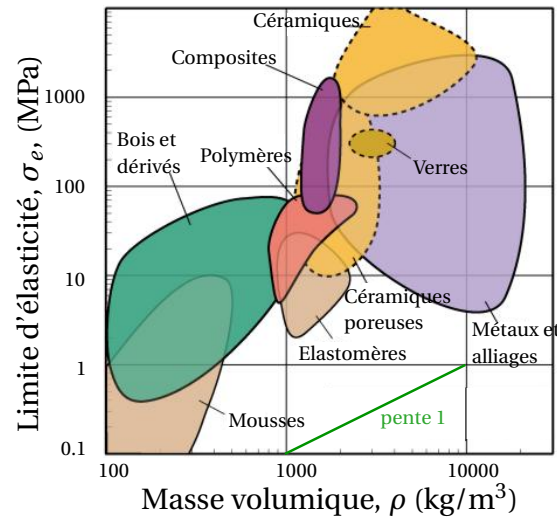


FIGURE 4. – Diagramme de sélection de matériaux. Représentation logarithmique de la limite d'élasticité σ_e en fonction de la masse volumique ρ (adapté de <http://www-materials.eng.cam.ac.uk>).

Maintenant, si l'on souhaite un réservoir à la fois léger et peu coûteux, l'optimisation devient multi-factorielle. On doit considérer le coût du réservoir,

$$C = cM = 2\pi sPR^3 \frac{\rho c}{\sigma_e}$$

où c est le prix massique du matériau. Si l'optimisation porte sur le prix du réservoir uniquement, on pourrait utiliser la technique précédente mais avec de nouveaux axes sur le diagramme des matériaux. Comment optimiser à la fois le prix et la masse du matériaux? Quels sont les poids relatifs des deux critères? Souhaite-t-on un matériau peu cher quitte à sacrifier un peu la légèreté? Dès qu'il y a deux propriétés à optimiser, l'ingénieur n'échappe pas à la construction d'une fonction coût, plus ou moins compliquée, avec des poids à déterminer. Dans notre exemple, il construira une fonction coût $F = \alpha M + \beta C$ dont il choisira les facteurs α et β en fonction des contraintes physiques, financières, réglementaires, environnementales, etc.

La notion d'optimalité en ingénierie est moins mathématisée qu'en physique/mécanique : la détermination des coefficients présents dans la fonction coût fait l'art du bon ingénieur.

En biologie évolutionnaire, l'optimalité est le fruit de la théorie de l'évolution de Charles Darwin (1859, p488),

And as natural selection works solely by and for the good of each being, all corporeal and mental endowments will tend to progress towards perfection.⁴

4. « Et comme la sélection naturelle marche seulement par et pour le bien de chaque être vivant,

Le processus de sélection naturelle est donc un algorithme d'optimisation (algorithme qui d'ailleurs a été copié par les informaticiens pour créer les optimisations évolutionnaires – lentement convergentes mais robustes).

L'écueil de l'ingénieur non-averti pourrait être le suivant : la sélection naturelle est en marche depuis des millions d'années, donc la nature est optimale - il suffit de copier! C'est parfois le cas mais une prise de recul s'impose. Premièrement, le processus évolutif est fait de l'accumulation de changements aléatoires infinitésimaux. Certains changements aléatoires ne sont pas critiques pour la survie de l'être vivant, ils peuvent être conservés sans pour autant être optimaux : la couleur des cheveux en est un exemple. Deuxièmement, le caractère adaptatif se fait vis-à-vis de conditions environnementales fluctuantes : ce qui était adapté à une période peut ne plus l'être mais avoir été conservé. Par exemple, la pilo-érection (chaire de poule) était une réaction pour gonfler le pelage humain et renforcer son caractère isolant pour lutter contre le froid. Aujourd'hui le pelage humain n'est plus assez dense pour remplir cette fonction, mais le mécanisme est conservé. Cette idée que certains traits aléatoires peuvent être conservés sans influencer les capacités de survie de l'animal – un physicien/mécanicien dirait que c'est un *optimum plat* par rapport à cette variable – est déjà présente chez Darwin (1859, p118),

When a variation is of the slightest use to a being, we cannot tell how much of it to attribute to the accumulative action of natural selection, and how much to the conditions of life.⁵

Enfin, pour les animaux sexués, la sélection naturelle s'accompagne de la sélection sexuelle, qui est souvent loin d'être optimale pour la survie. Prenons l'exemple de la roue du paon utilisée pour la parade nuptiale, plus elle est grande plus l'individu est désirable pour les femelles. Cependant plus la queue du paon est grande, plus elle est encombrante pour fuir en cas de danger.

La notion d'optimalité en biologie est beaucoup plus complexe à appréhender qu'en ingénierie et encore plus qu'en physique/mécanique : l'optimalité est réalisée envers plusieurs paramètres simultanément (parfois plusieurs dizaines), sans que l'on connaisse *a priori* le poids relatif de chacun de ces paramètres. De plus, l'être vivant évoluant dans un environnement fluctuant, la fonction coût optimisée par la sélection naturelle change au cours du temps.

La richesse de la bio-inspiration

Ces remarques sur les définitions disciplinaires de l'optimalité ne doivent pas empêcher le physicien/mécanicien de porter son intérêt sur des observations biologiques. Au contraire, être au fait de ce que veut dire être optimal pour un biologiste permet d'enrichir les questions auxquelles les modèles des physiciens/mécaniciens peuvent répondre. Cela permet également de comprendre que l'optimalité du biologiste est

tous les héritages physiques et comportementaux tendront à progresser vers la perfection. »

5. « Lorsque qu'une variation est de la moindre utilité pour un être vivant, nous ne pouvons pas dire combien provient de l'action cumulée de la sélection naturelle et combien des conditions de vie. »

éloignée de l’optimalité du physicien/mécanicien et qu’une réponse simultanée dans les deux disciplines n’est pas possible.

La bio-inspiration doit donc être vue comme un outil pour se poser des questions plutôt qu’un moyen pour trouver des réponses toutes prêtes. (i) Parfois les êtres vivants ont été confrontés à une problématique similaire à celle des êtres humains et l’observation de la nature permet d’avoir un *point de départ* pour les investigations scientifiques. *Point de départ*, car une copie à l’identique est souvent impossible : les procédés de fabrication, les matériaux, les dimensions, presque tout change. Entre l’observation des crochets des fruits de bardane et l’invention du Velcro, plusieurs années de recherche et de développement se sont écoulées. (ii) Le physicien/mécanicien peut essayer de préciser l’expression de la fonction coût que la sélection naturelle optimise : lequel des processus physiques ou biologiques est prépondérant ? Ou au contraire non-influent ? De cette manière notre connaissance de la biologie évolutionnaire s’affine et peut générer de nouvelles applications.

Choix des chapitres présentés

Le premier chapitre relate mes activités de recherche en mécanique des fluides sur les systèmes de gouttes marcheuses. Il couvre mes années de recherche de post-doctorat. Ce chapitre n’est pas directement lié à la mécanique bio-inspirée que je porte actuellement sur les systèmes modèles de muscle et de pulvinus. Cependant, la démarche de la recherche, l’utilisation d’un système modèle pour tester des hypothèses sur des systèmes plus complexes/moins accessibles, est identique à celles qui sont développées actuellement sur les maquettes de muscle et de pulvinus. Les outils utilisés, notamment en mécanique-physique non-linéaire sont utiles pour l’analyse des modèles bioinspirés que je développe.

Le second chapitre concerne la biomécanique des feuilles et des membranes pressurisées. Ce chapitre détaille les conséquences mécaniques de l’interaction entre un fluide interne et une membrane flexible externe. Les applications vont du rebond d’un ballon, aux chocs à la tête en passant par la mécanique des tissus de plantes turgescentes.

Le troisième chapitre recueille les maquettes d’actionneurs bio-inspirés que nous avons développées et leurs principaux résultats.

Enfin, je développe dans le quatrième chapitre mon projet de recherche à moyen terme et les perspectives scientifiques plus lointaines.

1. Un système modèle fluide pour la mécanique quantique ?

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Ce premier exemple de système modèle appartient à la classe des modèles physiques. Nous allons voir comment il nous permet d'explorer des limites inaccessibles à l'expérience directe. Ici les limites sont à la fois dues au paradigme quantique (onde de probabilité, non mesurable sauf en perturbant le milieu) et à l'absence de système de mesure¹.

1.1. Une dualité onde corpuscule à l'échelle macroscopique

La mécanique quantique : une description probabiliste du Monde. L'avènement de la mécanique quantique au cours du XX^{ème} siècle a bouleversé la vision du monde microscopique. À cette échelle, les particules ne sont plus représentées par des points matériels mais par une onde spatiale de probabilité Ψ . Cette description ondulatoire de la lumière et de la matière rend bien compte de l'expérience de diffraction par des fentes doubles, d'abord découverte par Young (1802) pour la lumière puis plus récemment pour des particules matérielles (Bach et al., 2013). Dans cette expérience des franges de diffraction apparaissent après le passage à travers deux fentes. Ces

1. Certaines expériences de mesures faibles on néanmoins été menées avec succès mesurant position et impulsion simultanément mais avec incertitude (le principe d'incertitude étant une limite du paradigme).

franges d'interférence correspondent à une variation spatiale de l'intensité (nombre de photons, d'électrons ou d'atomes ayant frappé le capteur localement), figure 1.1.

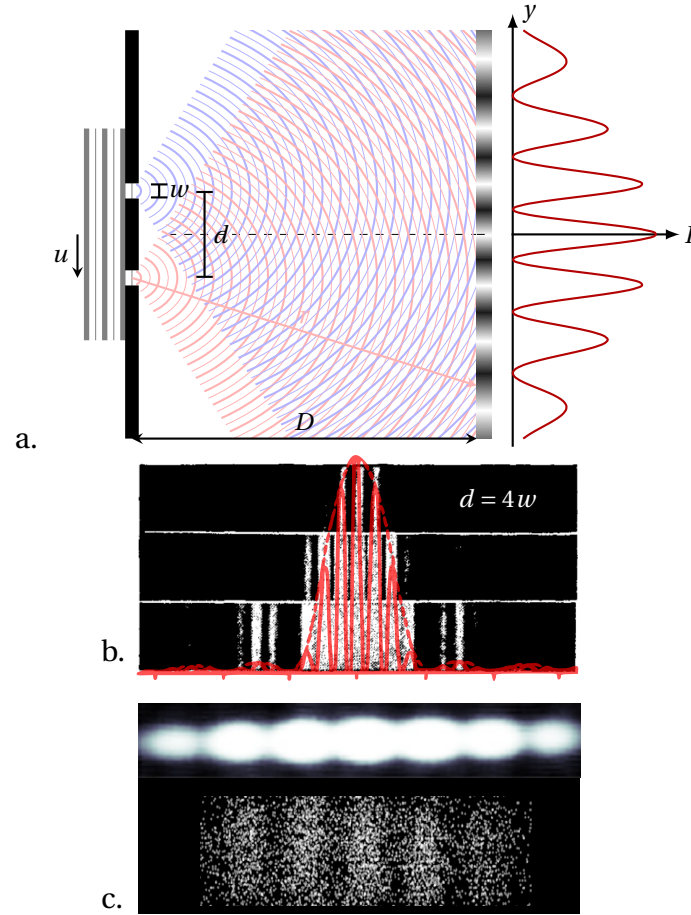


FIGURE 1.1. – **a.** Expérience de la diffraction d'onde à deux fentes. Une onde plane frappe un mur dans lequel sont percées deux fentes de largeur w et d'écartement d . Un écran est placé à une distance D et permet de visualiser la distribution spatiale des particules détectées. Des domaines réguliers d'intensité variable apparaissent : ce sont les franges d'interférence. **b.** Franges d'interférences de lumière. (adapté de Jenkins et al. (1957) avec $d = 4w$) **c.** Haut : Franges d'interférence d'électrons. Bas : Détection électron par électron avec 6235 électrons détectés (Bach et al. (2013) avec $d = 4.22w$)

La diffraction par deux fentes révèle les bizarreries quantiques. La modélisation de cette expérience est identique pour les photons ou les particules matérielles : une onde électromagnétique (une onde de probabilité pour les particules matérielles), occupant tout l'espace, traverse les deux fentes en même temps avant d'interagir avec elle-même. L'onde initiale, cohérente, est diffractée lors du passage par une fente. On suppose les deux fentes assez allongées pour considérer un problème 2D et l'analyse

ne considère qu'une seule tranche dans la direction z . Au niveau des fentes 1 et 2, l'onde lumineuse infinitésimale s'écrit $dh(t) = ae^{i\omega t} du$ avec a l'amplitude de l'onde, ω la pulsation de l'onde et u l'ordonnée orthogonale à la propagation de l'onde au niveau des fentes (voir figure 1.1a). Après le passage dans une fente de largeur w , l'onde est diffractée,²

$$dh(r, t) = dh(t) \frac{e^{-ikr}}{r} = \frac{a}{r} e^{i(\omega t - kr)} du$$

avec k le nombre d'onde. Au niveau du détecteur en (D, y) , le chemin parcouru est $r = \sqrt{D^2 + (y - u)^2}$. Si l'on se place dans les conditions où $D \gg y \gg w, d, u$ (diffraction de Fraunhofer) le chemin s'approxime par $r = D(1 + y^2/2D^2 - yu/D^2)$ ³.

$$dh(D, y, u, t) = dh(t) \frac{e^{-ik(D + y^2/2D - yu/D)}}{D} = \frac{a}{D} e^{i[\omega t - k(D + y^2/2D)]} e^{iky u/D} du$$

L'onde totale est la somme des ondes infinitésimales diffractées par les deux fentes (ondes bleue et rouge sur la figure 1.1a),

$$\begin{aligned} h(D, y, t) &= \int_{-w/2-d/2}^{w/2-d/2} dh(D, y, u, t) + \int_{-w/2+d/2}^{w/2+d/2} dh(D, y, u, t) \\ &= \frac{a}{D} e^{-i\omega t} e^{ik(D + y^2/2D)} \left(e^{iky d/2D} + e^{-iky d/2D} \right) \int_{-w/2}^{w/2} e^{iky u/D} du \\ &= 2 \frac{aw}{D} e^{-i\omega t} e^{ik(D + y^2/2D)} \cos\left(\frac{kdy}{2D}\right) \text{sinc}\left(\frac{kwy}{2D}\right) \end{aligned}$$

L'intensité perçue au niveau de l'écran est le module carré de l'onde,

$$I(D, y) = |h(D, y, t)|^2 = 4 \frac{a^2 w^2}{D^2} \cos^2\left(\frac{kdy}{2D}\right) \text{sinc}^2\left(\frac{kwy}{2D}\right)$$

Ce résultat classique des livres de cours d'optique, montre en quelques lignes la spécificité de la mécanique quantique. Si les particules ne passaient que par une seule fente, le résultat de la diffraction devrait être la somme des passages par chaque fente. Ce n'est pas le cas expérimentalement où l'on observe des interférences constructives et destructives localisant la détection de particules. Ce résultat prouve le caractère non local de la mécanique quantique, où l'onde passe par les deux fentes en même temps. Pour résumer cette idée en une ligne de calcul :

$$|h_1 + h_2|^2 \neq |h_1|^2 + |h_2|^2.$$

2. On considère que l'onde est réémise sphériquement dans tout l'espace. Le facteur $1/r$ permet de considérer que l'intensité lumineuse $dI = \int_{\theta, \phi} |dh|^2 r^2 \sin(\theta) d\theta d\phi = 4\pi a^2$ de cette source ponctuelle est constante lorsque l'on l'intègre sur une sphère de rayon r .

3. Dans la suite, le facteur de phase de l'onde étant multiplié par k , très grand pour une onde lumineuse, on conservera cette approximation pour la phase uniquement. On remplacera directement r par D dans le facteur d'amplitude.

La double solution, une interprétation non orthodoxe de la mécanique quantique.

La modélisation d'une particule de matière par une onde de probabilité reste pour le physicien réaliste difficile à accepter. D'ailleurs, lors des fondements de la mécanique quantique, lorsque Louis de Broglie postula la nature ondulatoire de la matière (de Broglie, 1925), il se gardait de définir la nature de l'onde :

J'ai intentionnellement laissé assez vagues les définitions de l'onde de phase et du phénomène périodique dont elle serait en quelque sorte la traduction ainsi que celle du quantum de lumière. La présente théorie doit donc plutôt être considérée comme une forme dont le contenu physique n'est pas entièrement précisé que comme une doctrine homogène définitivement constituée.

Dès 1924, il commencera à développer une théorie réaliste de la mécanique ondulatoire, la théorie de la double solution (de Broglie, 1927),

Ma conception se rapproche de celle qui a été brillamment soutenue par M. Born en ceci qu'elle conduit à considérer les solutions continues comme donnant des probabilités de présence, mais elle en diffère sur un point essentiel. Pour M. Born, en effet, il n'y a que des probabilités; le déterminisme des phénomènes individuels devrait être abandonné, la probabilité des phénomènes statistiques étant seule déterminée. Dans la manière de voir adoptée ici, au contraire, le point matériel est une réalité essentielle et son mouvement est entièrement déterminé comme étant celui d'une singularité de l'amplitude dans une onde qui se propage.

Cette vision d'un corpuscule surfant sur la phase de l'onde de probabilité formera l'interprétation de la mécanique quantique de Bohm-Vigier (Bohm et al., 1954).

Les gouttes marcheuses, l'espoir de réhabiliter la vision réaliste du Monde. En 2005, Yves Couder propose à ses étudiants de reproduire une expérience de lévitation d'une goutte sur un bain vibré. En jouant sur les paramètres, un étudiant remarque un déplacement spontané de la goutte. Les gouttes marcheuses sont nées (Couder; Protiere et al., 2005).

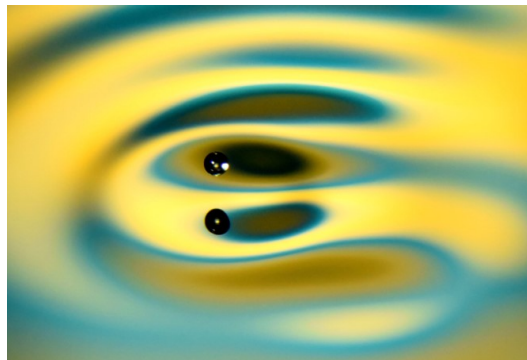


FIGURE 1.2. – Goutte marcheuse en lumière rasante. La goutte est entourée d'un chapelet d'ondes de Faraday.

Un marcheur est constitué d'une goutte d'huile silicone déposée sur un bain vibré

verticalement. Lorsque l'on augmente l'amplitude de la vibration, un chapelet d'ondes concentriques apparaît autour de la goutte, voir figure 1.2. Si l'on se rapproche encore du seuil de Faraday, le déphasage entre le rebond de la goutte et l'onde de Faraday est tel que la goutte rebondit sur une crête. Ce mouvement est instable et la goutte démarre spontanément dans une direction aléatoire. Après ce démarrage, la goutte progresse en ligne droite si elle n'est pas déviée par un obstacle ou un courant d'air.

Ayant posé le contexte anti-réaliste de l'interprétation quantique orthodoxe – une réalité difficile à accepter pour le physicien réaliste –, la découverte d'un objet macroscopique guidé par une onde pilote a relancé l'espoir de pouvoir tester la théorie de la double solution de de Broglie/Bohm, le système modèle étant extraordinairement proche de la vision originelle de Louis de Broglie (1926),

En résumé, le point matériel nous apparaît maintenant comme un système d'ondes stationnaires entourant un point source central.

Après de rapides succès en faveur d'une analogie profonde entre une particule quantique et un marcheur – dont le principal est la quantification des orbites d'un marcheur dans un puits de potentiel (Eddi et al., 2012; Harris & Bush, 2014; Oza et al., 2014; Labousse et al., 2016) – l'expérience de diffraction à deux fentes a modéré l'optimisme. Initialement, Couder & Fort (2006) ont mesuré un pattern de diffraction compatible avec la diffraction ondulatoire. L'expérience consistait à faire passer un marcheur à travers deux fentes physiques créées par des obstacles submergés⁴, voir figure 1.3. Les marcheurs sont lancés perpendiculairement aux fentes de manière analogue à l'expérience de diffraction à deux fentes.

4. Si les obstacles ne sont pas submergés, les ménisques formés par la ligne triple irradient le bain d'ondes capillaires parasites.

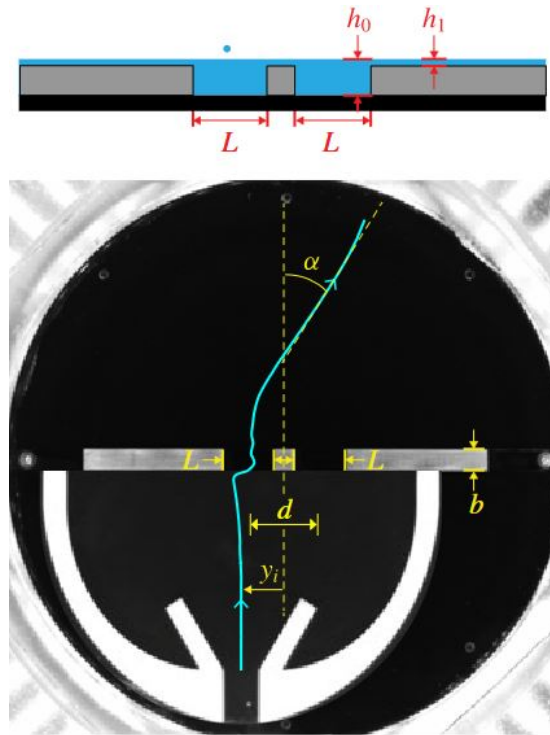


FIGURE 1.3. – Dispositif expérimental de la diffraction à deux fentes pour les marcheurs, d'après Pucci et al. (2018).

Rapidement un groupe de chercheurs de l'université de Copenhague, mené par Thomas Bohr, a critiqué ce résultat sur 3 points (Andersen et al., 2015) :

1. Le nombre de gouttes analysées est trop faible pour conclure à une différence entre un pattern de diffraction et une gaussienne.
2. Le motif de diffraction a été symétrisé.
3. Ils n'ont pas réussi à reproduire ce résultat sur leur montage expérimental.

En fait, il est clairement établi que la connaissance expérimentale du moment était trop faible pour réaliser correctement une expérience aussi sensible. Les trajectoires des gouttes sont courbées, aussi bien sur l'expérience de Couder que celle de Copenhague, mettant en évidence l'influence des bords de la cavité ou bien de courants d'air parasites. La température du bain et les tailles de gouttes sont aussi des sources de variabilité qui ne sont pas prises en compte dans ces deux expériences.

Cette expérience a été reprise avec beaucoup de soin par Guiseppe Pucci et al. (2018) dans le groupe de John Bush en limitant les courants d'air par un couvercle et en utilisant une même goutte plusieurs fois. Les résultats bruts montrent que le motif d'interférence n'est pas le motif des particules quantiques. Néanmoins, le profil de d'interférence n'est pas celui des particules classiques non plus, les bords de la fente influençant la trajectoire. On peut même observer des trajectoires influencées par la deuxième fente. Cette influence n'est pas due au passage d'onde par la seconde fente mais à un effet postérieur d'interaction avec les parois. Sans franges d'interférences,

impossible de convaincre la communauté scientifique que les marcheurs peuvent être un analogue hydrodynamique à la mécanique quantique.

Pourquoi n'observe-t-on pas de franges d'interférence pour des marcheurs ? La mécanique quantique offre des franges d'interférence car, les objets sont non-locaux, l'onde s'étale dans tout l'espace. Une connaissance plus approfondie des propriétés ondulatoires et du pilotage de la goutte est nécessaire pour comprendre si une diffraction est possible ou pas.

Une vue idéalisée des conditions d'obtention d'un champ d'ondes interférées pour un marcheur est présenté figure 1.4. Le marcheur incident progresse initialement en ligne droite avec autour de lui un champ d'onde étendu spatialement (figure 1.4a). Quand le marcheur est suffisamment proche de la barrière submergée, les ondes capillaires sont stoppées et ne peuvent passer qu'au dessus des deux fentes. Les ondes passant par les deux fentes sont diffractées par les bords des fentes et construisent un champ d'ondes interférées dans la partie bleue, figure 1.4b. Lorsque le marcheur est dans la zone d'interférences, il est dévié de sa trajectoire rectiligne : il est diffracté.

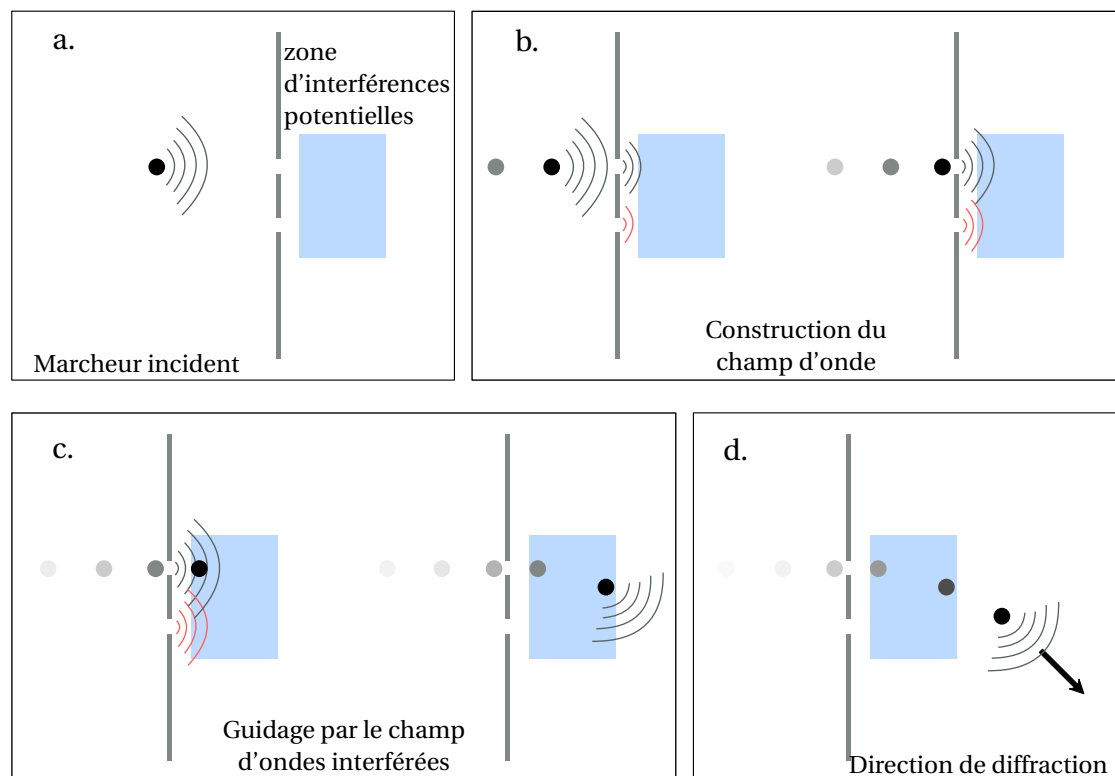


FIGURE 1.4. – Schéma *hypothétique* du passage d'un marcheur par deux fentes. Les ondes rouges correspondent aux ondes de Faraday excitées par le passage de l'onde capillaire dans la seconde fente. La possibilité d'observer des interférences dépend de la présence des ondes de la seconde fente (rouge).

Cette hypothèse n'a jamais été réalisée expérimentalement mais elle permet de gui-

der nos travaux de recherche. Notons les principales conditions pour la construction d'un champ d'ondes interférées :

1. Il faut que les ondes ayant traversé la barrière ne soient pas trop amorties.
2. Il faut que l'extension spatiale du champ d'onde soit suffisante pour passer à travers les deux fentes.

L'absence d'observation de diffraction pour des marcheurs, jusqu'à ce jour, tient-elle au choix des conditions expérimentales ?

La première partie de ce chapitre est consacrée à la description de l'extension spatio-temporelle du champ d'onde. Les conditions minimales pour observer des vraies interférences sont discutées. La seconde partie de ce chapitre est consacrée à une configuration expérimentale propice à l'observation d'une diffraction : la diffraction de Kapitza-Dirac.

1.2. L'interférence à deux fentes est-elle possible pour les marcheurs ?

Les marcheurs sont accompagnés d'un champ d'onde spatialement et temporellement étendu. Ils peuvent interagir avec des objets à distance à travers leur champ d'onde (Harris ; Brun et al., 2018). Pour déterminer la possibilité d'observer des interférences, nous avons quantifié le champ d'onde d'un marcheur à travers une solution visqueuse des équations de Navier Stokes d'un bain vibré (Tadrist ; Shim et al., 2018).

1.2.1. Généralité sur les marcheurs

Le bain d'huile support des gouttes marcheuses est vibré verticalement. Cette vibration verticale entraine une modification de la gravité, $g(t) = g [1 + \Gamma \cos(\Omega t)]$ où Ω est la pulsation de la vibration. Lorsque l'amplitude de la modification de gravité Γ est suffisante, des ondes stationnaires apparaissent à la surface : ce sont les ondes de Faraday (figure 1.5a). Lorsque le seuil de Faraday est franchi, les ondes de Faraday inondent toute la cavité et leur position est prescrite par les bords.

Pour un fluide newtonien, le seuil d'apparition des ondes, le seuil de Faraday, est largement dépendant de la viscosité ν du fluide (Tadrist ; Shim et al., 2018),

$$\Gamma_F = \frac{4\nu k_0 \Omega}{g} - \frac{\Omega^2}{2g k_0} \left(\frac{4\nu k_0^2}{\Omega} \right)^{3/2} + O \left(\left(\frac{4\nu k_0^2}{\Omega} \right)^{5/2} \right) \quad (1.1)$$

avec k_0 la solution de la relation de dispersion sans viscosité, $g k_0 + \sigma k_0^3 / \rho = \Omega^2 / 4$.

Symbole	Paramètre
ν	viscosité cinématique
γ	tension de surface
ρ	Masse volumique
g	Accélération de la gravité
k_0	Nombre d'onde fluide non visqueux
k_F	Nombre d'onde de Faraday
Γ	Amplitude de l'accélération de forçage
Γ_F	Accélération au seuil de Faraday
$\mathcal{M}$	Distance au seuil de Faraday
M_e	Mémoire (en périodes de forçage)
D	Coefficient de diffusion des ondes de Faraday
J_0	Fonction de Bessel de première espèce

TABLEAU 1.1. – Table des paramètres pour discuter du modèle onde-corpuscule des gouttes marcheuses.

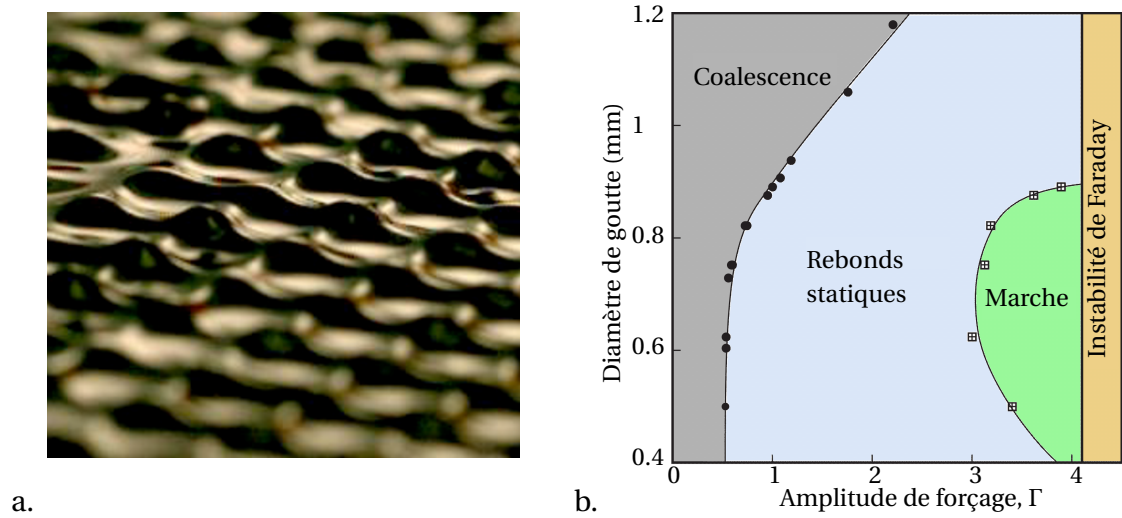


FIGURE 1.5. – a. Ondes stationnaires de Faraday. Ces ondes stationnaires apparaissent spontanément lorsque le seuil est franchi. Pour les marcheurs les plus étudiés, avec une huile silicone à 20 Cst et une fréquence d'excitation à 80 Hz, les ondes de Faraday apparaissent pour $\Gamma = 4,155$. b. Diagramme de phase simplifié des comportement de goutte sur bain vibré en fonction du diamètre de goutte et de l'amplitude de la vibration verticale Γ , d'après Antonin Eddi et al. (2011). Des diagrammes bien plus complets existent aujourd'hui.

Pour obtenir un marcheur viable, le seuil de Faraday n'est pas franchi, voir figure 1.5b. Les ondes capillaires sont amorties inversement à la distance au seuil : plus on est proche du seuil de Faraday, plus le temps de vie des ondes de Faraday est important.

La distance au seuil s'écrit

$$\mathcal{M} = \frac{\Gamma_F}{\Gamma_F - \Gamma},$$

et le temps de vie des ondes capillaires générées par un impact sur le bain est proportionnel à cette quantité. Plus précisément, on caractérise le champ d'onde d'un marcheur par sa mémoire M_e , qui correspond au nombre de périodes de Faraday, $2\pi/\Omega$, qu'il faut pour oublier la présence d'un marcheur à ce point,

$$M_e = \frac{\mathcal{M}\Omega}{8\pi\nu k_0^2} \left(1 + \left(\frac{\nu k_0^2}{\Omega} \right)^{1/2} + \left(\frac{\nu k_0^2}{\Omega} \right) + O\left(\left(\frac{4\nu k_0^2}{\Omega} \right)^{3/2} \right) \right)$$

1.2.2. Ondes consécutives à un impact unique

La caractérisation d'un impact unique montre qu'il est la superposition d'une onde capillaire propagative et d'un champ d'ondes de Faraday amorties, voir figure 1.6. La théorie visqueuse, permet de décrire efficacement le champ d'onde spatio-temporel (r, t) d'un impact unique (Tadrist; Shim et al., 2018),

$$\zeta_1(r, t) \propto \sqrt{\frac{2\pi}{D\Omega(t-t_i)}} J_0(k_F r) \cos\left(\Omega t + \frac{\pi}{4}\right) \times \exp\left[\frac{t-t_i}{M_e T_F} - \frac{r^2}{2D(t-t_i)}\right] \quad (1.2)$$

où $T_F = 2\pi/\Omega$ est la période de Faraday, $k_F = 2\pi/\lambda_F$ est le nombre d'onde de Faraday, t_i le temps à l'impact, J_0 la fonction de Bessel de première espèce d'ordre 0 et D est le coefficient de diffusion des ondes de Faraday,

$$D = \frac{(g + 3\sigma k_0^2/\rho)^2}{2\nu k_0^2 \Omega^3} \left(1 + \sqrt{\frac{\nu k_0^2}{\Omega}} + O\left(\frac{4\nu k_0^2}{\Omega} \right) \right)$$

Les mesures de champ d'onde sont effectuées par corrélation d'image avec un schlieren synthétique (Moisy et al., 2009). La mesure brute correspond au gradient du champ d'onde, qui est comparée qualitativement au gradient radial du champ d'onde $\partial_r \zeta_1(r, t)$. La comparaison qualitative entre les mesure directes, figure 1.6a, et la théorie, figure 1.6b, montre un bon accord. Les différences sont la présence d'une onde capillaire propagative de fort gradient (diagonale verte) et le fait qu'avant le passage de cette onde, le fluide n'est pas informé de l'impact (gradient nul dans le coin en haut à droite, figure 1.6a).

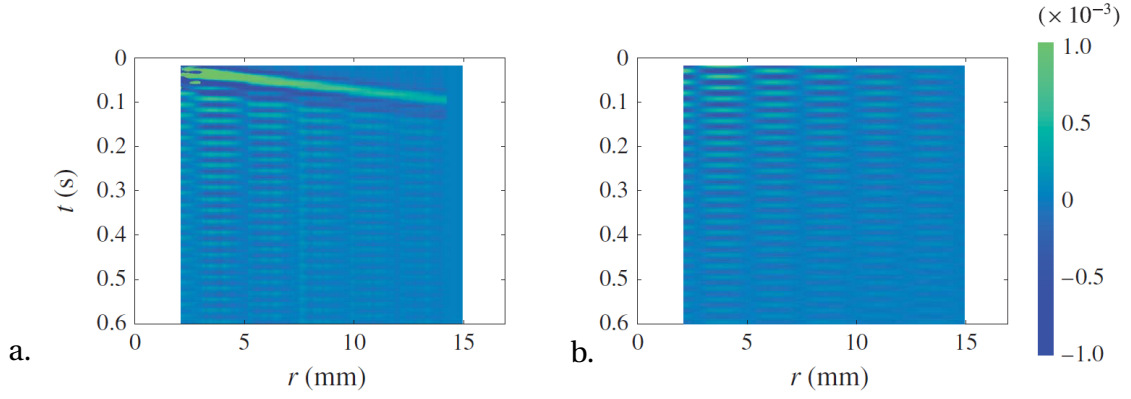


FIGURE 1.6. – Evolution spatio-temporelle du gradient radial du champ d'ondes evanescentes généré par un seul impact (Tadrist; Shim et al., 2018). a. Mesures du gradient du champ d'onde via la technique de reconstruction de F. Moisy (Moisy et al., 2009). b. Gradient du champ d'onde prédit par la théorie visqueuse. L'onde capillaire propagative n'est pas prise en compte (barre oblique verte). Avant le passage de cette onde, le fluide n'est pas informé de l'impact (coin en haut à droite des mesures expérimentales).

Les comparaisons quantitatives entre le gradient de champ d'onde déterminé par la théorie visqueuse et les mesures, figure 1.7 mettent notamment en évidence un déphasage de $+\pi/4$ entre l'onde de Faraday et la vibration de forçage, principalement du aux effets visqueux. Un second apport de la théorie est le terme diffusif dans l'exponentielle en $-r^2/2D(t - t_i)$.

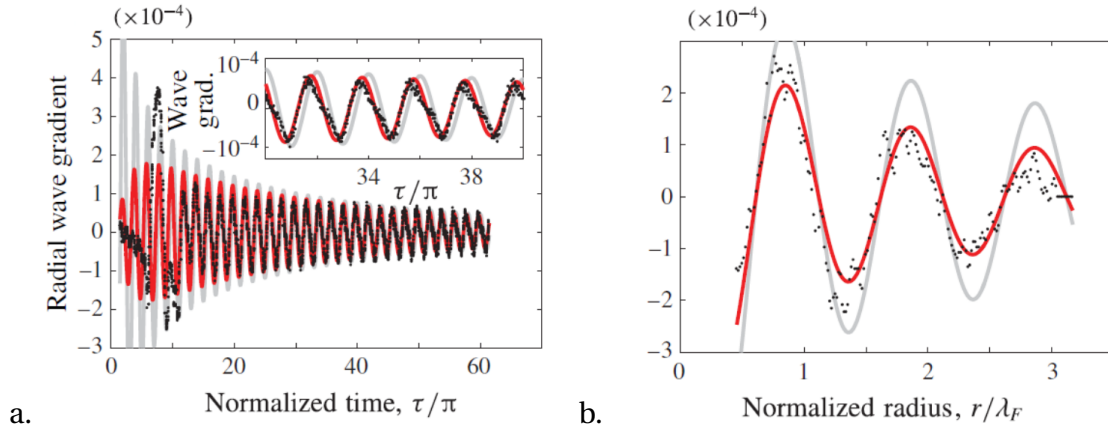


FIGURE 1.7. – Comparaison quantitative entre l'onde evanescente de Faraday consecutive à un impact. Noir : mesures directes. Rouge : théorie visqueuse (Tadrist; Shim et al., 2018). Gris : théorie à basse viscosité. a. Mise en évidence du déphasage temporel ($+\pi/4$) b. Amplitude du gradient de champ d'onde à temps fixé ($t=0,233$ s).

Enfin, la définition de la mémoire M_e , l'amortissement temporel des ondes de

Faraday, a été vérifiée expérimentalement par deux groupes indépendants (Tadrist; Shim et al., 2018; Ellegaard et al., 2024).

1.2.3. Le champ d'onde des marcheurs

Le champ d'onde d'un marcheur est simplement la somme des ondes générées par chaque impact,

$$\zeta_w(r, t) = \sum_n \zeta_1(r - r_n, t - t_n).$$

En l'absence d'obstacle et pour des mémoires pas trop élevées, on peut déterminer une solution approchée de la forme du champ d'onde d'un marcheur,

$$\zeta_w(r, t) \propto J_0 \left(k_F \left| r + \frac{r}{\sqrt{v^2 + 2 \frac{4(g+3\sigma k_0^2/\rho)^2}{\omega^4 \mathcal{M}}}} v \right| \right) e^{-r/l(\theta)}$$

Cette forme permet de définir une longueur caractéristique $l(\theta)$ de l'extension spatiale du marcheur : c'est la taille typique de sa non-localité. L'angle polaire θ traduit le fait que le champ d'onde n'est pas étendu de la même manière dans toutes les directions ; il est beaucoup plus court devant le marcheur et plus étendu derrière lui par exemple. Le marcheur n'interagit plus avec des obstacles ou des ondes situées au delà de quelques $l(\theta)$ ⁵,

$$l(\theta) = \frac{4(g + 3\sigma k_0^2/\rho)^2}{8\omega^3 v k_0^2 \left(v \cos \theta + \sqrt{v^2 + 2 \frac{4(g+3\sigma k_0^2/\rho)^2}{\omega^4 \mathcal{M}}} \right)}$$

D'un point de vue expérimental, cette extension caractéristique de la non-localité des marcheurs est fortement liée la viscosité du bain à la surface duquel ils évoluent : $l(\theta)$ est inversement proportionnelle à la viscosité (ν). Lorsque la viscosité tend vers 0, la non-localité des marcheurs devient infinie.

Cette limite n'est pas possible expérimentalement avec les marcheurs « historiques » à 20 cSt mais des observations de marcheurs à faible viscosité ont été réalisées par Protière (2007) avec de l'huile de viscosité 5 cSt. A cette viscosité, les gouttes marcheuses sont petites et très sensibles aux courants d'air. Une configuration expérimentale intéressante serait de générer des gouttes à 20 cSt et de les faire interagir avec un bain de faible viscosité.

On remarque aussi qu'en faisant diverger le paramètre de distance au seuil, $\mathcal{M}$, l'extension spatiale du marcheur ne diverge pas.

5. Dans le papier original (Tadrist; Shim et al., 2018), la distance caractéristique $l(\theta)$ est erronée. Le coefficient 2 à l'intérieur de la racine carrée est manquant. Je remercie Mogens Levinsen pour avoir pointé cette erreur.

Le groupe de Copenhague, a montré que cette distance caractéristique de non-localité des marcheurs ne tient pas bien pour des mémoires élevées ($M_e \gg 1$). Ils ont préféré mettre en évidence la hauteur d'onde à 3% du maximum comme longueur caractéristique de non-localité, $l_{3\%}$. Ils obtiennent alors des dimensions caractéristiques de non-localité plus importantes que celles prédites par la présente théorie. Après discussion avec Mogens Levinsen, qui a réalisé des simulations numériques sur des gouttes allant jusqu'à 1 cSt, l'extension spatiale est toujours loin d'être infinie.

Cette non-localité limitée des marcheurs a des conséquences sur la possibilité d'observer des interférences discutées dans le paragraphe suivant.

1.2.4. Des conditions expérimentales pour observer des interférences

Pour espérer observer une diffraction de marcheur, le champ d'onde postérieur aux fentes doit être composé des ondes ayant traversé les deux fentes. Cette situation est schématisée figure 1.4. Pour des marcheurs, cette situation n'est pas facile à réaliser expérimentalement à cause de :

1. l'extension spatiale finie des marcheurs,
2. de la mémoire finie du système,
3. des dimensions finies de la zone expérimentale,
4. et des conditions de propagation de l'onde capillaire à travers des obstacles submergés, notamment au niveau des fentes.

L'extension spatiale des marcheurs est quantifiée par la distance d'amortissement des ondes d'un marcheur libre $l(\theta)$. Grossièrement, il faut que les ondes ayant traversé la seconde fente lorsque le marcheur traverse la première fente ne soient pas trop amorties. Si l'écartement des fentes est supérieur à $l(\theta)$, il y a peu de chances de pouvoir observer des ondes ayant traversé les deux fentes. Une première condition nécessaire pour observer des interférences de marcheur est donc : $l(0) \gg d$ où d est la distance entre le centre de chaque fente.

La mémoire du système est quantifiée par l'amortissement temporel du système $M_e T_F$. Grossièrement, il faut que les ondes ayant été générées en amont des fentes n'aient pas été trop temporellement amorties lorsque le marcheur pénètre dans la zone d'interférences. Cette condition signifie que le temps de traversée des fentes ($\simeq d/v$) doit être petit devant l'amortissement temporel du système. Une seconde condition nécessaire pour observer des interférences de marcheur est donc : $M_e T_F \gg d/v$.

Les effets de bord sont générés lorsque la dimension de la zone d'intérêt, R est proche de la dimension de l'extension spatiale des marcheurs. La troisième condition nécessaire est : $l(0) \ll R$ (ou bien $l_{3\%} \ll R$).

Les conditions de propagation de l'onde capillaire lorsqu'il y a des obstacles est moins bien connue. Notamment, si la barrière est trop épaisse ou bien les fentes trop étroites, on peut augmenter la dissipation au moment du passage des fentes de manière remarquable. Les observations de champ d'onde de marcheurs traversant les

deux fentes montrent que les ondes traversant la seconde fente sont très amorties et irradient très faiblement, voire pas du tout, l'espace postérieur aux fentes (Rode et al., 2019).

Avec les contraintes expérimentales du système modèle, on ne peut actuellement pas explorer les limites intéressantes pour la comparaison quantique. Pour imaginer une diffraction en s'affranchissant des limites énoncées ci-dessus, une configuration plus propice est l'interaction directe du marcheur avec une onde stationnaire.

1.3. Une configuration expérimentale plus confortable : la diffraction de Kapitza-Dirac

Les résultats ci-dessous ont été collectés lors de mes derniers mois de chargé de recherche du FNRS au micro-fluidics lab (2020), dirigé par Tristan Gilet (Université de Liège, Belgique). Récemment, l'équipe américaine de recherche sur les marcheurs a publié des résultats expérimentaux similaires sur l'interaction des marcheurs avec une onde stationnaire (Primkulov et al., 2025).

La diffraction de Kapitza-Dirac se conçoit simplement : on envoie un flux de marcheurs sur une onde stationnaire perpendiculaire à la trajectoire de départ du marcheur. Cette configuration a pour bénéfice de ne pas mettre en jeu des obstacles immergés et donc d'éviter l'inconnu de la réflexion des ondes de pression et de l'amortissement lors du passage dans les fentes. Cette configuration expérimentale n'est pas facile à produire : il est difficile de générer une onde stationnaire d'amplitude donnée avec le système actuel.

Le bénéfice principal de cette expérience est que l'on peut pour la première fois comparer la diffraction d'une particule quantique et celle d'un marcheur. En effet la diffraction de Kapitza-Dirac prévoit que la probabilité de présence entre le pic central et les pics latéraux change avec l'amplitude de l'onde stationnaire. L'analogie va-t-elle tenir ?

A contre-pied des marcheurs classiques, nous avons choisi de travailler avec un bain d'huile moins profond ayant un seuil de Faraday plus élevé. L'onde stationnaire est générée par une tranchée où le bain d'huile est plus profond et où le seuil de Faraday est franchi. Le schéma expérimental est présenté figure 1.8. Une des difficultés expérimentales a été d'asservir l'amplitude de vibration sur l'amplitude de l'onde transversale. Spécifiquement, il a fallu mettre en place un dispositif de mesure temps réel de l'amplitude de l'onde transversal par corrélation d'image (technique utilisant un schlieren synthétique, dite technique de Moisy).

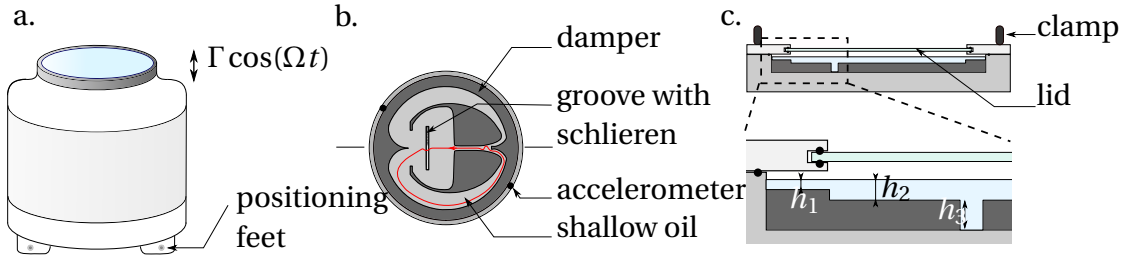


FIGURE 1.8. – Montage expérimental de l'expérience analogue à Kapitza-Dirac pour des gouttes marcheuses. Le bain d'huile est placé sur le dessus de pot vibrant et nivelé horizontalement grâce à des pieds de positionnement ($10 \mu\text{m m}^{-1}$). b. Vue de dessus du bain d'huile avec une trajectoire de gouttelette typique (rouge). Les zones peu profondes sont indiquées en gris foncé, la zone expérimentale en gris clair et la rainure en blanc. Un Schlieren synthétique est placé dans la rainure pour la mesure de la hauteur des vagues. c. Vue latérale du bain d'huile avec $h_1 = (1,16 \pm 0,02)$ mm, $h_2 = 2,16$ mm et $h_3 = 5$ mm.

Les trajectoires brutes des marcheurs montrent que l'interaction avec l'onde stationnaire dévie les marcheurs de leur mouvement rectiligne, figure 1.9. Plus précisément, les gouttes sont déviées selon 3 directions préférentielles : une trajectoire rectiligne et une avec un angle $+\theta$ et une symétrique avec un angle $-\theta$.

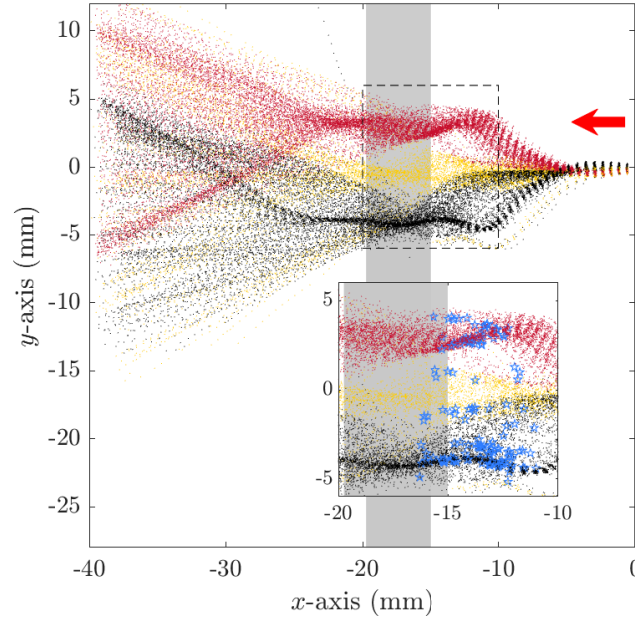


FIGURE 1.9. – Trajectoires des gouttes marcheuses traversant une onde stationnaire longitudinale.

L'expérience de Kapitza dirac quantique peut donner un résultat similaire selon

l'amplitude de l'onde stationnaire transversale, voir figure 1.10a. Cependant une des propriétés essentielles de l'expérience quantique est que selon l'amplitude de l'onde transversale, le nombre de pics et leur intensité relative varient, figure figure 1.10b et c.

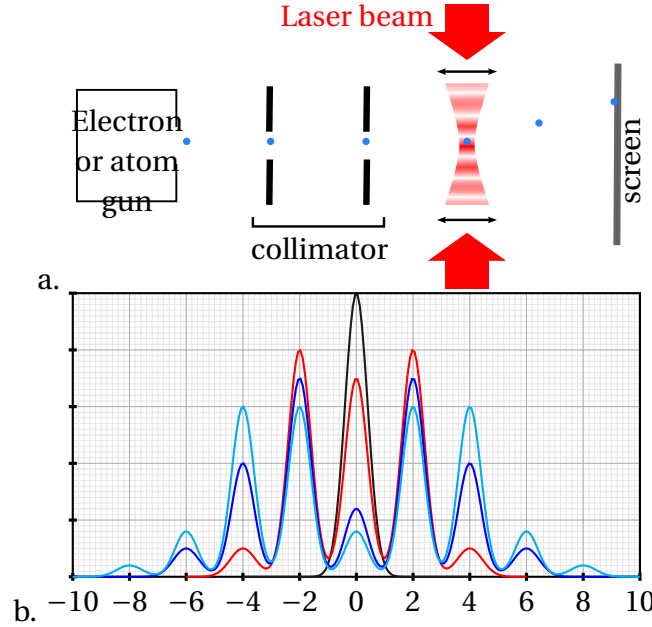


FIGURE 1.10. – a. Dispositif expérimental de diffraction de Kapitza-Dirac. b. Figures de diffraction de Kapitza-Dirac d'après Gould et al. (1986). Les couleurs indique l'intensité de l'interaction. Noir : $Vt/\hbar=0$; Rouge : $Vt/\hbar=1.69$; Bleu : $Vt/\hbar=2.33$; Cyan : $Vt/\hbar=2.84$.

L'asservissement de l'amplitude de l'onde stationnaire est compliqué à réaliser car nous devons travailler au dessus du seuil d'une instabilité hydrodynamique. Les mécanismes non-linéaires limitant l'amplitude de l'onde n'étant pas bien maîtrisés et offrant empiriquement une gamme d'amplitude de forçage limitée, $\Gamma \in [\Gamma_F, 1.001\Gamma_F]$. De plus des variations infimes de température peuvent changer la viscosité de l'huile du bain, décaler le seuil de Faraday et induire des comportements chaotiques (Tadrist; Gilet et al., 2020). Néanmoins, il a été possible de mesurer l'amplitude de l'onde avant le passage d'une goutte marcheuse et d'en déduire des figures de diffraction en fonction de l'amplitude de l'onde longitudinale, voir figure 1.11.

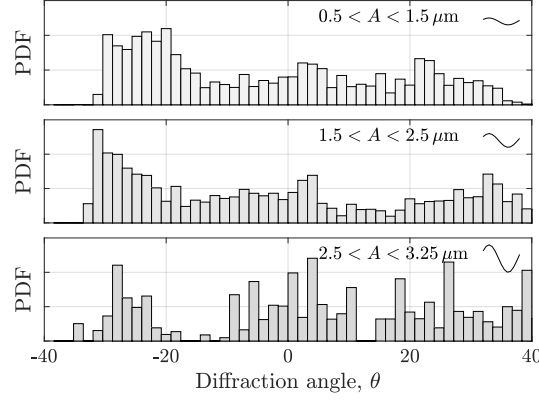


FIGURE 1.11. – Figures de diffraction des gouttes marcheuses traversant une onde stationnaire longitudinale en fonction de l’amplitude de l’onde. Haut : $0.25 < A < 1,25 \mu\text{m}$. Centre : $1.25 < A < 2,25 \mu\text{m}$. Bas : $2.25 < A < 3,25 \mu\text{m}$

Les résultats montrent que l’on retrouve toujours une distribution à 3 pics. La position des pics peut varier ainsi que leur intensité relative selon les configurations. Ces figures de diffraction pourraient être assez proches de celles de l’expérience quantique avec un paramètre d’interaction $Vt/\hbar \simeq 1.7$. Cependant, l’accès à des figures de diffraction en fonction de la longueur d’onde nous permet de comparer quantitativement à l’expérience quantique.

Ne connaissant pas l’analogue de la constante $\hbar$ pour les marcheurs, nous avons essayé d’ajuster la prédiction quantique du rapport d’amplitude du pic central par rapport aux pics latéraux sur les mesures avec les gouttes marcheuses (Tadrist & Gilet, 2026). La prédiction quantique est tracée en ligne pleine et les données expérimentales en pointillés sur la figure 1.12. La comparaison est sans appel, la tendance des probabilités de déviation dans le pic central vs pics latéraux est très différente.

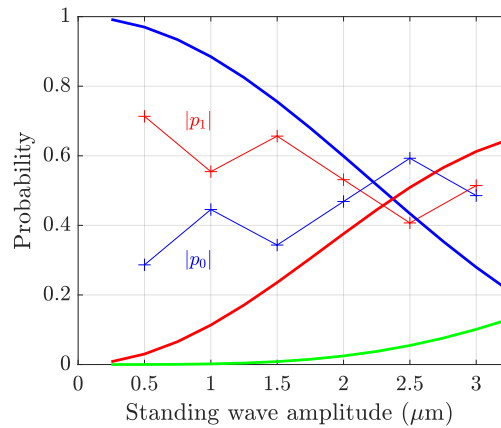


FIGURE 1.12. – Comparaison qualitative de la déviation des gouttes marcheuses / des particules quantiques dans les deux premiers pics de diffraction. Trait plein : prédiction quantique. Pointillés : mesure sur les marcheurs.

Cette expérience montre une limite à l'analogie entre mécanique quantique et gouttes marcheuses. La dualité onde particule de marcheurs offre une figure de diffraction quantifiée avec 3 pics mais différente de celles obtenues dans une configuration similaire avec une particule quantique. Cette expérience montre que le mode d'interaction entre la particule et l'onde est différent dans les deux systèmes.

Les résultats rapportés par Primkulov et al. (2025) avec des ondes beaucoup plus importantes que celles étudiées ici (240 μm) montre des figures de diffraction à 4 pics que je n'ai jamais observées.

1.4. Limites du marcheur comme modèle à la mécanique quantique

Les marcheurs sont différents des particules quantiques principalement par leur mode de pilotage et leur extension spatiale non-infinie. Même en réalisant des conditions expérimentales favorables comme stipulées au paragraphe précédent, il est peu probable que l'on puisse obtenir des motifs de diffraction identiques à ceux des photons ou des atomes. Ces deux différences essentielles sont l'amplitude locale des ondes et le pilotage de la particule par l'onde.

Le pilotage

Notons que la vision orthodoxe de la mécanique quantique n'envisage pas qu'une particule soit pilotée par l'onde de probabilité. La seule interprétation de la mécanique quantique qui envisage l'existence de la particule quantique tout au long de son trajet est la théorie de l'onde pilote de de Broglie-Bohm. Cette théorie ne s'écarte pas des résultats de la mécanique quantique : l'onde de probabilité $\Psi(r, t) = A(r, t)e^{i\phi(r, t)/\hbar}$ est existante et conforme à l'équation de Shrödinger. Ici, pour une particule libre, sans potentiel extérieur,

$$i\hbar \frac{\partial \Psi(t, \vec{r})}{\partial t} = -\frac{\hbar^2}{2m} \Delta \Psi(t, \vec{r}).$$

Cette onde de probabilité guide la particule. On suppose que la particule est déposée aléatoirement sur l'onde puis est guidée sur celle-ci :

$$v = -\frac{1}{m} \nabla \phi$$

La particule surfe sur *la phase de l'onde de probabilité* sans agir sur cette onde. Dans le cas des marcheurs, il y a une rétro-action du marcheur sur l'onde et de l'onde sur le marcheur,

$$\zeta_w(r, t) = \sum_n \zeta_1(r - r_n, t - t_n) \quad \text{et} \quad v = \alpha \nabla \zeta_w$$

Ce couplage n'existe pas dans la théorie de l'onde pilote. De plus dans la théorie de l'onde pilote, la particule surfe sur la phase de l'onde et pas sur son amplitude au contraire des marcheurs qui surfent sur l'amplitude de l'onde capillaire.

Amplitude des ondes

L'amplitude des ondes est la seconde source de différence entre le marcheur et les particules quantiques. Les particules quantiques étant parfaitement non-locales (extension infinie du champ d'onde), leur guidage a lieu partout : même si l'amplitude de l'onde est extrêmement faible, il peut y avoir un guidage efficace. Au contraire des marcheurs dont le champ d'onde est plus important au niveau du marcheur et l'effet rétro-actif nécessaire pour le guidage.

La différence de la forme du pilotage et de la rétroaction avec l'onde, entre les marcheurs et les particules quantiques, ne permet pas aujourd'hui d'envisager une analogie convaincante entre les deux systèmes. Néanmoins, il est possible que des améliorations du système expérimental puissent permettre d'observer de la diffraction de marcheurs compte tenu des conditions nécessaires énoncées ci-dessus.

1.5. Qu'est-ce qu'une bonne analogie en physique ?

Les physiciens aiment les analogies. Elles permettent (i) de pouvoir appliquer des résultats connus dans un autre domaine de la physique et (ii) de façonner son intuition en réfléchissant sur un objet plus familier.

L'analogie en physique fonctionne bien lorsque les équations de base décrivant un système sont identiques ou très similaires. On peut alors identifier deux à deux les variables, par exemple en hydraulique et en électricité (ex : pression/tension), ou expliquer des traits biologiques les équations chimiques de réaction/diffusion pour les motifs des pelages de léopard.

Rechercher une analogie basée sur la ressemblance de deux systèmes est très risqué, surtout lorsque les équations de base sont différentes. Terminons sur une note positive : Les marcheurs ne semblent pas se comporter comme les particules quantiques mais leur étude nous a permis de mieux comprendre la dualité onde-corpuscule et permis d'insister sur les spécificités de la mécanique quantique.

2. Mécanique vers Biologie. Quelques modèles physiques pour comprendre les fonctions, développement et évolution des feuilles d'arbre.

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Ce second chapitre expose comment les outils d'ingénieurs et notamment les artefacts physiques représentant des objets biologiques permettent de comprendre l'effet des contraintes physiques sur leur fonctionnement, leur développement ou leur évolution. Cet exposé traite des feuilles d'arbre, mon principal sujet d'intérêt biologique jusqu'à présent. La première partie de ce chapitre présente les feuilles d'arbres et les questions biologiques qui s'y rapportent. Dans les 3 sections suivantes, j'expose quelques résultats que j'ai obtenu avec une approche biomécanique et avec des systèmes idéalisés. La dernière partie de ce chapitre, plus générale, est une tentative pour montrer la puissance pédagogique et de persuasion des systèmes modèles en biomécanique des plantes.

2.1. Une feuille n'est pas un panneau solaire

Les feuilles sont des objets communs : hormis dans les déserts, elles habillent tous les végétaux. Les feuilles sont des objets bidimensionnels où l'une des dimensions est très petite devant les deux autres. Parfois, pour les conifères à aiguilles par exemple, les feuilles sont unidimensionnelles (deux dimensions sont très petites devant la troisième).

Cette caractéristique géométrique des feuilles est partagée par pratiquement tout le règne végétal (hormis quelques cactus). Cependant, les autres caractéristiques géométriques des feuilles sont peu partagées : les feuilles varient en taille sur 4 décades, de 2 mm pour le cyprès (*Cupressus Sempervirens*), jusqu'à 25 m pour le grand raphia (*Raphia Regalis*). Les plus grandes feuilles à limbe continu sont celles de la rhubarbe géante (*Gunnera manicata*) avec des feuilles atteignant les 2 m de diamètre. De même, les formes de feuilles sont très différentes avec des feuilles simples (un limbe idéalisé par un plan et un pétiole flexible. e.g. Cerisier, abricotier, peuplier, érable, etc.), les feuilles pennées (une tige centrale avec de nombreuses folioles latérales, e.g. Acacia, fougères, etc.), les feuilles palmées (des folioles partant toutes d'un même point central, e.g. Marronnier) et les feuilles sessiles (un limbe attaché directement à la tige, sans pétiole e.g. Blé, poireau, etc.), figure 2.1.



FIGURE 2.1. – Différentes formes de feuilles observées dans la nature. a. feuille simple
b. feuilles composées (Gauche : feuille pennée, Droite : feuille palmée)
c. feuille sessile.

2.1.1. Fonction des feuilles

La fonction principale des feuilles est de collecter l'énergie lumineuse environnante, la transformer en énergie chimique à travers le procédé de photosynthèse et d'exporter ces produits vers les autres parties de la plante à travers le réseau phloémien. La réaction de photosynthèse s'écrit,



Cette réaction chimique nécessite de l'eau qui est amenée jusqu'aux feuilles par le réseau xylémien. Le xylème et le phloème constituent le réseau vasculaire de la plante. La physique des flux à l'intérieur de chaque réseau est différente. Le xylème transportant l'eau parfois sous pression négative (théorie de la cohésion-tension) à partir

d'une dépression d'origine capillaire au niveau des stomates¹ des feuilles (K. Jensen et al., 2022). Au contraire, les flux de sève élaborée dans le phloème sont déclenchés par des différences de pression osmotique : les concentrations en sucres sont plus importantes au niveau des feuilles où ils sont produits que dans les autres parties de la plante où ils sont puisés. Pour contrôler cet écoulement d'origine osmotique, un peu d'eau est injectée par voie symplastique tout le long du phloème. Ce mécanisme a été initialement proposé par Ernst Münch (Munch, 1926; Münch, 1930) et démontré expérimentalement avec un système modèle puis *in vivo* (Kaare H Jensen et al., 2011; Kåre Hartvig Jensen et al., 2012).

2.1.2. Quelques facteurs de stress biotiques et abiotiques

Les feuilles sont aussi soumises à des facteurs de stress environnementaux (abiotiques) comme les hautes températures ou le vent. Ces facteurs de stress influent sur les caractéristiques mécaniques des feuilles.

Le vent applique un effort aérodynamique sur le feuillage des arbres pouvant endommager les feuilles, les arracher de l'arbre ou bien même casser les branches. Les feuilles sont flexibles, ce qui leur permet de ployer sous le vent et de réduire l'effort aérodynamique ressenti, voir figure 2.2. Ce mécanisme s'appelle la reconfiguration² (Alben et al., 2004; Gosselin et al., 2010; de Langre et al., 2012). La traînée d'un objet flexible évolue proportionnellement à $U^{2+\alpha}$ où α est appelé coefficient de Vogel. Le coefficient de Vogel vaut $-2/3$ pour une poutre élastique et est généralement proche de -1 avec une légère dépendance sur la géométrie considérée. La flexibilité offre une protection qui est d'autant plus efficace que les vents sont puissants.

1. Les stomates sont des petites ouvertures dans la cuticule des feuilles qui peuvent s'ouvrir ou se fermer sur demande. Les stomates sont constituées de deux cellules de garde, chacune en forme de boudin cylindrique. Lorsque ces cellules sont dégonflées, les boudins sont jointifs et la stomate est fermée. Au contraire lorsque les boudins sont gonflés, ils se tendent et se courbent ouvrant ainsi la stomate aux échanges gazeux.

2. Considérons une poutre de section au vent hw et de rigidité en flexion EI , la force de traînée s'écrit $F = \frac{\rho U^2 hw}{2}$ où ρ est la masse volumique du fluide. Pour une poutre flexible la dimension h varie. On peut déterminer la loi de variation de h directement en équilibrant le moment de la force aérodynamique avec le moment de retour élastique : $EI\mathcal{C} = Fh$. En considérant la courbure de la poutre élastique, $\mathcal{C} \simeq 1/h$, on obtient $EI/h \simeq \rho U^2 h^2 w \rightarrow h \propto (EI/\rho U^2)^{1/3}$. La force de traînée n'est plus proportionnelle à U^2 comme pour un objet rigide mais à $U^{2-2/3}$. Le coefficient de Vogel d'une poutre élastique est $-2/3$.

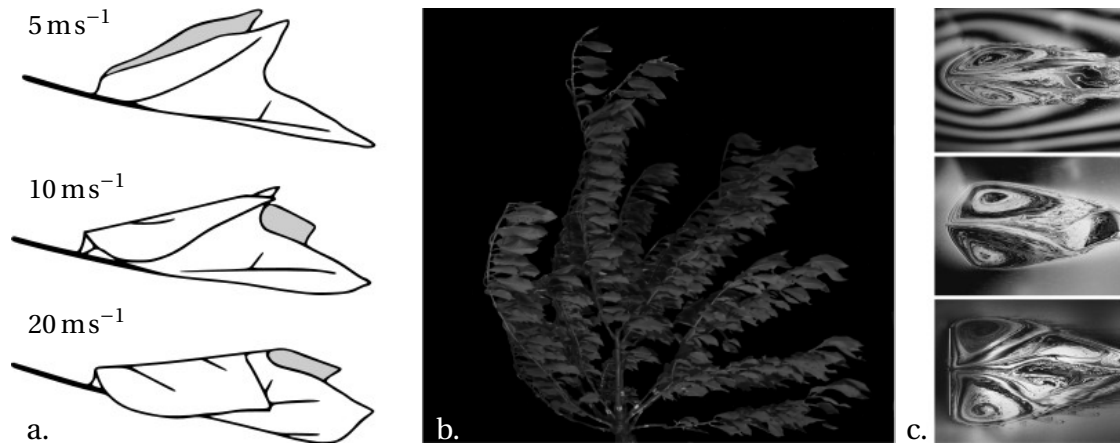


FIGURE 2.2. – a. Reconfiguration d'une feuille de tulipier de Virginie (*Liriodendron tulipifera*), d'après Vogel (1989) b. Reconfiguration d'un cerisier en soufflerie (Tadrist; Saudreau et al., 2018). c. Reconfiguration d'une plaque élastique dans un écoulement 2D (Alben et al., 2004).

Les hautes températures ou l'excès d'irradiation empêchent la réaction de photosynthèse (Hopkins et al., 1995). En conséquence les plantes ont développé de nombreuses parades pour éviter la surchauffe :

1. Adaptation de la biomachinerie moléculaire.
2. Utilisation de la ressource en eau pour refroidir par évaporation (environ 99% de l'eau utilisée par un arbre est perdue par évaporation)³.
3. Utilisation du mouvement du feuillage dans le vent pour (i) augmenter les échanges thermique (instabilité de flottement des feuilles) et (ii) créer un environnement lumineux dynamique en favorisant un éclairage de la surface foliaire des feuilles sous jacentes. Cet environnement lumineux dynamique permet de partager la ressource lumineuse et d'éviter la saturation lumineuse des feuilles supérieures de la canopée et de limiter leur température (Roden, 2003).
4. Production d'un mouvement passif. La pression de turgescence est liée au potentiel hydrique du sol. En cas de sécheresse, le potentiel hydrique du sol est bas, la pression de turgescence diminue. Les pétioles deviennent plus flexibles, les stomates se ferment et les feuilles pointent vers le sol. Ce changement d'orientation permet de diminuer l'interception lumineuse et de limiter l'utilisation d'eau pour les refroidir (Gonzalez-Rodriguez et al., 2016).
5. Production d'un mouvement actif. Certaines plantes sont dotées de pulvini, les moteurs végétaux. Ces actionneurs fonctionnent par déformation et permettent de modifier l'orientation des feuilles d'arbre. Pour les arbres de l'espèce *Robinia*

3. Certains biologistes indiquent que l'évaporation permet de refroidir les feuilles. Notons cependant qu'une partie des biologistes voient l'évaporation comme un mal nécessaire : les arbres doivent capter du CO₂ et ces échanges gazeux impliquent une ouverture des stomates et donc une évaporation inévitable, voir (Hopkins et al., 1995). Dans tous les cas, l'évaporation est un élément essentiel du bilan thermique d'une feuille.

Pseudo-acacia, les feuilles sont déployées lorsqu'elles sont à l'ombre et se replient au soleil, voir figure 2.3.

TRAVAUX FUTURS

Créer un dispositif de mesure transportable et automatisé pour déterminer l'évolution des distributions d'angle foliaires à l'échelle de quelques minutes. En effet, ces distributions d'angle foliaires sont importantes pour (i) déterminer la séquestration de CO₂ par un couvert végétal en période de sécheresse et (ii) Déterminer les taux d'évaporation réels des arbres et prédire leurs risques d'embolie. Ces enjeux sont importants pour la gestion des couverts forestiers à l'ère du réchauffement du climat avec des été chauds et secs.

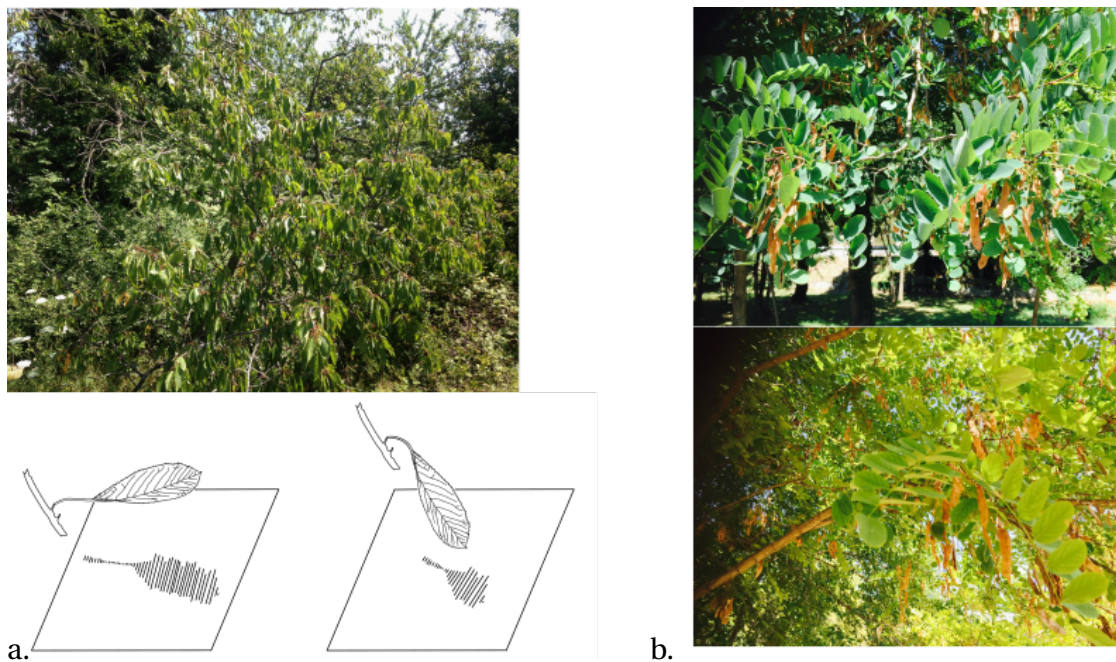


FIGURE 2.3. – a. Protection solaire par fanage b. Protection solaire par mouvement actif.

Stress biotiques Enfin les plantes sont aussi soumises à des stress comme la compétition inter-espèce pour la lumière, la défense contre les herbivores (insectes et mammifères) et les pathogènes. Notons que certaines plantes se sont dotées de mouvement actifs pour se protéger des herbivores, comme *Mimosa Pudica*.

Les champignons sont des pathogènes microscopiques qui se développent dans les gouttes d'eau à la surface des feuilles. Pour éviter le dépôt des pathogènes, certaines feuilles sont dotées de poils ou de revêtements qui les rendent hydrophobes (e.g. le lotus, (Bhushan et al., 2006)). D'un point de vue dispersion des pathogènes, la

flexibilité des feuilles est préjudiciable : lors d'une pluie, elle permet le rebond de certaines gouttes vers le haut, la feuille agissant comme un trampoline élastique (Gilet & Bourouiba, 2015).

2.2. Un exemple de modèle mécanique pour comprendre l'évolution : Comment la biomasse est-elle allouée dans une feuille ?

Les modèles mécaniques peuvent expliquer certains traits évolutifs lorsque la pression de sélection est d'origine physique. Prenons l'exemple de l'allocation de biomasse dans une feuille entre la partie plane servant à capter le soleil et la veine centrale assurant le support.

La forme des feuilles est dépendante de leur fonction principale et de tous les facteurs de stress qui forment autant de pressions de sélection. On peut tout de suite noter que l'optimalité face à un facteur de stress peut être préjudiciable pour un autre, par exemple être flexible pour éviter d'avoir une importante traînée dans le vent peut mener à la sélection de feuilles pas assez rigides pour être autoportantes.

La plupart des ports foliaires nécessitent d'allouer de la biomasse pour orienter et maintenir les feuilles face au soleil. Comment les feuilles allouent-elles la biomasse disponible ? Vaut-il mieux créer beaucoup de petites feuilles ou bien une seule grande feuille ?

Pour répondre à ce problème, la feuille est idéalisée par un limbe pesant supporté par une veine centrale cylindrique. La feuille a pour longueur l , largeur w et épaisseur t . La veine centrale a pour diamètre d . Le matériau constitutif de la feuille est homogène, isotrope et élastique avec un module d'Young E et une masse volumique ρ . On suppose que la feuille est encastrée horizontalement dans une branche à une de ses extrémités et permet une déflexion d'angle θ sous l'effet de la gravité g , voir figure 2.4.

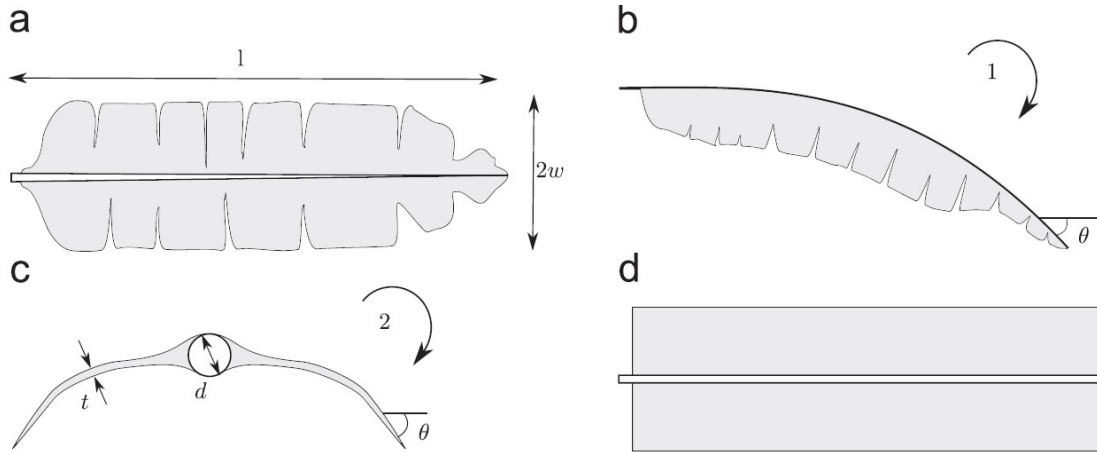


FIGURE 2.4. – Vues idéalisées d’une feuille de monocotylédone géante. (a) Vue de dessus. (b) Vue latérale. La feuille pend sous l’effet de son propre poids. θ est son plus grand angle d’inclinaison. Cette direction de courbure est notée 1. (c) Vue en coupe de la feuille. Là encore, la feuille pend sous l’effet de son propre poids et θ est son plus grand angle d’inclinaison dans cette direction. Cette direction de courbure est notée 2. (d) Vue de dessus de la feuille idéalisée utilisée pour le modèle.

Pour simplifier les raisonnements, nous allons décrire la mécanique en loi d’échelle (Pour les préfacteurs voir Tadrict & Darbois-Textier (2016)). La déflexion de la feuille dans la direction de la veine centrale (direction 1 sur la figure 2.4b) est donnée par l’équilibre du moment de courbure et celui du poids du limbe,

$$EI \frac{d\theta}{ds} = \rho g (w l t) l \quad \text{En approximant } \frac{d\theta}{ds} \simeq \frac{\theta}{l}, \Rightarrow \frac{\rho g w t l^3}{E d^4} = \theta \quad (2.1)$$

Avec un raisonnement similaire, on obtient pour la direction 2, figure 2.4c,

$$\frac{\rho g w^3}{E t^2} = \theta \quad (2.2)$$

On souhaite maximiser la surface de la feuille, $S = 2lw$ sous la contrainte d’autosupport, équations (2.1, 2.2) et pour un volume de biomasse donné $V = 2wlt + \pi d^2 l / 4$. Ce problème de minimisation sous contraintes est résolu via le formalisme des multiplicateurs de Lagrange, voir (Tadrict & Darbois-Textier, 2016). On obtient les lois d’échelles d’évolution des paramètres morphologiques de la feuille en fonction de ses paramètres mécaniques, E, ρ, g , d’autosupport θ et du volume de biomasse considéré V ,

$$t \propto \left(\frac{E\theta}{\rho g} \right)^{-7/20} V^{9/20} \quad (2.3)$$

$$w \propto \left(\frac{E\theta}{\rho g} \right)^{1/10} V^{3/10} \quad (2.4)$$

$$l \propto \left(\frac{E\theta}{\rho g} \right)^{1/4} V^{1/4} \quad (2.5)$$

$$d \propto \left(\frac{E\theta}{\rho g} \right)^{-1/8} V^{3/8} \quad (2.6)$$

Des mesures des différents coefficients du modèles ont été faites de manière systématique sur des feuilles de monocots géants (palmiers) et montrent un bon accord avec les lois d'échelles présentées ci-dessus excepté pour l'épaisseur des feuilles où d'autres facteurs bio-physiques peuvent rentrer en jeu, figure 2.5.

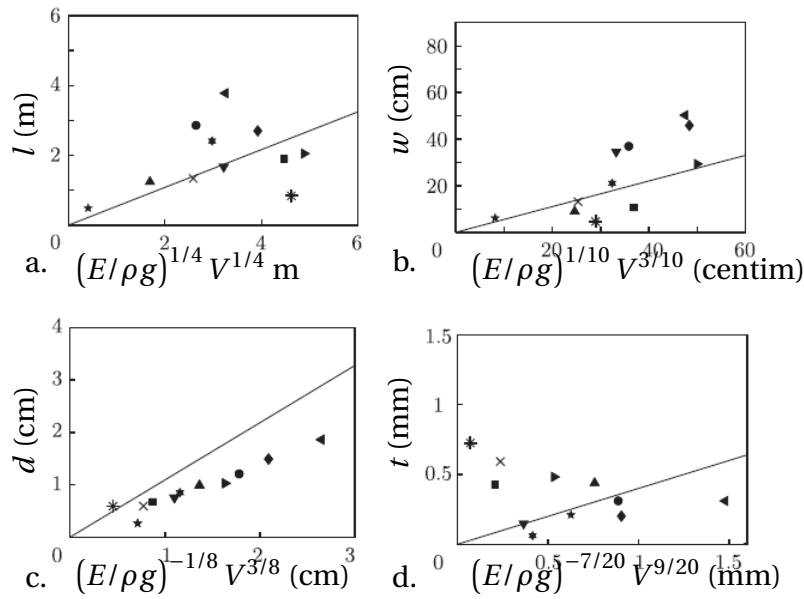


FIGURE 2.5. – Comparaison (parity plot) entre les données expérimentales et le modèle d'allocation de biomasse sous la condition d'autosupport. a. Épaisseur, équation (2.3) b. Largeur, équation (2.4) c. Longueur, équation (2.5) d. Diamètre de la veine centrale, équation (2.6)

Ce modèle mécanique montre que quelque soit le volume V de biomasse considéré, il existe une forme de feuille optimisant l'allocation de biomasse. Les feuilles mesurées suivent assez bien les lois d'échelles prédites par le modèle. Si l'on s'intéresse maintenant à une maximisation du volume fonctionnel par rapport au volume support, on

doit maximiser le ratio,

$$\frac{V_{\text{fonctionnel}}}{V_{\text{support}}} = \frac{2wlt}{\pi d^2 l} \propto V^{1/4} \quad (2.7)$$

Plus les feuilles sont grandes ($V \gg 1$), plus le rapport volume fonctionnel, volume de support est favorable. Ce gain vers les grandes feuilles est assez mou (puissance 1/4) et est limité par d'autres phénomènes physiques comme l'hydraulique des feuilles (Kaare H. Jensen et al., 2013) ou les risques de déchirures. Pour les petites feuilles, la séparation volume fonctionnel / volume support n'est pas aussi claire car certaines des cellules de support réalisent la photosynthèse. Enfin, les plantes mettent en place des stratégies de second ordre comme le raidissement par courbure de Gauss (Barois et al., 2014; Taffetani et al., 2019) pour les feuilles enroulées, la structuration des veines portantes et la présence de veines secondaires. Aujourd'hui la discussion sur la taille des feuilles est toujours active avec vraisemblablement un trade-off entre hydraulique et mécanique (Levionnois et al., 2021).

En résumé, ce modèle très simplifié de feuille auto-portante nous permet de comprendre l'optimum faible que représente la taille des feuilles, ce qui permet la grande variété de tailles observées dans la nature, du millimètre à plusieurs mètres.

2.3. Artefacts de feuilles pour les interactions vent, pluie et bourgeons

Les feuilles idéalisées ne ressemblent pas à des feuilles d'arbre classiques. Elles essentialisent une de leur propriété, par exemple, forme et élasticité pour flotter dans le vent ou bien l'organisation des veines par pliage. La figure 2.6 montre quatre exemples de feuilles idéalisées construites pour étudier le flottement et la reconfiguration au vent des feuilles larges, l'interaction avec la pluie et le repli des feuilles dans les bourgeons.

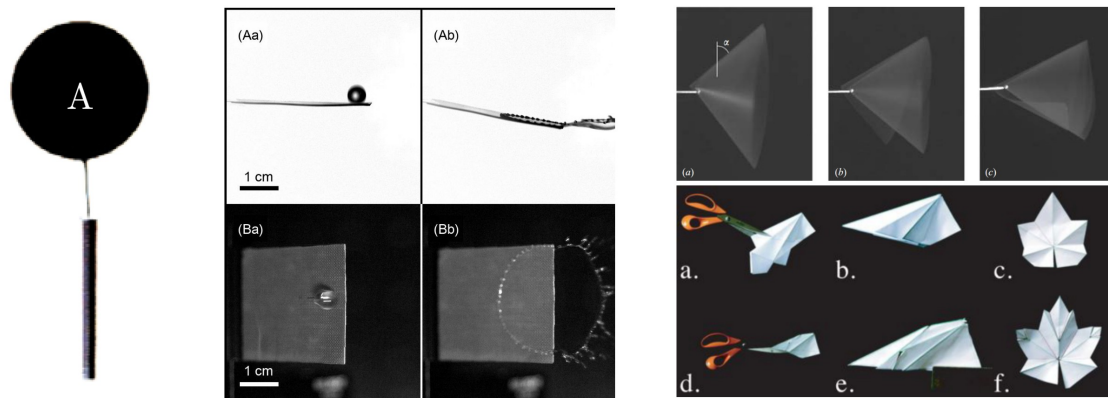


FIGURE 2.6. – Quelques artefacts de feuilles d'arbre. Gauche : Artefact de feuille pour l'étude du flottement au vent (haut droit) (Tadrist; Julio et al., 2015). Centre : Artefact de feuille pour l'étude de l'interaction avec des gouttes de pluie (Gilet & Tadrist, 2025). Haut Droit : Artefact de feuille pour l'étude de la reconfiguration de feuilles large (Schouveiler et al., 2006). Bas droite : Artefact de feuille pour l'étude de la morphogénèse des feuilles – loi de remplissage des bourgeons (Couturier et al., 2011).

Ces artefacts de feuilles sont réalisés en matériaux inertes pour éviter la variabilité biologiques. Ils permettent de montrer l'effet physique, géométrique ou mécanique le plus important pour une feuille dans certaines conditions.

Ces modèles de feuilles ne sont pas concurrents mais complémentaires : on ne peut pas remplacer le modèle de repliement dans un bourgeon par celui d'interaction avec la pluie. Chaque modèle insiste sur un élément unique sans prendre en compte la complexité biologique.

2.4. Artefacts de tissus mous de plantes : comprendre le lien entre raideur et turgescence

2.4.1. Modèles de rigidité cellulaire

L'intérêt pour les tissus mous des plantes est porté par les problèmes concrets de conservation des fruits et légumes, de modélisation du développement des plantes et par un outil : la microscopie à force atomique.

Le développement de la microscopie à force atomique a permis de déterminer des courbes force/déplacement à l'échelle cellulaire, dépendant à la fois de la pression de turgescence et des propriétés élastiques de la cellule. La détermination des contributions élastiques et de pression reposent sur une idéalisation de la cellule comme une coque sphérique élastique et le contact de la pointe du microscope comme ponctuel. La modélisation a été effectuée sur un système modèle : une sphère élastique

préssurisée avec un contrôle en pression (comportement isobare du fluide) déformée par une pointe quasi ponctuelle Vella et al. (2012).. Ce modèle d'indentation de cellule est macroscopique, voir figure 2.7a. Expériences et simulations ont montré un comportement linéaire de la relation force-indentation. De plus, un modèle de coques peu indentées a été proposé, basé sur les équations de Föppl-Von Karmàn. Une expression analytique linéaire de la relation force-indentation est donnée avec un préfacteur dépendant de la pression de gonflage. Néanmoins, si l'indentation n'est pas ponctuelle, le comportement linéaire de la relation force-indentation est perdu (Lacorre et al., 2024). Ce second exemple est aussi lié à l'utilisation d'un objet macroscopique (une balle élastique) pour idéaliser des contacts plans entre cellules. Dans ce travail nous avons montré que le comportement fluide influe sur la rigidité aux grandes déformations de manière non linéaire. Les différentes lois empiriques sont données dans la table 2.1. Elles montrent un raidissement ajouté important selon le comportement fluide.

Comportement fluide	Relation force-déplacement
Isobare	$F = \frac{Eta}{1 - \nu^2} \left(0.17 + 3.8 \left(\frac{Pa(1 - \nu^2)}{Et} \right)^{4/5} \right) \left(\frac{\delta}{a} \right)^{4/3}$
Isotherme / osmotique	$F = \frac{Eta}{1 - \nu^2} \left(0.17 + 3.8 \left(\frac{P_0 a(1 - \nu^2)}{Et} \left(1 + 0.22 \left(\frac{\delta}{a} \right)^2 \right) \right)^{4/5} \right) \left(\frac{\delta}{a} \right)^{4/3}$
Adiabatique	$F = \frac{Eta}{1 - \nu^2} \left(0.17 + 3.8 \left(\frac{P_0 a(1 - \nu^2)}{Et} \left(1 + 0.44 \left(\frac{\delta}{a} \right)^2 \right) \right)^{4/5} \right) \left(\frac{\delta}{a} \right)^{4/3}$
Isochore	$F = \frac{Eta}{1 - \nu^2} \left(0.17 + 3.8 \left(\frac{P_0 a(1 - \nu^2)}{Et} + 0.63 \left(\frac{\delta}{a} \right)^2 \right)^{4/5} \right) \left(\frac{\delta}{a} \right)^{4/3}$

TABLEAU 2.1. – Principales lois empiriques de contact avec un mur en fonction du comportement fluide. En bleu, le coefficient de raidissement supplémentaire par rapport au cas isobare.

Ici, l'apport de l'approche par systèmes modèles est indispensable : les cellules sont des objets petits, très mous, difficiles à mesurer et à analyser. Travailler sur des objets plus grands permet de déterminer l'évolution des forces physiques en jeu pourvu que les équations décrivant les deux systèmes soient identiques.

TRAVAUX FUTURS

Géométrie du contact.

Considérer n'importe quelle forme d'indenteur : plan, conique, cylindrique et sphérique. Pour l'indenteur sphérique, on peut faire varier le rayon de courbure pour passer continûment d'un contact ponctuel à un contact conforme.

2.4.2. Modèles de rigidité tissulaire

De manière surprenante, il existe peu de références sur la rigidification des tissus mous des plantes en fonction de la pression de turgescence des cellules. La première publication à traiter ce problème sous un angle mécanique est Nilsson et al. (1958) pour la partie théorique et Falk et al. (1958) pour la partie pratique. Les chercheurs essayaient de comprendre l'évolution de la rigidité de frites de pomme de terre en fonction du potentiel hydrique de la solution dans laquelle elles étaient conservées. Le potentiel hydrique de la solution était changé de manière contrôlée en ajoutant du manitol. Les expériences consistaient à déterminer la rigidité des frites de pomme de terre par vibration. Les chercheurs ont montré que la rigidité augmentait avec la pression de turgescence selon une loi affine,

$$Y = \frac{3(7-5\nu)}{10(1+\nu)} \frac{Et}{(1-\nu)a} + 3 \left(1 + \frac{7-5\nu}{20(1+\nu)} \right) P \quad (2.8)$$

où Y est le module d'Young apparent du tissu, (E, ν) le module d'Young et de Poisson de la paroi cellulaire, a le diamètre de la cellule sphérique et t l'épaisseur de la cellule. Les préfacteurs devant le module d'Young et la pression variant peu selon la forme de cellule considérée. Le modèle mathématique proposé colle remarquablement bien aux mesures expérimentales (Nilsson et al., 1958). Ce modèle s'appuie sur des hypothèses importantes :

1. **Géométrie.** Les cellules de forme sphérique se déformant en ellipsoïde lors de la vibration de la frite
2. **Thermodynamique.** Un comportement isochore des volumes intra-cellulaires.
3. **Mécanique.** Une loi de comportement linéaire pour la paroi cellulaire.

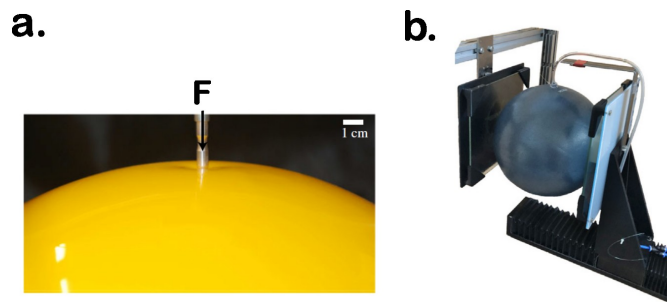


FIGURE 2.7. – Artefacts de cellules. a. Artefact de cellule avec contact ponctuel (type pointe d'AFM) (Vella et al., 2012). b. Artefact de cellule avec contacts plans (Lacorre et al., 2024).

Ces hypothèses doivent être discutées.

1. **Géométrie.** On idéalise généralement les cellules comme des sphères. Chez les végétaux matures, il existe une paroi cellulaire rigide et les cellules sont en contact plan rigide les unes avec les autres. Néanmoins, les résultats de Nilsson

ont été étendus à des formes anguleuses comme des décaèdres en changeant peu le résultat précédent.

2. **Thermodynamique.** Le comportement isochore est une bonne approximation pour des frites vibrées à des fréquences élevées (quelques Hertz). Les mouvements d'eau de nature poro-élastiques n'ayant pas le temps de s'établir sur une période. Au contraire, les mouvements rencontrés chez les plantes sont souvent lents ainsi que les changements environnementaux qui les accompagnent (une sécheresse change le potentiel hydrique du sol à l'échelle des jours et des semaines). Les parois cellulaires étant poreuses, les transformations fluides ne sont pas isochores mais plutôt osmotiques⁴.
3. **Mécanique.** Une loi de comportement peut toujours être linéarisée autour d'un point de fonctionnement. Néanmoins, les parois cellulaires des plantes présentent des lois de comportement fortement non-linéaires, figure 2.8, et il est peu probable que l'on puisse modéliser des elongations de quelques dizaines de pourcents avec une loi linéaire.

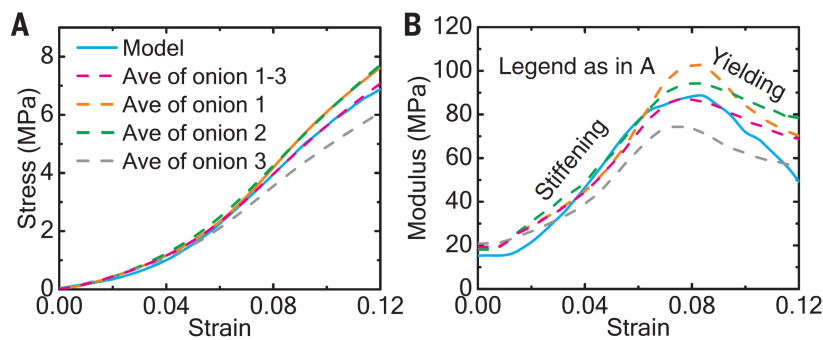


FIGURE 2.8. – Exemple de mesure de courbes force-déplacement pour des parois cellulaire d'oignon, d'après (Zhang et al., 2021). La loi de comportement perd sa linéarité après une déformation de 1% et devient très raidissante.

Malgré les écueils des hypothèses simplificatrices énoncées ci-dessus, ce résultat est le meilleur état de l'art disponible pour modéliser, avec une expression simple, des tissus mous de plantes soumis à des changements de pression osmotiques (Faisal et al., 2010; Beauzamy et al., 2015; Gonzalez-Rodriguez et al., 2016). Le système modèle est directement un objet biologique et l'absence d'un système modèle se fait sentir pour son côté didactique et intuitif.

Dans ce contexte, nous avons repris les travaux sur les solides cellulaires pressurisés en débutant par la construction d'un système modèle (Tadrist; Mammadi et al., 2022).

4. La thermodynamique des transformations osmotiques pour les fluides (solutions diluées) est identique à la transformation thermodynamique isotherme des gaz. Pour chacun des deux cas, le produit PV est conservé. pour les gaz $PV \propto T$. Pour une solution diluée, P est proportionnel à la concentration en soluté, le produit $PV \propto N$ est donc proportionnel au nombre de molécules de soluté présent à l'intérieur de la membrane cellulaire et qui ne peuvent la franchir (N est constant).

Ce système modèle est obtenu par moulage d’une matrice polymère et s’est révélé difficile à réaliser⁵. Néanmoins ces tests préliminaires ont été surprenants : à contre pied des prédictions du modèle de Nilsson, les modules d’Young équivalents se sont effondrés avec la pression.

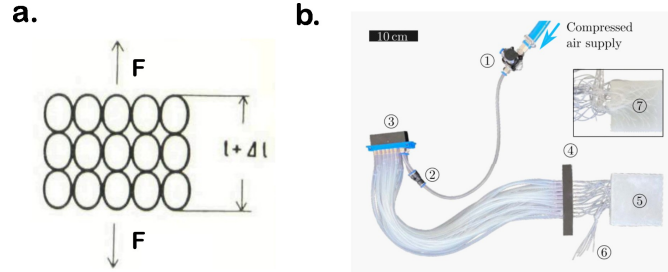


FIGURE 2.9. – Idéalisation de solides cellulaires pressurisés. a. Déformation de cellules sphériques en ellipsoïdes à volume constant, d’après Nilsson et al. (1958). b. Solide cellulaire pressurisé monocouche à 49 cellules (Tadrist; Mammadi et al., 2022).

Le modèle physique a permis de remettre en question le modèle proposé par Nilsson. Avec des cellules cubiques et de l’élasticité linéaire, l’effet de la pressurisation avec un comportement fluide isobare est assouplissant, figure 2.10.

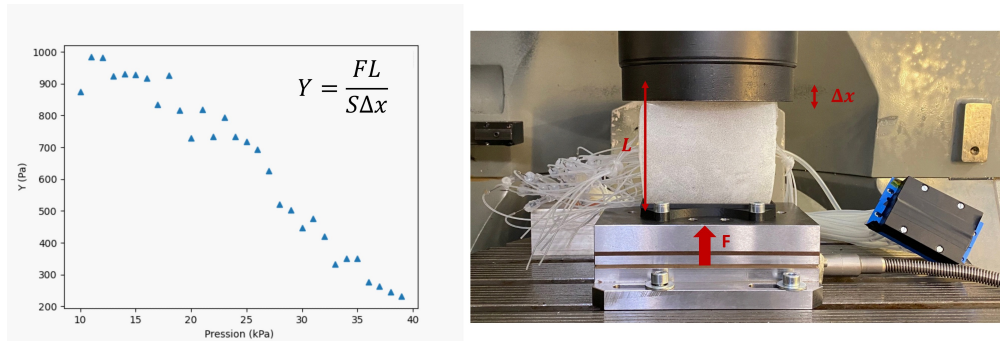


FIGURE 2.10. – Courbe typique d’évolution de la rigidité d’un solide cellulaire pressurisé avec la pression en régime linéaire, comportement fluide isobare.

Ce résultat est-il contre-intuitif? Pas vraiment. Si l’on simplifie au maximum une cellule unique, en représentant les faces comme des ressorts de raideur k et en ajoutant une pression interne, figure 2.11. La pression à pour effet de mettre les ressorts en traction et d’augmenter les dimensions de l’échantillon $\ell = \ell_0(1 + \alpha P)$. Les raideurs des ressorts quant à elles ne changent pas. Les efforts F du ressort déplacé d’une longueur x autour de sa position d’équilibre pressurisée permettent de déduire l’effet

5. Une technique par impression 3D de moule en hydrogel inspiré de celle d’Antonio Ibarra et al. (2022) a permis d’obtenir une précision de fabrication et une répétabilité meilleure.

de la pression sur le module d'Young :

$$F = kx \Rightarrow \underbrace{\frac{F}{\ell^2}}_{\sigma} = \underbrace{\frac{k}{\ell}}_Y \underbrace{\frac{x}{\ell}}_{\varepsilon} \Rightarrow Y \simeq Y_0(1 - \alpha P) \quad (2.9)$$

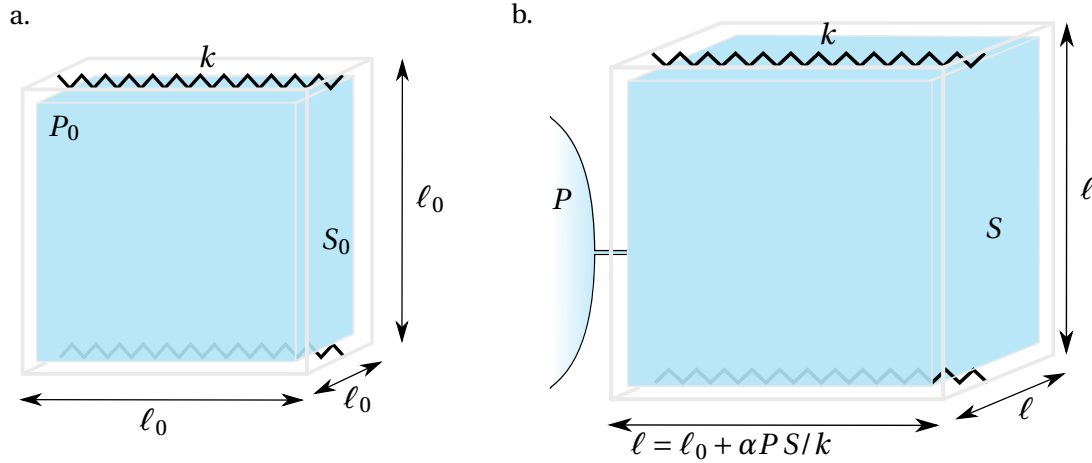


FIGURE 2.11. – Schéma physique de l'assouplissement des solides cellulaires pressurisés. Suite au gonflement, les dimensions sont affectées d'un facteur $1 + \alpha P$ mais les raideurs sont identiques. L'effet linéaire d'un ajout de pression est de diminuer le module d'Young apparent en « diluant » le matériel élastique.

Les résultats par modélisation éléments finis (Code_Aster, EDF) et avec un modèle analytique à cinématique simplifiée confirment que la raideur apparente du système⁶ diminue en élasticité linéaire. Au premier ordre, si la loi de comportement du matériau constituant les murs cellulaire est linéaire, c'est l'augmentation de volume du matériau qui induit une diminution du module d'Young apparent.

2.4.3. Turgescence et raideur, une relation plus complexe qu'il n'y paraît.

Si la forme des cellules est changée au profit de cellules polygonales, la non-linéarité des forces de contact observée sur des membranes sphériques a une origine différente :

6. La raideur apparente du matériau est la raideur telle que mesurée par l'expérimentateur : un morceau parallélépipède du solide pressurisé de dimensions pressurisées L_p, w_p, t_p est testé en traction. La force mesurée F est reliée à l'incrément de longueur x par la loi de Hooke, $F = \frac{Y w_p t_p}{L_p} x$, où Y est le module d'Young apparent. Ce module d'Young apparent dépend de la pression à travers de la raideur F/x et de l'augmentation des dimensions du parallélépipède due à la pression. De manière surprenante, on peut avoir une augmentation de la raideur globale mais une diminution du module d'Young apparent, ceci à cause des variations de dimension du solide cellulaire pressurisé.

elle ne dépend plus l'évolution de la forme du contact mais uniquement des non-linéarités liées au processus thermodynamique du fluide et aux lois de comportement hyperélastiques des matériaux des murs cellulaires. C'est le cas pour des cellules cubiques.

Les propriétés homogénéisées des solides cellulaires pressurisés à cellules cubiques, présentent un comportement similaire : une diminution de la raideur au début de la pressurisation puis une augmentation lorsque l'on arrive dans la zone de raidissement à l'extension du matériau.

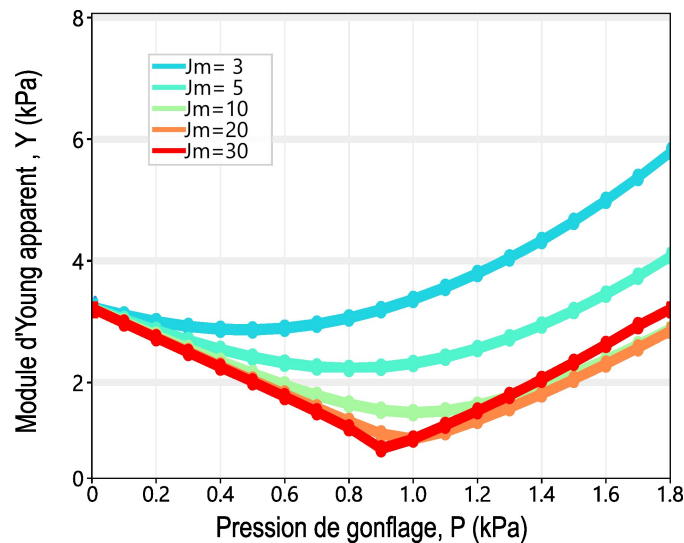


FIGURE 2.12. – Exemple de mesure de courbes force-déplacement pour des parois cellulaire d'oignon, d'après (Zhang et al., 2021).

TRAVAUX FUTURS

Déterminer une courbe maîtresse pour le raidissement des solides cellulaires pressurisés. La possibilité d'une courbe maîtresse n'est pas évidente car le problème physique comprend 4 paramètres sans dimension indépendants.

TRAVAUX FUTURS

Rôle de la cuticule Pour pouvoir conclure par rapport à la raideur des tissus mous des plantes, il faut pouvoir estimer l'action de la couche extérieure (cuticule). Le tissu n'est pas infini mais limité par une peau élastique mais plus rigide, la cuticule⁷ Un système modèle est en cours de développement dans le groupe de Philippe Marmotant avec des pois chiches et un tube élastique.

TRAVAUX FUTURS

Déterminer comment les motifs de pressurisation permettent de changer les propriétés apparentes du matériau (Modules d'élasticité anisotropes et auxétisme).

2.5. Autres succès remarquables de systèmes modèles pour comprendre et rendre intuitif la physique des plantes

Le développement des systèmes modèles pour les plantes s'est accéléré avec le développement de la millifluidique et de la microfluidique. Les techniques de micro-fabrication ont permis de créer des artefacts de cellules ou de vaisseaux pour étudier des phénomènes aux échelles de quelques dizaines de microns.

Les systèmes modèles millifluidiques ont permis de confirmer certains mécanismes comme celui de Münch (écoulement des sèves élaborées des feuilles vers les racines), figure 2.13. Ce travail mené dans l'équipe de Kaare Jensen montre des déviations entre le mécanisme de Münch idéal et les mesures réelles (Comtet et al., 2017). Ces modèles physiques peuvent être étudiés systématiquement en variant certains paramètres, par exemple la porosité entre phloème et xylème pour mettre en évidence l'optimalité d'une configuration.

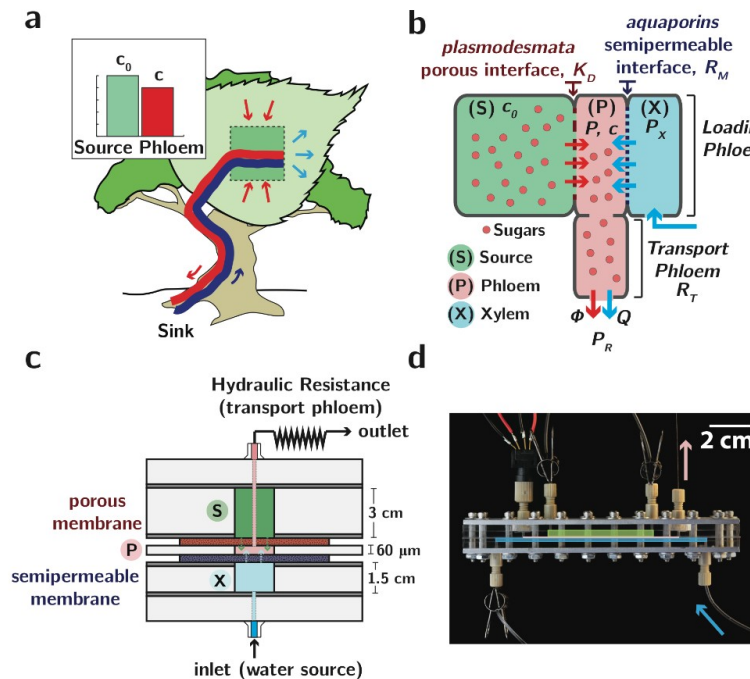


FIGURE 2.13. – Système modèle pour le mécanisme de Münch, d'après (Comtet et al., 2017).

L'utilisation de systèmes modèles a permis de mieux comprendre comment écouter la présence d'embolie chez les arbres en déterminant les signatures sonores d'une cavitation fluide entourée d'un milieu souple (Vincent et al., 2012), figure 2.14a. Toujours pour comprendre l'embolie chez les gymnospermes, des ponctuations aréolées (*torus-margo pit pores*) ont été reproduites, figure 2.14b, et ont montré des comportements rapides et non linéaires permettant de réguler les flux d'eau au sein d'un arbre.

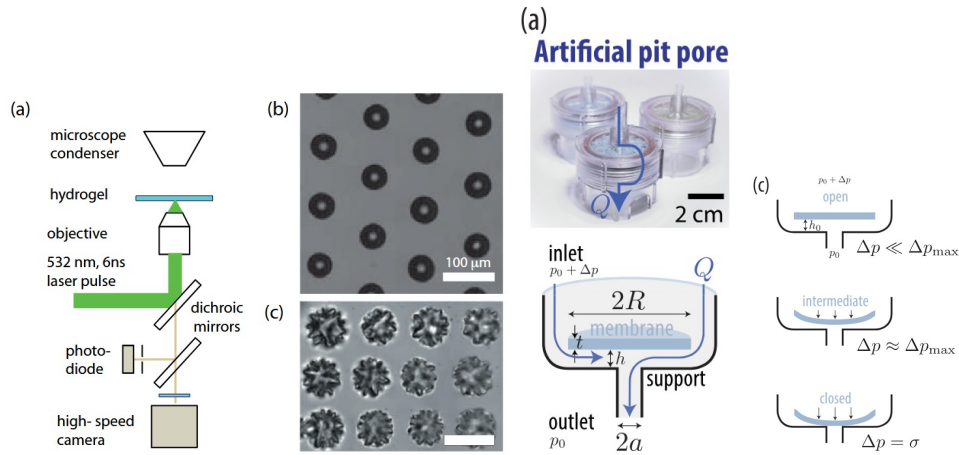


FIGURE 2.14. – a. Système modèle pour comprendre l'émission ultrasonore d'une cavité cavitante dans un solide mou (Vincent et al., 2012). b. Système modèle pour comprendre le fonctionnement des ponctuations aréolées, *torus-margo pit pores*, chez les gymnospermes (Park et al., 2021).

Enfin, les systèmes modèles permettent aussi d'envisager l'étude de l'optimalité de traits biologiques (densité de stomates, de veines) par rapport à la physique mise en jeu, figure 2.15.

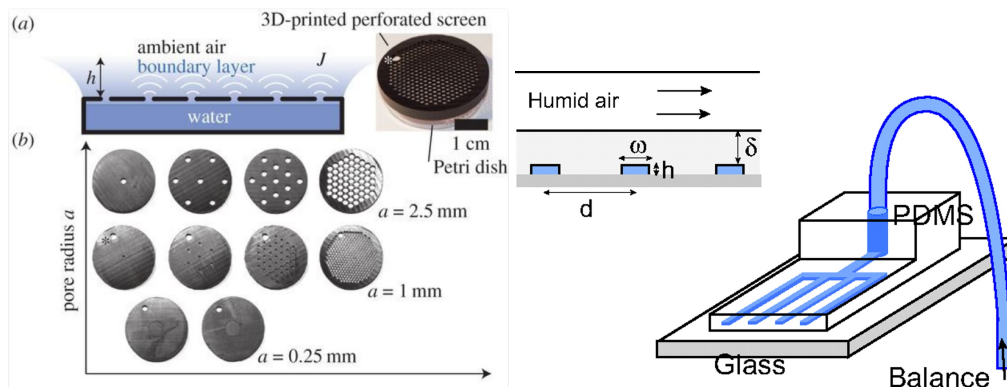


FIGURE 2.15. – a. Système modèle pour la densité optimale des stomates (Zwieniecki et al., 2016) et sa relation au flux évaporé. b. Système modèle pour la densité optimale de veines sur une feuille et sa relation au flux évaporé (Noblin et al., 2008).

Conclusion sur les systèmes modèles

En biomécanique des plantes, l'apport des modèles physiques est précieux pour l'investigation contrôlée d'un paramètre physique. Nous avons vu que ces systèmes modèles sont très variés et reposent sur des techniques de fabrication différentes, de l'origami à la cavitation déclenchée par laser.

Leur capacité didactique et de persuasion ne doit pas nous faire oublier que la biologie reste complexe et que le caractère optimal d'un trait doit toujours être regardé en considérant les multiples pressions de sélection.

Ces systèmes modèles sont intéressants pour comprendre le monde biologique, ils sont souvent détournés pour créer des systèmes innovants. Par exemple le système ayant permis de reproduire les ponctuations aérolées des conifères a été envisagé pour créer un réseau fluide à flux programmable (Park et al., 2021).

Dans le prochain chapitre nous allons voir quelques exemples de systèmes inspirés de la nature pour l'ingénierie. La littérature sur le sujet est importante, parfois se réclamant de la bioinspiration en n'ayant qu'un lointain lien avec la biologie. Pour éviter une longue liste d'exemple, je présenterai 3 exemples d'actionneurs bioinspirés que j'ai développés au cours de mon parcours scientifique.

3. Biologie vers Ingénierie : Actionneurs bio-inspirés.

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Ce chapitre regroupe quelques exemples de maquettes ou d'actionneurs inspirés d'êtres vivants auxquels j'ai participé. Le premier actionneur est un préhenseur capillaire inspiré du Chrysomèle à gros ventre (*Gastrophysa viridula*). Le second est une maquette de muscle squelettique. Le troisième est un actionneur par déformation inspiré du pulvinus de *Mimosa pudica*. Reproduire la nature est un défi technologique : nous sommes quasi incapables de former la matière aux échelles de la nature. Pourtant ces tentatives de reproduction nous apprennent beaucoup sur les limites de modélisation et nos difficultés à fabriquer les actionneurs.

3.1. Gripper capillaire inspiré des insectes

Défi industriel : attraper de manière précise, fiable et rapide des petits éléments (taille inférieure au millimètre) avec application à la fabrication microélectronique.

Observation biologique : Le chrysomèle à gros ventre peut se déplacer à l'envers sur des surfaces lisses. son mode d'adhésion est capillaire : il sécrète une infime quantité de liquide sous ses pattes.

Gripper bio-inspiré : Le gripper bioinspiré consiste en une partie texturée imprimée en 3D à l'échelle sub-micronique (nanoscribe). Ce doigt texturé est plongé dans un liquide pour mouiller ses aspérités. Lors du retrait du doigt, une partie du liquide est capturé par le doigt : la ligne triple du liquide s'ancre sur les textures imprimées en 3D. Une fois chargé, le doigt se positionne au niveau de l'objet à manipuler. La capture a lieu par contact : Les forces capillaires sont suffisantes pour vaincre le poids de l'objet (ce qui implique un objet suffisamment petit).

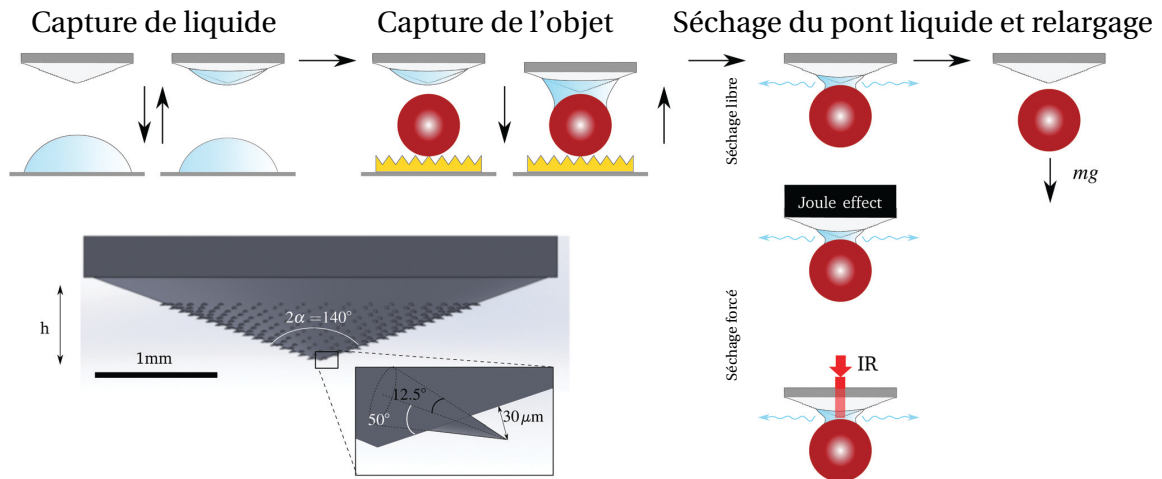


FIGURE 3.1. – Schéma de fonctionnement du gripper capillaire. La séquence est en 4 temps : (1) Le gripper se charge en liquide pour l'adhésion capillaire. (2) Capture de l'objet par contact avec la goutte de fluide. (3) Déplacement de l'objet. (4) Relargage par évaporation libre ou forcée. La surface du gripper capillaire est texturée pour maîtriser l'ancrage de la ligne triple.

Une fois capturé, l'objet peut être déplacé à volonté avec des accélérations modérées (plus l'objet est petit plus ces accélérations peuvent être grandes). Le relâchement de l'objet n'est pas facile : en l'absence d'autres forces, un relargage inertiel peut être envisagé pour les objets les plus massifs. Nous avons opté pour un relargage par évaporation du liquide par chauffage laser infra rouge. Cette technique permet de déposer les objets avec précision.

Une vidéo du fonctionnement du gripper est disponible sur [ce supplementary material](#), (Iazzolino et al., 2020).

3.2. Maquette de muscle

Défi industriel : Pour assurer sécurité et fiabilité, les actionneurs critiques sont conçus avec des marges de sécurité et sont dédoublés. En cas de défaut, l'actionneur redondant prend le relai. Parfois, la redondance n'est pas permise (encombrement, coût, poids ou consommation énergétique). Ces contraintes sont fortes pour les applications aéronautiques et spatiales.

Sur un hélicoptère, par exemple, si l'actionneur de commande vol (orientation des pâles) commence à se détériorer avec des pertes de performances raisonnables (résilience aux dégâts), ce défaut peut être détecté et indiquer au pilote de se poser pour réparation.

Le paradigme actuel de conception des actionneurs (action centralisée, monobloc, surdimensionnement, asservissement complexes) ne permet pas de briser les limites pour la résilience aux dommages.

Observation biologique : À l'échelle du nanomètre, la production de force est

assurée par les moteurs moléculaires de la myosine. Chaque paire d'unité contractile tire ses filaments voisins dans des directions opposées, générant un raccourcissement microscopique dans le sarcomère. Les sarcomères (environ 3 microns de long) sont connectés en série pour former une fibre musculaire de plusieurs centimètres. Les fibres sont placées en parallèle avec différents angles de pennation, selon la fonction musculaire. A contrario des actionneurs industriels actuels, le muscle squelettique propose une architecture décentralisée, à multiples chemins d'effort, à pilotage multi-échelle, réalisant des mouvements fluides et peu sensibles aux dommages.

Matérialisation de la maquette : Pour tester la résilience aux dommages d'un assemblage série et parallèle d'actionneurs, nous avons construit une maquette de muscle mimant le raccourcissement élémentaire des sarcomères. Le fonctionnement de la maquette de muscle suit fidèlement le cycle de réduction de l'ATP grâce à une crémaillère symbolisant les emplacements des ponts actine-myosine. Le détail de la maquette est présenté figure 3.2.

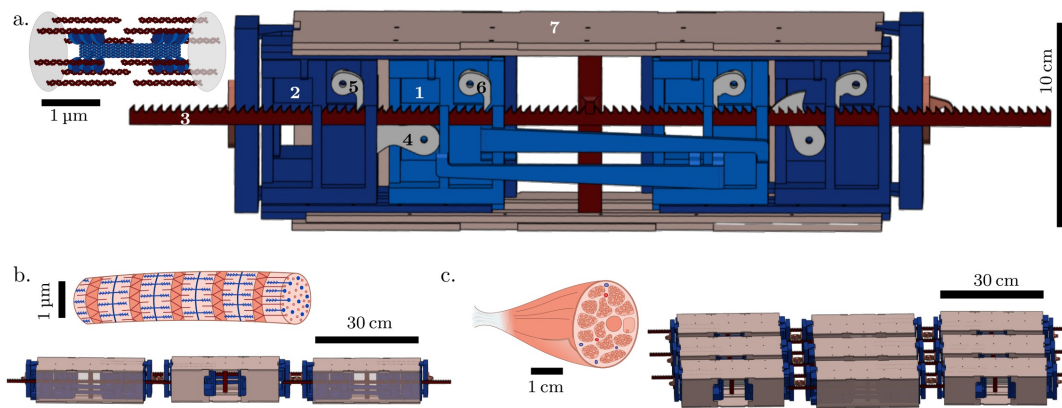


FIGURE 3.2. – Maquette de muscle au 1 :100 000. a. Vue 3D de la maquette de sarcomère composée de deux têtes de myosine face-à-face. La tête de myosine est composée de deux parties mobiles indépendantes (1 et 2). Chaque partie possède une patte biomimétique (dent 5 et 6) qui se fixe sur la fibre d'actine (tige de crémaillère 3). La course de la tête de myosine est assurée par un bras mécanique (4). La maquette du sarcomère est délimitée par un fascia artificiel (7). b. Les unités de sarcomères sont empilées en série pour produire une maquette de fibre. c. Les myofibrilles sont empilées en parallèle pour créer un muscle biomimétique.

Cette maquette a permis de démontrer que l'empilement d'actionneurs élémentaires en série et en parallèle permet de ne pas connaître d'effondrement des performances : les performances diminuent proportionnellement aux dégâts à la micro-échelle.

Deux suites sont envisagées pour ce travail :

TRAVAUX FUTURS

Déterminer un algorithme de contrôle des micro-actionneurs pouvant s'adapter aux dégâts. Ces algorithmes adaptatifs sont utilisés par les roboticiens (Cully et al., 2015) et font appel à des outils d'apprentissage par renforcement.

La plupart de ces algorithmes fonctionnent bien sur simulateurs, le défi est de réaliser un jumeau numérique de notre actionneur pour l'entraînement avant d'implémenter les algorithmes d'adaptation aux dégâts.

L'autre aspect intéressant soulevé par la fabrication de cette maquette est la possibilité de réaliser un actionneur répondant aux spécifications de l'industrie de l'air et de l'espace.

TRAVAUX FUTURS

Vers l'industrialisation d'un actionneur hiérarchique inspiré du muscle. Cet actionneur à multi degrés de liberté permet :

1. Résilience : la multiplicité d'actionneurs élémentaires permet de compenser la défaillance locale d'un actionneur élémentaire en adaptant le motif de sollicitation.
2. Démultiplication d'effort : la disposition des actionneurs élémentaires perpendiculaire aux supports permet une démultiplication des efforts (effet géométrique).
3. Fluidité du mouvement : La sollicitation progressive des actionneurs élémentaires permet un pilotage simple pour produire des mouvements fluides.

3.3. Pulvinus de mimosa Pudica artificiel

Les plantes bougent. Cette affirmation peut paraître cocasse : il n'y a guère que les ouvrages de science fiction qui donnent vie à des armées végétales d'Ents ou animent des saules cogneurs. Hors de la fiction, la plupart des mouvements végétaux que l'on observe sont principalement dus à l'effet du fluide autour des feuilles flexibles : le vent fait flotter les feuilles d'arbres et les marées ploient les forêts de Kelp. Pourtant, les mouvements actifs sont bel et bien présents, mais on les observe mal : les échelles de temps de l'observation humaine (quelques secondes) sont dissociées par deux ordres de grandeur des échelles de temps de mouvement des plantes (quelques heures). La plupart des mouvements des plantes sont des mouvements de croissance et on les observe par exemple avec les plantes d'appartement dirigeant leurs tiges et orientant leurs feuilles à la recherche de la lumière. Les mouvements par croissance sont complexes, ils créent des contraintes résiduelles dans les matériaux et une littérature abondante existe à leur sujet, depuis la modélisation de la croissance

cellulaire (modèle de lockart (Quiros et al., 2022)) jusqu'à des modèles plus macroscopiques de modélisation des formes de couvert végétal en gérant les poids relatifs du phototropisme et du gravitropisme (Moulia et al., 2022).

Certaines espèces de plantes préservent la possibilité d'un mouvement à l'état mature. Ces mouvements actifs sont générés par déformation d'un pulvinus, un petit coussinet foliaire à la base des feuilles. Les déformations à l'origine du mouvement proviennent d'un changement de pression osmotique à l'intérieur des cellules.

Ces mouvements actifs permettent aux plantes de s'ajuster à leur environnement, par exemple d'éviter une trop forte illumination solaire pour *Robinia Pseudo-acacia*, ou se protéger des prédateurs pour *Mimosa pudica*.

Ces mouvements induits par un changement osmotique peuvent être lents ou rapides selon les espèces¹. Les pulvinus sont présents chez plusieurs espèces éloignées sur l'arbre phylogénétique (Tadrist; Mammadi et al., 2022). Cet éloignement des pulvinus sur l'arbre phylogénétique montre que (i) soit cette évolution a été préservée au cours de l'évolution, (ii) soit il y a eu convergence évolutive : la génération de mouvement par choc osmotique est apparue plusieurs fois au cours de l'évolution.

Défi industriel 1 : Créer un matériau cellulaire pressurisé ayant des propriétés mécaniques ajustables. Les propriétés mécaniques variées peuvent être (i) la rigidité, (ii) l'anisotropie, (iii) le module de poisson.

Ce type de matériau peut être employé pour réaliser des liaisons ajustables en terme de raideur par exemple pour des liaisons robotiques.

Défi industriel 2 : Créer un actionneur cellulaire pressurisé versatile et résilient. Cet actionneur mécanique peut générer des déplacements en 3D avec une très large gamme d'amplitudes et de directions.

Cet actionneur souple fonctionne par déformation, sans pièce mobile. Il peut apparaître comme une solution pour des applications où des mouvements variés sont nécessaires. Par exemple pour l'interaction directe avec un humain, où sa flexibilité et sa souplesse sont des atouts.

Matérialisation de la maquette : Il est difficile de reproduire la disposition et les formes des cellules d'un pulvinus (il y a environ 350 000 cellules dans un pulvinus de 150 mm³). Le tissu parenchymatique du pulvinus a été idéalisé par un solide cellulaire pressurisé, la pression étant contrôlable pour chaque cellule individuellement. L'idée de cette maquette est de réaliser des mouvements complexes en contrôlant de nombreux degré de liberté (chaque case représente un degré de liberté que l'on peut gonfler ou non).

1. les mouvements les plus rapides ont été mesurés chez *mimosa pudica* avec des temps caractéristiques inférieurs à la seconde

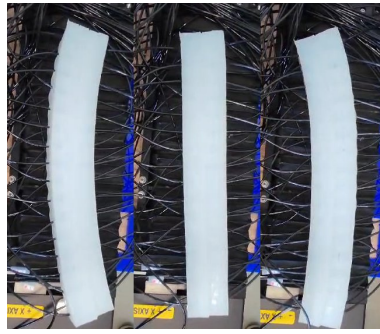


FIGURE 3.3. – Exemple d'actionneur type de pulvinus réalisé en polymères hyperélastique (Ecoflex 50, Smooth-on). Dépendant des cases gonflées, plusieurs mouvements sont possibles (ici, courbé à droite ou à gauche).

TRAVAUX FUTURS

Le défi est de réaliser un cube de 8x8x8 cases pilotables individuellement. Ce cube permettra de réaliser le premier matériau aux propriétés mécaniques entièrement ajustables (anisotropie, rigidité, coefficient de Poisson).

TRAVAUX FUTURS

Réaliser un actionneur par déformation de 64 cases avec un pilotage intelligent capables de s'adapter aux cases défaillantes (apprentissage par renforcement).

3.4. La modélisation biologique comme inspiration pour l'ingénierie

Ces expériences de reproduction de la nature pour l'ingénierie nous ont appris une chose : au delà des faux-semblant, la reproduction des mécanismes mis en place par la nature est difficile. Il faut mettre en place des instruments de pointe (technologie d'impression 3D biphotonique pour le gripper capillaire), miniaturiser et démultiplier des éléments de base (maquette de muscle), maîtriser des processus de fabrication à l'échelle microscopique pour des objets macroscopique (pulvinus artificiel).

L'intérêt principal de ces tentatives est de montrer la présence de ces barrières technologiques et nous inviter à les dépasser en inventant de nouvelles techniques. Être confronté à ces barrières permet d'isoler les concepts de base qui sont intéressants pour l'ingénierie, par exemple pour le muscle artificiel, c'est la redondance à la micro-échelle qui est importante. L'aspect "crémaillère des ponts de liaison actine myosine" – au delà d'être difficile à réaliser – ne semble pas être le concept clé qui permet la résilience musculaire. La première maquette de muscle nous a permis d'identifier les

concepts intéressants et d'écarter les solutions trop complexes et spécifiques à un contexte biologique. La nouvelle version d'actionneur biomimétique (dépôt de brevet en cours) met en avant la redondance à la micro-échelle, des solutions techniques simples, à mettre en oeuvre avec des outils actuels, mais s'écarte de la géométrie du muscle.

4. Projet scientifique : biomécanique des feuilles & Bioinspiration.

4.1. Moyen terme (à 5 ans) : biomécanique des feuilles et actionneurs multi degrés de liberté bioinspirés.

Mon parcours scientifique m'a permis d'explorer des domaines scientifiques très différents, à la frontière entre science fondamentale et appliquée. À moyen terme, mes activités vont se centrer sur deux applications : la biomécanique des feuilles et les actionneurs multi-degrés de liberté. Les questions scientifiques d'intérêt ont été marquées comme « travaux futurs » dans les deux chapitres précédents. Je propose dans ce chapitre une vision unifiée de ces travaux de recherche pour l'innovation. L'approche envisagée est mécanique-physique en s'appuyant, à chaque fois que c'est possible, sur des systèmes modèles idéalisés.

4.1.1. Genèse du projet

Ce projet a germé en expérimentant. Je prélevais des feuilles et mesurais les rigidités des pétioles (petite tige flexible qui relie le limbe de la feuille aux branches) pour comprendre l'effet de la gravité sur le port foliaire. Après la cueillette, les pétioles devenaient de plus en plus souples au cours du temps. Cet assouplissement provient de la perte de turgescence dans les cellules. Avec des yeux d'inventeur, je voyais un matériau à rigidité pilotable à la demande.

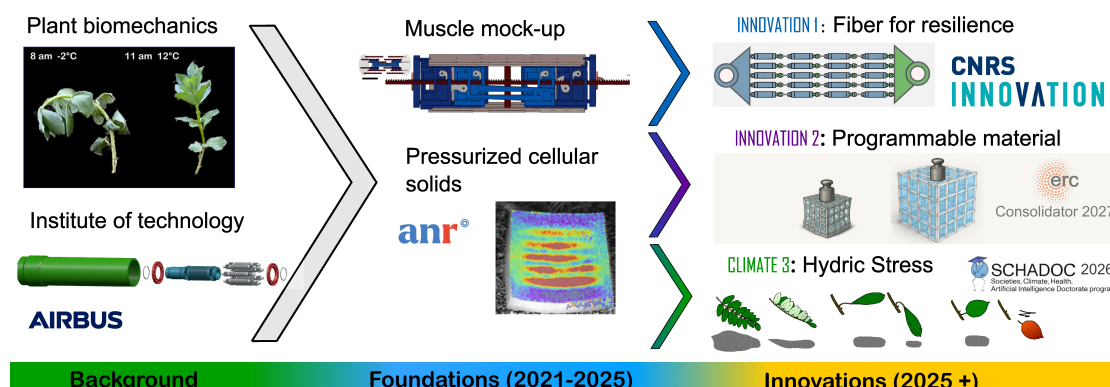
En intégrant un institut universitaire de technologie, j'ai été mis au contact d'industriels de l'aéronautique et de l'espace (Airbus, Thales) qui ont des problèmes concrets de masse et de fiabilité des actionneurs. La redondance permet d'assurer la sécurité des aéronefs mais elle pèse lourd, coûte cher et diminue l'autonomie en vol. Se passer de redondance sans rogner sur la sécurité serait un atout industriel majeur. Actuellement, il manque un actionneur instrumenté dont les performances ne s'effondrent pas en cas de dommage. En détectant un défaut sur l'actionneur, l'aéronef peut se poser pour être réparé : plus besoin de redondance. Les actionneurs biologiques sont intéressants sur ce point : le muscle est résilient, quelques courbatures n'empêchent pas de bouger ; un pulvinus, même excisé à 50%, permet le mouvement d'une feuille. Avec

de tels actionneurs, plus besoin de redondance pour assurer la sécurité des aéronefs.

En se basant sur le savoir-faire acquis sur une première maquette de muscle, nous avons créé un nouvel actionneur fibré résilient (INNOVATION 1) - projet financé par CNRS innovation. Le brevet vient d'être déposé.

À la manière des pulvini, un matériau cellulaire pressurisé (INNOVATION 2) est en cours de développement basé sur les résultats et outils développés au laboratoire : (i) un code numérique permettant de prédire la rigidité macroscopique et les mouvements. Outre l'intensité de la pression, ce modèle permet aussi de choisir les motifs de pressurisation pour changer l'anisotropie / l'auxétisme du matériau. (ii) Le développement au laboratoire d'une technique de fabrication de solide cellulaire pressurisé par impression 3D d'hydrogel et silicone.

Enfin le modèle de variation de rigidité des milieux cellulaires pressurisés peut être utilisé pour comprendre les adaptations des plantes à la sécheresse. En période de manque d'eau certaines plantes fanent ou se replient. Avec le modèle de rigidité de solide cellulaire pressurisé développé au laboratoire, nous pouvons prendre en compte la rétroaction du port foliaire sur le taux d'évaporation d'une parcelle. Cette rétroaction peut permettre d'affiner les prédictions d'embolie des arbres et permettre aux décideurs d'évaluer finement l'impact du changement climatique sur la gestion des couverts forestiers (CLIMAT 3).



4.1.2. Synopsis scientifique et industriel

Constat industriel : la redondance a un coût financier, énergétique, massif et stérique

Standards actuels de sécurité et de fiabilité : surdimensionnement et redondance. Pour assurer sécurité et fiabilité, les actionneurs critiques sont conçus avec des marges de sécurité et sont dédoublés. En cas de défaut, un actionneur redondant prend le relai. Parfois, la redondance n'est pas permise pour des raisons d'encombrement, de coût, de masse embarquée ou de consommation énergétique. Ces contraintes sont fortes pour les applications aéronautiques et spatiales.

Pour fixer les idées, la figure 1 donne un exemple de duplication d'actionneur en aéronautique : un vérin hydraulique utilisé sur les commandes de vol d'hélicoptère. Le piston vert peut être actionné par deux paires de cavités indépendantes. Si une cavité fuit, la paire de cavités est défaillante mais l'autre paire prend le relai.

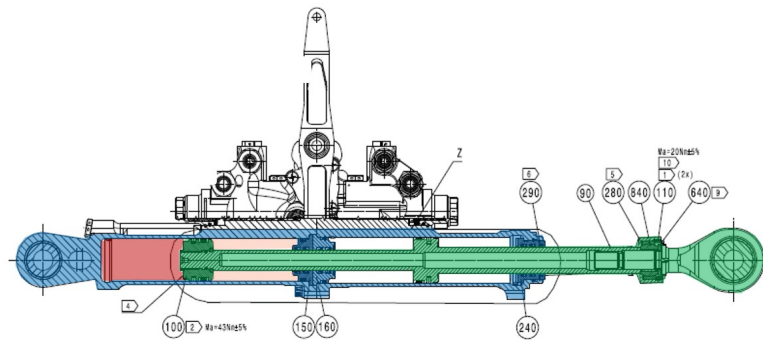


FIGURE 4.1. – Exemple de redondance utilisée dans l'industrie aéronautique. Vérin hydraulique dupliqué utilisé sur les commandes de vol d'hélicoptère. Bleu : corps du vérin. Vert : piston. Les chambres de pression (rouge et orange) sont dupliquées sur la droite.

Continuer à fonctionner malgré un défaut pour éviter la redondance. Sur un hélicoptère, par exemple, si l'actionneur de commandes de vol (orientation des pâles) commence à se détériorer avec des pertes de performances raisonnables (résilience aux dégâts), ce défaut peut être détecté et indiquer au pilote de se poser pour réparation.

Les actionneurs multi-échelle architecturés (e.g. le muscle) offrent des degrés de liberté pour la résilience. Le paradigme actuel de conception des actionneurs (action centralisée, monobloc, surdimensionnement, asservissement complexes) ne permet pas de briser les limites pour la résilience aux dommages. À contrario, les actionneurs biologiques (e.g. le muscle squelettique chez les animaux, le pulvinus chez les végétaux) possèdent une architecture décentralisée, à multiples chemins d'effort, réalisant des mouvements fluides et peu sensibles aux dommages. Les actionneurs multi-degrés de liberté permettent de s'adapter aux dégâts à la micro-échelle en répartissant les efforts sur les actionneurs voisins. Le nombre élevé de degrés de liberté permet d'ajuster

les séquences de micro-actions pour des déplacements macroscopiques fluides.

Ce projet de recherche vise à développer et maturer 2 actionneurs industriels résilients pour éviter la redondance et les surcoûts associés (financier, masse, ressources, énergie, autonomie).

Axes de recherche et d'innovation

INNOVATION 1 : RÉSILIENCE FIBRÉE

Créer un actionneur résilient pour l'industrie en remplaçant la redondance macroscopique par la redondance microscopique.

INNOVATION 2 : MATÉRIAU CELLULAIRE PILOTABLE

Créer un actionneur unique composé de cellules adjacentes pouvant réaliser des mouvements complexes et/ou changer ses propriétés mécaniques à la demande (rigidité, anisotropie, module de Poisson).

Basé sur les résultats déjà acquis sur les matériaux cellulaires pressurisés, dont le champ d'application inclut les tissus mous des plantes, le troisième axe traite de l'adaptation du couvert végétal au manque d'eau dans le contexte du réchauffement climatique global.

CLIMAT 3 : STRESS HYDRIQUE

Modéliser les effets de la modification du port foliaire en période de sécheresse pour affiner les prédictions du taux d'évaporation et les risques d'embolie à l'échelle d'une forêt.

État-de-l'art

Redondance & résilience en ingénierie

Redondance : (latin : *redundantia*, trop plein, excès. De *red* + *unda*, re + eau qui a donné *reundare* : déborder). Caractère d'un message, d'un code, présentant des répétitions, des éléments excédentaires par rapport au nombre de signes strictement nécessaires à la transmission de l'information et permettant ainsi une compréhension et une fiabilité meilleures¹. En ingénierie : duplication (voire triplication des systèmes) pour prévenir la défaillance de l'un d'entre eux.

Résilience : pas de définition évidente. Du latin *salio* (sauter) et préfixe *re-* : rebondir. Première utilisation par Francis Bacon (1626) en anglais. Passage au français au début du XX^e siècle (en 1906 sous la forme *résélie* puis dès 1911 sous sa forme actuelle). Le mot résilience n'a pas une acception unique en français. Chaque domaine scientifique a sa propre définition : en écologie, la capacité d'un écosystème à recouvrer d'une perturbation, en psychologie, la capacité d'un individu à faire face à des situations adverses, en mécanique, l'énergie maximale par unité de volume qui peut être absorbée sans dégât permanent (déformation élastique ou plastique).

1. Centre National de Ressources Textuelles et Lexicales (<https://www.cnrtl.fr/lexicographie/redondance>)

Pour les actionneurs, la résilience peut avoir deux sens différents (Perrier et al., 2022) :

1. la capacité d'un actionneur à récupérer ses fonctionnalités après un dégât - par exemple un actionneur fait en élastomère auto-cicatrisant.
2. la capacité d'un actionneur à continuer à fonctionner face à des conditions adverses/des dégâts.

La résilience et la redondance en ingénierie sont deux concepts visant la fiabilité des systèmes qui seront envisagés par les ingénieurs avec différentes méthodes, par exemple l'AMDEC (Analyse des Modes de Défaillances, de leurs Effets et de leur Criticité) ou le SSR (Structural System Reliability).

État de l'art sur les actionneurs biologiques

Nous avons montré dans un travail préliminaire que la résilience aux dégâts des actionneurs biologiques est bien meilleure que celle des actionneurs industriels (Perrier et al, 2022). La sélection naturelle explore le monde des possibles de manière stochastique, de proche en proche. Les essais de la nature sont rarement parfaits et il est nécessaire de considérer les convergences évolutives face à une pression de sélection forte, comme de bons candidats pour la bio-inspiration. Quels sont les points de convergence entre la structure d'un muscle squelettique (animal) et celle d'un pulvinus (végétal) ?

Muscle squelettique : Le muscle est un actionneur hiérarchique constitué de faisceaux de myofibres placées en parallèle et reliées ensemble au niveau du tendon. Ces myofibres sont constituées de sarcomères placés les uns à la suite des autres, en série. Chaque sarcomère est constitué de fibres d'actines et de myosine glissant l'une par rapport à l'autre. Des moteurs moléculaires (pont de liaison actine-myosine) permettent cette contraction en consommant de l'énergie chimique.

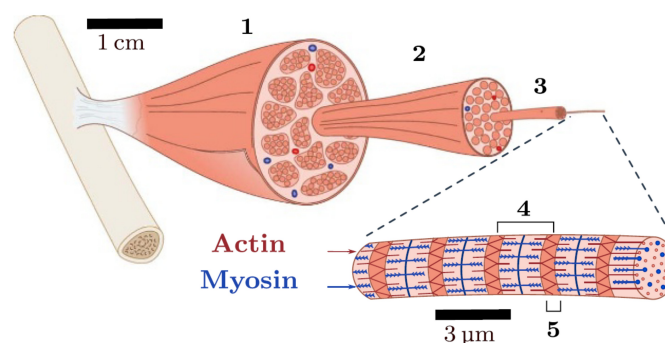


FIGURE 4.2. – Structure du muscle. 1. Muscle. 2. Myofibre. 3. Myofibrille. 4. Sarcomère. 5. Bande Z.

Pulvinus : Les plantes créent du mouvement par un écoulement d'eau contrôlé par osmose. Le pulvinus (Fig. 3a) est un coussinet à la base d'une feuille qui permet un mouvement réversible, indépendant de la croissance, et qui modifie l'orientation de

la feuille. Le pulvinus est constitué de cellules à parois minces et flexibles organisées autour d'un noyau de tissus vasculaires (xylème pour le transport de l'eau et phloème pour le transport de la sève), voir Fig. 3b.

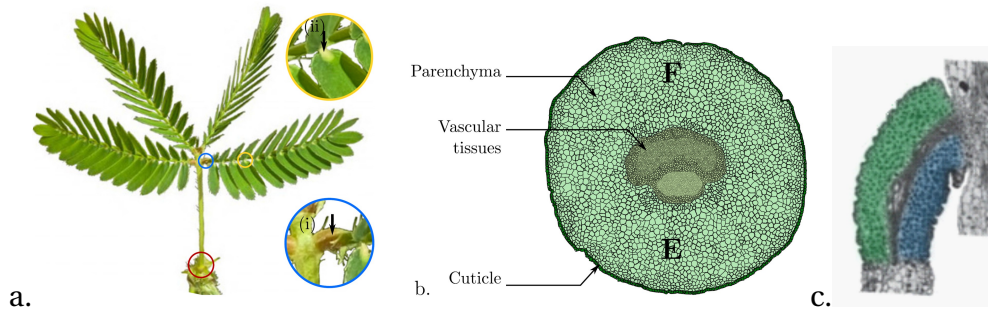


FIGURE 4.3. – a. Les pulvini sont situés à trois niveaux sur une feuille de *Mimosa pudica*. (Rouge) pulvinus de la tige, (Bleu) pulvinus du rachis et (Jaune) pulvinus du foliole. b. Coupe transversale simplifiée d'un pulvinus de *Mimosa pudica* au repos, avec les principaux éléments structurels : couche externe (cuticule), cellules motrices (parenchyme) et phloème/xylème (tissus vasculaires). E : Extensor. F : Flexor. c. Section d'un pulvinus de *Mimosa Pudica* (adapté de Campbell biology). Bleu : Flexor, cellules flasques. Vert : Extensor, cellules turgides.

Nous avons montré que parmi les angiospermes (plantes à fleurs), des plantes appartenant à des familles distinctes (oxalidacées, fabacées, marantacées) ont développé - ou conservé - un pulvinus, montrant son intérêt pour le biomimétisme (Tadrist, 2022).

Éléments de convergence structurelle entre muscle et pulvinus

Bien que très différents et issus d'histoires évolutives complexes, le muscle et le pulvinus partagent (i) une architecture multi-échelle avec des actionneurs élémentaires nombreux (sarcomères pour le muscle; cellules de parenchyme pour le pulvinus); (ii) des liens flexibles entre les actionneurs élémentaires pour répartir les efforts sur plusieurs chemins; (iii) une bonne résilience aux dégâts (continue à fonctionner malgré des actionneurs élémentaires défectueux). Ces éléments de convergence évolutive sont l'inspiration des actionneurs innovants que nous allons créer pour l'industrie.

État de l'art sur les actionneurs hiérarchiques

La mise en série et en parallèle d'un grand nombre d'actionneurs élémentaires a pour noms : actionneurs hiérarchiques, tableau d'actionneurs, ou actionneurs à multi-degrés de libertés. Ces actionneurs hiérarchiques ont prouvé leur utilité pour la résilience aux micro-dommages (Perrier, 2024), l'efficacité énergétique, la fluidité du mouvement. La mise en parallèle et en série des actionneurs élémentaires requiert des liens élastiques pour répartir les efforts à travers plusieurs chemins.

Industrialisation des actionneurs hiérarchiques : une frontière jamais franchie. Actuellement aucune version industrielle d'actionneur hiérarchique n'existe. Les pro-

totypes n'ayant pas été développés jusqu'au niveau industriel [B1, B2]. Raisons identifiées : (i) Les chemins d'effort passaient à travers l'actionneur limitant les forces admissibles. (ii) Les flexibilités inter-actionneurs sont difficiles à placer en limitant le nombre de pièces mobiles.

Actionneurs souples inspirés des plantes : une miniaturisation qui demeure un challenge industriel. Les défis de l'industrie 4.0 pour créer des actionneurs souples résistants aux dommages. Il existe une vaste littérature sur les actionneurs inspirés des plantes et utilisant divers phénomènes physiques pour mimer la turgescence (Li et Wang, 2017). La tendance actuelle chez les actionneurs baromorphes consiste à créer une géométrie complexe pouvant générer avec un seul régulateur de pression plusieurs formes complexes. La partie inspirée des plantes de ce projet prend à contre-pied cette tendance : nous considérons une géométrie simple mais comprenant un grand nombre d'actionneurs que nous combinons pour créer des mouvements et des formes complexes.

Nous avons montré qu'une membrane sphérique est rigidifiée par la pression et que le mode de pressurisation (isobare, osmotique, adiabatique, isochore) change la rigidité de la membrane (Lacorre, 2024). Cependant cette rigidification est un effet géométrique : sur un tissu composé des cellules cubiques on observe l'effet inverse (assouplissement avec la pression). Dépasseant la mécanique linéaire, nous avons montré que la rigidification chez les plantes provient d'un effet du matériau du mur cellulaire dont le comportement en traction est non-linéaire. Ce résultat a remis en cause le mécanisme de rigidification par turgescence admis jusqu'alors (Nilsson, 1958). Nous avons également montré que le pattern de pressurisation au sein du matériau permettait de faire varier l'anisotropie, la rigidité ainsi que le module de Poisson (Lacorre, 2025 en préparation pour PNAS). Ces résultats forment la base des axes « **INNOVATION 2** » et « **CLIMAT 3** ».

La limite physique pour un métamatériau pneumatique est la miniaturisation. La miniaturisation pneumatique implique des résistances importantes à l'écoulement fluide et donc une faible bande passante. La miniaturisation passe par une technique d'électro-osmose déjà maîtrisée pour la réalisation d'écrans braille ou de dispositifs tactiles. La technique actuelle permet de réaliser des valves millimétriques générant des pressions de l'ordre de quelques centaines de kPa. Aujourd'hui, cette technique est réalisée sur substrat rigide (figure 5) et le défi est de la réaliser sur un substrat flexible. L'électroosmose doit également être optimisée pour créer facilement des pressions élevées avec des basses tensions électriques (inférieures à 60 V) et des cellules souples.

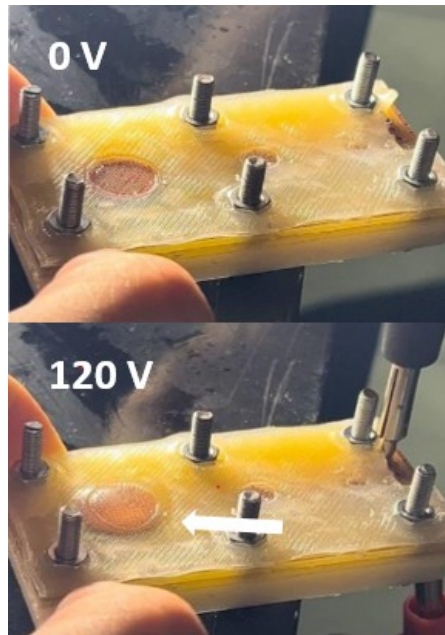


FIGURE 4.4. – Pressurisation par électro-osmose réalisée au laboratoire.

État de l'art sur la transpiration des plantes et le port foliaire

L'interception lumineuse est clé pour la photosynthèse. La photosynthèse est efficace entre 0 et 40°C, au-delà de 40°C le rendement photosynthétique s'effondre et à partir de 50°C les feuilles se dégradent. La température de surface des feuilles dépend du bilan d'énergie : la feuille reçoit du rayonnement solaire, elle en réémet une partie comme un corps noir, elle échange de la chaleur par convection et elle transpire. L'évaporation permet de refroidir efficacement les feuilles pour garantir un bon rendement photosynthétique.

Lorsque le potentiel hydrique est bas, les stomates se referment et l'arbre ne peut plus se refroidir de manière active. L'arbre peut encore modifier la manière dont le rayonnement est reçu en modifiant le port foliaire. L'interception lumineuse d'un couvert fait intervenir la distribution d'angle d'inclinaison des feuilles, $P(\varphi)$ et la densité de surface foliaire, LAI (Leaf Area Index). 3 stratégies ont été mises en évidence : la fanaison, le repli actif et le dépouillement foliaire.

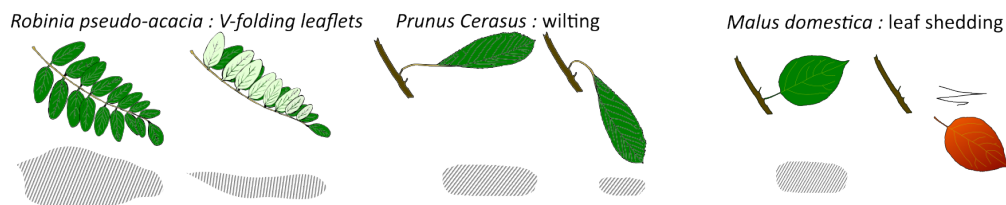


FIGURE 4.5. – Stratégies d'évolution du port foliaire face à la sécheresse. a. Repli actif. b. Fanaison passive. c. Dépouillement. Les aires grisées correspondent aux surfaces d'interception lumineuse (ombre).

En se basant sur les résultats acquis de comportement des tissus mous des plantes (solide cellulaire pressurisé) en fonction de la turgescence, nous quantifierons les changements de port foliaire et les conséquences en termes de survie en période de sécheresse.

Défis scientifiques et techniques

INNOVATION 1 : RÉSILIENCE FIBRÉE (TRL 2 VERS TRL 4 / 6)

- (i) Gérer les flexibilités inter-actionneurs;
- (ii) Gérer le pilotage de 200 actionneurs élémentaires redondants (commun innovation 2);
- (iii) Éviter les chemins d'efforts à travers les actionneurs élémentaires.

INNOVATION 2 : MATÉRIAU CELLULAIRE PILOTABLE (TRL 2 VERS TRL 4)

- (i) Miniaturiser le dispositif avec la création d'une cellule d'électroosmose souple.
- (ii) Gérer le pilotage de 200 actionneurs élémentaires redondants (commun innovation 1) pour le mouvement et les propriétés mécaniques;
- (iii) Réduire les contraintes d'utilisation (haut voltage, changer le liquide dipolaire par de l'eau).

CLIMAT 3 : STRESS HYDRIQUE

- (i) Construire un dispositif automatisé pour mesurer les variations de port foliaire lors de sécheresse et acquérir des données en environnement contrôlé et environnement réel.
- (ii) Modéliser les changements de port foliaire sur la relation entre potentiel hydrique / transpiration / rendement photosynthétique.

4.1.3. Programme de recherche et d'innovation

INNOVATION 1 : RÉSILIENCE FIBRÉE

FINANCEMENT ACQUIS : CNRS INNOVATION (2026-2028)

FINANCEMENT ENVISAGÉ : CO-DÉVELOPPEMENT AVEC AIRBUS HELICOPTERS (2028+)

COLLABORATEUR(S) : JULIEN CHAVES-JACOB (ISM, AMU)

2026-2028 : Créer un actionneur fibré constitué d'une matrice d'actionneurs élémentaires et protéger la solution mécatronique et logicielle (brevet).

- (i) Industrialisation : Les fibres supportent les charges mécaniques, la transmission de puissance électrique et l'information de la commande. Les chemins d'efforts ne transitent pas par les actionneurs mais par les fibres.
- (ii) Résilience : Beaucoup de degrés de liberté (≈ 200) pilotés par un algorithme de commande adaptatif.

WP1 : Design (1.1 Dimensionnement et Digital Twin, 1.2 Design électronique et Communication)

WP2 : Manufacture – Fabrication du prototype d'actionneur électromécanique.

WP3 : Learning - Apprentissage profond de la motricité sur simulateur numérique.

WP4 : Evaluate – Évaluation des performances du prototype.

2028+ : Co-développement envisagé avec Airbus Helicopters autour des commandes de vol (orientation des pâles) du drone hélicoptère VSR700.

- WP1 : Re-design for VSR700 integration.
- WP2 : Fatigue lifespan and resilience in laboratory.
- WP3 : Tests in aeronautical conditions.

Le co-développement est envisagé avec Airbus Helicopters car (i) l'entreprise possède le savoir-faire pour l'obtention d'accréditation de pièces mécaniques et d'actionneurs auprès de l'EASA et FAA, (ii) l'entreprise possède des installations pour les tests en fatigue et (iii) l'équipe de recherche a un passé de collaboration avec l'entreprise.

INNOVATION 2 : MATÉRIAU CELLULAIRE PILOTABLE

Financement envisagé : ERC CONSOLIDATOR (2027) / ADVANCED (>2027)

Collaborateur(s) : ANDERS THORIN (CEA), PAUL LACORRE

2022-2027 : Comprendre et mimer un milieu cellulaire pressurisé.

- WP1 : Model. Modélisation de la rigidification induite par pression. Effets de la géométrie des cellules, des comportements fluides, du comportement des murs cellulaires (strain-hardening), des patterns de pressurisation (tâche complétée à 90%).
- WP2 : Mimic. Développement d'une technique de fabrication par impression 3D pour réaliser un matériau cellulaire pilotable de 512 cellules indépendantes avec énergie pneumatique. (Actuellement sous étude de brevetabilité, tâche complétée à 80%).
- WP3 : Move. Réalisation, test et pilotage (IA) d'un matériau cellulaire pressurisé optimisé pour les mouvements par déformation (tâche complétée à 50%).

2027+ : Créer un métamatériau cellulaire pressurisé pilotable.

- WP1 : Miniaturize. Réduire la taille des cellules à pressuriser en utilisant une technique d'électroosmose. Créer des valves d'électro-osmose souples.
- WP2 : Optimize. Réduire les contraintes de fonctionnement (haut voltage, liquide dipolaire spécial, collage, résistance à la fatigue), Réduire les contraintes de fabrication et d'entretien.
- WP3 : Actuate. Contrôler jusqu'à 10^4 cellules indépendantes, détecter les cellules défectueuses et ajuster l'algorithme de pressurisation.
- WP4 : Characterize. Déterminer les conditions environnementales limites de fonctionnement de l'actionneur pour l'industrie (température, pression, rayonnement, fatigue, etc.)

2029+ : Valorisation de l'innovation (i) dépôt de propriété intellectuelle, (ii) soit co-développement avec un industriel (non-identifié), soit lancement d'une start-up en cours d'ERC.

CLIMAT 3 : STRESS HYDRIQUE

Financement envisagé : SHADOC (MARIE CURIE / AMU) (2026-2029) ; ADEME (2028-2031)

Collaborateurs : JAN PISEK (UNIVERSITY OF TARTU, ESTONIA), MARC SAUDREAU (INRAE)

Le but de ce projet est d'affiner les prédictions bio-climatiques : taux d'évaporation réel des couverts, taux d'embolie (mort d'un arbre) et rendement photosynthétique en tenant compte de l'adaptation des arbres à la sécheresse (rétroaction des plantes).

Les mouvements actifs et passifs des feuilles permettent-ils d'augmenter les rendements photosynthétiques / l'efficacité de l'utilisation de l'eau / de résister aux sécheresses et périodes de canicule? Applications aux modèles de croissance d'arbre et climatiques de long terme pour les zones touchées par la sécheresse (e.g. la côte méditerranéenne).

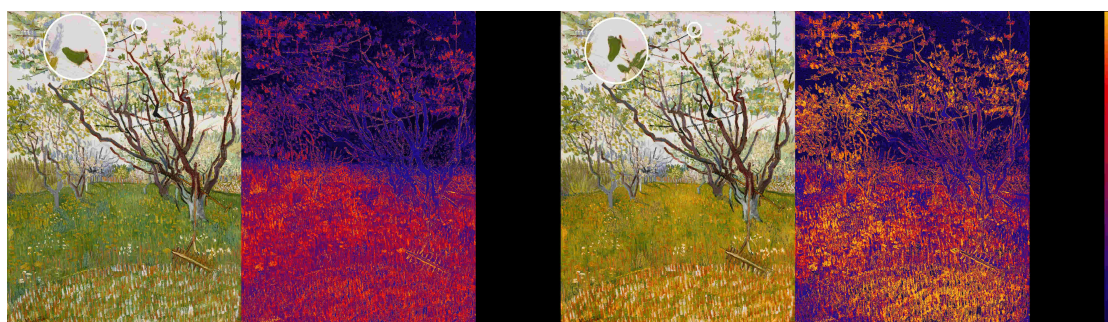


FIGURE 4.6. – Résumé graphique du projet Sitienti Arbores : Verger en fleur par Vincent van Gogh (1888). Caméra visible et thermique. À gauche : conditions bien arrosées. Les arbres et l'herbe atténuent l'élévation de la température par la transpiration. Les feuilles sont correctement orientées pour intercepter la lumière. À droite : Conditions de sécheresse. L'augmentation de la température du sol et du feuillage empêche une photosynthèse efficace. L'herbe qui se dessèche jaunit et les feuilles se flétrissent (voir zoom).

2026-2029 : Mesurer et modéliser le port foliaire en fonction des paramètres biomécaniques des feuilles d'arbre et du potentiel hydrique du sol.

- WP1 : Measure. Créer un automate de mesure d'inclinaison d'angle des feuilles par photographie (Technique Pisek). Acquisition de données en environnement contrôlé et en environnement réel.
- WP2 : Model. Modéliser bilans radiatifs des feuilles et en déduire les taux d'évaporation en fonction des distributions d'angle d'inclinaison des feuilles. (Modèle INRAE : RATP)

2028-2031 : Photosynthèse et transpiration en condition de sécheresse.

- WP1 : Model. Modéliser le rendement photosynthétique et la transpiration en fonction de (i) le potentiel hydrique du sol, (ii) les propriétés biomécaniques des feuilles, (iii) l'espèce d'arbre.
- WP2 : Map. Affiner les cartes de risque d'embolie des arbres en fonction de l'intensité de sécheresse en tenant compte de la rétroaction des arbres en turgescence (fermeture des stomates & changement de port foliaire). (Modèle INRAE :

SUREAU)

4.2. Valorisation des travaux de recherche

INNOVATION 1 : RÉSILIENCE FIBRÉE

BREVET : 2026 (ARCHITECTURE MÉCANIQUE ET SOLUTION MÉCATRONIQUE)

CO-DÉVELOPPEMENT : AIRBUS HELICOPTERS 2028+

La valorisation de cette partie du projet est déjà entamée. Nous avons sécurisé un soutien de CNRS Innovation (instrument de financement : prématuration) pour développer et protéger notre invention qui débutera en 2026. Ce soutien prématuration nous amènera à développer un actionneur de laboratoire (TRL3).

Nous envisageons ensuite de continuer à développer cet actionneur en partenariat avec Airbus Helicopters autour du cas d'étude du VSR 700. Le VSR700 est un drone hélicoptère de charge utile 150 kg réalisé pour la Marine Nationale. Nous envisageons ce cas d'étude car il est plus aisé à mettre en œuvre (charges, dimensions, autorisations, risques) que sur un hélicoptère classique.

INNOVATION 2 : MATÉRIAU CELLULAIRE PILOTABLE

BREVET : 2028/2029 (DISPOSITIF D'ÉLECTRO-OSMOSE SOUPLE ET MINIATURISÉ)

BREVET : 2029+ (MATÉRIAU CELLULAIRE PILOTABLE)

CRÉATION D'UNE START-UP : 2029+

Une première demande de brevetabilité est en cours sur le dispositif d'impression 3D de matériau cellulaire pressurisé pneumatique.

Nous comptons réaliser une cellule électroosmotique souple et la breveter en 2028/2029. De ce brevet découlera une valorisation de l'invention avec la création d'une start-up. Dans la suite, nous comptons aussi breveter un matériau mou composé de milliers de cellules (2029+).

Les cas d'application de ce matériau pilotable ne sont pas encore définis, mais on peut envisager plusieurs applications : médicales (orthèses, literie), mécaniques (actionneur résilient, liaisons à raideur variables, robotique souple) ou militaires.

CLIMAT 3 : STRESS HYDRIQUE

SOCIÉTAL :

- Rédaction d'un rapport sur le risque d'embolie forestière selon les couverts forestiers pour l'ADEME.
- Diffusion open science du dispositif de mesure des distributions d'angles d'inclinaison des feuilles.

ACADÉMIQUE : AJOUT DE FONCTIONS AUX CODES INRAE (RATP ET SUREAU)

Ce troisième axe de recherche, complémentaire à l'axe 2, n'est pas une innovation industrielle mais aide à répondre aux enjeux climatiques actuels en aidant les organismes de gestion de l'environnement (ADEME) à affiner leurs prédictions sur l'évolution des forêts et leurs recommandations sur leur gestion.

4.3. A plus long terme (> 10 ans) : les dissipateurs bioinspirés.

Enfin l'activité actionneurs bio-inspirés à son pendant du côté des dissipateurs mécaniques bio-inspirés, notamment pour les chocs. Comme pour les actionneurs les dissipateurs de chocs existent chez les animaux (les boeufs musqués par exemple pour les chocs à la tête) et chez les plantes (les écorces d'agrumes par exemple). L'approche mécanique-physique avec des systèmes modèles est peu considérée dans cet espace de science très complexe.

Les casques souples sont utilisés comme protection dans différents sports (rugby, boxe, arts martiaux) cependant aucune protection vis-à-vis des dommages au cerveau n'est démontrée. Pourquoi sont-ils inefficaces? Comment protéger le cerveau des chocs? Quelle est la grandeur physique à minimiser? Comment relier une grandeur physique liée à l'impact macroscopique aux dégâts biologiques?

4.3.1. Questions scientifiques

1. Quelles sont les principales stratégies de protection au choc à la tête chez les animaux? Les crânes des animaux subissant des chocs à la tête sont-ils plus épais? Ces animaux ont-ils des cerveaux plus petits, des cervicales plus rigides, plus de liquide cébrospinal? Pour protéger des chocs à la tête, vaut-il mieux rajouter un casque ou renforcer les muscles cervicaux qui relient la tête au reste du corps?
2. Comment la taille de la tête et le poids du corps augmentent-ils les dégâts au cerveau (allométrie)?
3. Le fluide autour du cerveau (liquide cébrospinal) et les méninges permettent-ils d'amortir les sollicitations inertielles?
4. Où et comment a lieu la dissipation d'énergie? La géométrie de la tête, notamment les zones anisotropes (corps calleux) et les zones remplies de liquides (ventricules) localisent-ils les dégâts au sein du cerveau?
5. Biomimétisme : Peut-on puiser dans la nature des solutions nouvelles pour la protection des chocs?

Quelques animaux ont été identifiés pour leur mode de vie (alimentation, reproduction, défense) qui les expose à recevoir des chocs à la tête. La dimension de la tête R de ces animaux varie sur deux décades, du centimètre au mètre, voir Table 4.1.

Parmi les animaux étudiés pour les chocs à la tête, il y a le gecko, un lézard arboricole qui peut tomber des arbres et subir un choc à la tête lors de la chute; des oiseaux, le pivert, le pic glandivore qui frappent les arbres à la recherche de nourriture, le fou du cap qui heurte à haute vitesse l'océan plusieurs fois par jour pour pêcher; certains ovins et caprins (dont les béliers, la chèvre, le moufflon canadien, le céphalophe de Maxwell et le buffle d'Afrique qui se donnent des coups de tête frontaux lors du rut. Les plus impressionnants sont les boeufs musqués (qui bien que dénommés

Animal	M (g)	R (cm)	M_B (g)	V_{CSF} (mL)	h (mm)	V_0 (ms ⁻¹)
Pic glandivore	200	1	0.18	-	0.080	7
Gecko	40 – 70	2	0.17	0.13	1.5	6
Fou du cap	2600	6	-	-	-	3 – 24
Homme	70 10 ³	10	1400	125	6.5	5
Bœuf Musqué [†]	315 10 ³	25	500	-	40	16
Baleine à Bec	5 10 ⁶	50	1800	-	-	4.5
Cachalot	50 10 ⁶	200	8000	-	-	4.5

TABLEAU 4.1. – Animaux soumis à des chocs réguliers à la tête au cours de leur existence. Les tailles de tête varient du centimètre jusqu'au mètre. Les vides correspondent aux données manquantes de la littérature. M Masse du corps de l'animal, R Rayon typique de la tête, M_B Masse du cerveau, V_{CSF} Volume de liquide cébrospinal, h épaisseur du crâne et V_0 vitesse maximale de l'animal (chute ou course). [†] Communication personnelle de Pamela Groves (Institute of Artic Biology, University of Alaska Fairbanks).

vulgairement bœufs, sont des ovins). Deux mâles de 315 kg environ se ruent l'un sur l'autre à une vitesse maximale de 16.6 m/s pour un choc frontal parfaitement dissipatif. À chaque ruade, le corps de chaque bœuf dissipe 40 000 joules en quelques dizaines de millisecondes, soit des puissances de l'ordre de quelques mégawatts!

4.3.2. Résultats préliminaires : Lois allométrique du choc à la tête.

L'animal de masse M impacte avec la tête un solide infiniment massif et rigide à une vitesse V_0 . La dynamique de l'indentation x du crâne peut être modélisée par une force linéaire de réponse à la déformation d'une coque élastique (Audoly et al., 2005),

$$M\ddot{x} = F \quad \text{avec} \quad F = C_0 \frac{Eh^2}{R} x, \quad (4.1)$$

$C_0 = 4/3(1 - \nu^2)$ où ν est le coefficient de Poisson du matériau. C_0 est proche de l'unité et on le négligera par la suite pour ce raisonnement en loi d'échelle. Pour l'Homme, avec des valeurs typiques des os du crâne, $E = 5$ GPa, $h = 6,5$ mm et $R = 10$ cm, on trouve une raideur de la boîte crânienne Eh^2/R de l'ordre de 2,1 MNm⁻¹. L'intégration de l'équation différentielle permet d'obtenir l'indentation maximale et ainsi l'accélération maximale générée par le choc,

$$x^{\max} = \left(\frac{MV_0^2 R}{Eh^2} \right)^{1/2} \quad \text{et} \quad \ddot{x}^{\max} = \left(\frac{Eh^2 V_0^2}{MR} \right)^{1/2}. \quad (4.2)$$

Un critère de résistance du crâne est qu'il ne doit pas créer une indentation impor-

tante devant la taille de la tête, $x^{\max} < \alpha R$ avec $\alpha \simeq 0,2$, en condition de choc normal. Ce critère nous permet d'obtenir la loi d'épaisseur du crâne,

$$h \simeq \sqrt{\frac{MV_0^2}{\alpha^2 ER}}. \quad (4.3)$$

Maintenant pour déterminer la loi d'épaisseur du crâne chez les mammifères, on peut remarquer que la taille de la tête est isométrique $R \propto M^{1/3}$ (Fitch, 2000) et que les vitesses de courses sont presque indépendantes de la masse de l'animal (Garland, 1983). On trouve donc

$$h \propto M^{1/3}. \quad (4.4)$$

Si l'on reporte l'épaisseur du crâne des animaux en fonction de la masse de l'animal, Figure 4.7, l'équation (4.4) est en bon accord avec les données. Cette relation, bien que dérivée par des considérations mécaniques, correspond à une isométrie de construction.

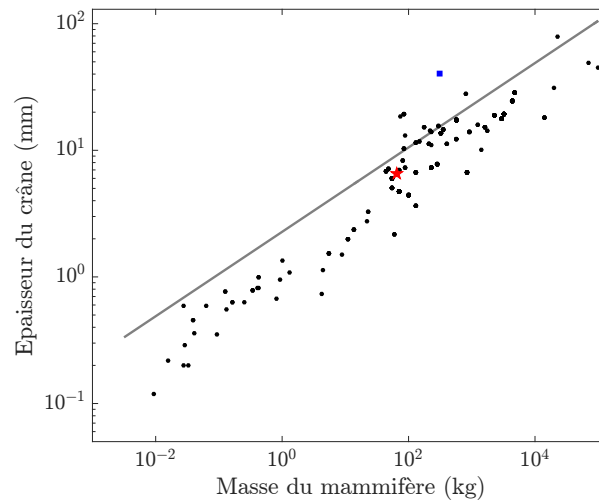


FIGURE 4.7. – Évolution de l'épaisseur du crâne en fonction de la masse de l'animal chez les mammifères. L'équation (4.3) est représentée en trait plein avec le module d'Young typique de l'os du crâne $E = 5 \text{ GPa}$, $V_0 = 5 \text{ m s}^{-1}$ et $R = \beta M^{1/3}$ with $\beta = 0,024 \text{ m/kg}^{1/3}$. L'étoile rouge correspond à l'Homme moderne et le carré bleu au boeuf musqué.

Cette analyse par loi d'échelle (allométrie) montre simplement que la construction crânienne est compatible avec une hypothèse de dimensionnement vis-à-vis du choc. La notion d'optimum est exclue ici, il faudrait analyser les bénéfices (ici résister au choc) par rapport aux risques (investir trop de biomasse ou bien avoir un crâne trop lourd à porter, ce qui est critique pour les oiseaux par exemple) pour pouvoir discuter cette notion.

Conclusion

Qu'est-ce qu'un bon système modèle ?

Il n'y a pas une réponse unique à cette question. Différents systèmes modèles peuvent être efficaces pour décrire un phénomène complexe, parfois ils sont complémentaires ; c'est d'ailleurs souvent le cas des approches par modèles réduits et codes numériques. Pour caractériser l'aérodynamique d'une aile d'avion par exemple, les modèles numériques sont très employés, peu chers, facile à mettre en oeuvre et leur robustesse a fait ses preuves. Par contre, il est parfois impossible de décrire à la fois la géométrie d'une aile de 10 m et l'état de surface de l'aile à l'échelle de quelques microns (rugosité). Les temps de calculs seraient beaucoup trop longs. Pourtant cet état de surface peut déclencher le phénomène de crise de traînée. Ici, le modèle numérique est très bon pour décrire l'aérodynamique de l'aile d'avion mais il peut être complété avec un modèle réduit de rugosité contrôlée pour décrire la transition de régime dans la couche limite. Il est plus facile de décrire ce qu'est un mauvais système modèle :

1. **Simplification.** Le système modèle ne simplifie pas le système réel, ou y apporte de nouvelles contraintes.

Exemple : Un code numérique prenant en compte la géométrie globale de l'aile d'avion et les écoulement à l'échelle micrométrique serait beaucoup trop coûteux.

2. **Physique.** Le système modèle ignore une partie de la physique du phénomène étudié.

Exemple : Un code numérique unique aura du mal à décrire la transition laminaire/turbulent dans la couche limite de l'aile d'avion.

3. **Similitudes.** Le système modèle ne respecte pas les similitudes – ou bien, les similitudes conservées ne sont pas celles pilotant le phénomène à observer.

Exemple : Sur un modèle réduit d'aile d'avion pour test en soufflerie, on n'ajuste pas la magnitude de la rugosité.

Choisir un système modèle : plutôt numérique, physique ou vivant ?

Le choix d'un système modèle doit être effectué en ayant à l'esprit : le temps de développement, la disponibilité des moyens d'essais et de mesure (leur originalité peut impliquer des développements plus longs), le coût total de l'expérience, la maturité

des technologies employées. Enfin, lorsqu'il s'agit d'expérience sur des êtres vivants modèles, les principes d'éthique doivent être respectés.

L'expérimentation physique ou sur un être vivant a un avantage important en terme de crédibilité : l'observateur est habitué à l'observation directe et doute rarement de ses sens. Lorsque l'effet recherché est clair, l'observateur est souvent convaincu. Pour le code numérique, c'est un peu différent, surtout avec l'avènement des technologies de manipulation graphique par intelligence artificielle, qui finiront par rendre les résultats numériques sujets à caution.

Quelques réflexions philosophiques sur le modèle

L'essence d'un modèle est une question philosophique de baccalauréat. Pour caricaturer, le monde se sépare entre philosophes réalistes qui pensent qu'il existe une réalité sous-jacente à un modèle théorique ; et philosophes anti-réalistes qui supposent qu'un modèle n'est un objet théorique rassemblant des observations sensorielles dans un cadre cohérent.

Cet affrontement entre réalistes et antiréalistes peut paraître anecdotique en mécanique classique, les théories se basant sur des descriptions assez fidèles de ce que nous percevons. Mais c'est bien cet affrontement philosophique qui a eu lieu à la création de la mécanique quantique, séparant les tenants d'un monde physique réel ne laissant pas la place au hasard et l'interprétation probabiliste de la mécanique quantique (école de Copenhague) où ne compte que le résultat issu du modèle. Ce débat est incarné par la célèbre phrase d'Einstein "Dieu ne joue pas aux dés" (congrès de Solvay, Belgique, 1927).

La plupart des physiciens classiques (mécaniciens inclus) sont réalistes : ils décrivent le monde tels qu'ils l'imaginent. Au contraire des mécaniciens quantiques qui décrivent le monde au moyen d'un modèle efficace qui rassemble toutes leurs observations dans un cadre théorique cohérent. C'est cet affrontement idéologique qui a soulevé l'intérêt pour les gouttes marcheuses : enfin un système dual onde-corpuscule où l'on voit ce qu'il se passe !

Les systèmes modèles sont tous basés sur une vision réaliste du monde : « les choses doivent se passer de manière similaire à ce que j'imagine, donc si j'ai bien imaginé le phénomène, je pourrai acquérir des résultats plus facilement avec ce système équivalent ». Ils sont souvent très efficaces. Que l'on soit réaliste ou anti-réaliste, ces modèles sont un outil nécessaire à l'exploration du monde.

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A. Curriculum vitae

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Areas of Interest

Plant biomechanics, Fluid-structure interaction, Fluid mechanics, Elasticity, Nonlinear physics, Mechanical actuators, Resilience of actuators.

Employment

Since 2020	Associate Professor at Aix-Marseille University, Institute of Movement Sciences (ISM), Bio-inspired systems team. France
2017–2020	FNRS Research Fellow, microfluidics lab, University of Liège, Belgium
2015 – 2017	Postdoctoral Researcher, University of Liège, Belgium
2011–2015	PhD Candidate & teaching assistant, École Polytechnique, France
2007–2011	Normalien student, École Normale Supérieure of Paris-Saclay, France

Education

2015	PhD in fluid-structure interaction and plant biomechanics, " <i>Mechanics of foliage in wind : Multi-scale approach and biological consequences</i> ", École Polytechnique, France. Highest honors
2011-2015	PhD candidate, École Polytechnique & INRAE Clermont-Ferrand, France. Supervisors : E. de Langre & M. Sautreau.
2011	Master 2 of history and philosophy of sciences, high honors, Sorbonne university, France.
2010	Master 2 of physics of liquids and soft matter, highest honors, University Paris VI.
2007-2011	Physics Department (Normalien), École Normale Supérieure de Caen, France.

Bibliometry

Web of Science (October 2025) : h-index : 11 ; citations : 318

Google scholar (October 2025) : h-index : 12 ; citations : 453.

Publications (Journal Citation Reports)

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Publications in proceedings

1. Leblanc, O., **Tadrisk, L.**, Moreau, N., Brun, B., Gilet, T., & Hackens, B. An Affordable Pedagogical Setup for Wave-particle Duality and Applications to Chaotic Stadium Cavity. *15th International Conference on Advanced Technologies, Systems and Services in Telecommunications (TELSIKS)* (pp. 398-403). *IEEE*. **2021**, Nis, Serbia.
2. L. A. Iazzolino, Y. Tourtit, A. Chafaï, T. Gilet, P. Lambert and **L. Tadrisk**, *Pick and release of micro-objects : a motion-free method to change the conformity of a capillary contact*, *MARSS Conference* **2019**, Helsinki, Finland.
3. **L. Tadrisk**, L. Motte, O.Rahli, L. Tadrisk, Preliminary study of adhesion forces of an evaporating capillary bridge, *13-th International Conference on "Two-Phase Systems for Space and Ground Applications"*, **2018**, Xi'an, China.
4. **L. Tadrisk**, M. Saudreau, E. de Langre, External loading changes Leaf Inclination Angle : Effects on direct light interception, *Functional Structural Plant Model*, **2013**, Saariselka, Finland.
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Patents and Invention Disclosure

1. Tadrisk, L.& Chaves-Jacob, J. (2025). Potentia Ex Fibrarum : Fibrous actuator for damage resilience (Invention disclosure, no. 2023-inv-0000013).SATT sud-est, Aix-Marseille University, France.
2. Tadrisk, L. (2014) Helmet integrated impact measurement device, enveloppe Soleau.

Oral communications

Invited talks

1. Tartu Observatory, University of Tartu, Estonie, 23/09/2024. *Biomechanics of leaves : wind, rain and sun*.

2. Liphy, University of Grenoble, France, 16/01/22. *Making actuators resilient : Can we draw inspiration from Nature?*.
3. Emmanuel de Langre day, Ecole Polytechnique, France, 17/05/22. *Moving leaves*.
4. Carnot Star Institute, Aix-Marseille University, online, 20/04/2021. *Bioinspired actuators and dampers*.
5. Icube, University of Strasbourg, France, 10/2019. *Preliminary works on bio-inspired dampers*.
6. LMGC, University of Montpellier, France, 11/2018. *Macroscopic wave particule duality : why tunnelling should be unpredictable?*.
7. Institut Jean Lamour, Lorraine University, France, 04/2018. *Some results about wave mechanics*.
8. Liphy, University of Grenoble, France, 07/2017. *Exotic fluid structure interactions : walking droplets and balls*.
9. LBA, University Gustave Eiffel, France, 11/2016. *Some examples of exotic fluid-structure interactions : plants, walking droplets and balls. Transition towards impacts*.
10. LBMC, University Gustave Eiffel, France, 09/2016. *Some examples of exotic fluid-structure interactions : plants, walking droplets and balls. Transition towards impacts*.

International meetings

- 2025 [T] Tadrist, L., Gilet, T. Diffraction of walking droplets by a standing wave. **Droplets 2025**, Liège, Belgium.
- 2024 [T] Tadrist, L., Lacorre, P., Fiore, L. Macroscopic mechanical properties of pressurized cellular solids. **EMMC 2024**, Madrid, Spain.
- 2022 [T] Tadrist, L., Mammadi, Y., Diperi, J., Linares, J-M., Deformation and mechanics of a biomimetic pulvinus. **Plant biomechanics 2022**, Lyon, France.
- 2019 [T] Tadrist, L., Gilet, T., Schlagheck, P., Bush, J. M. W., Where ‘unpredictability’ shades off : the tunnelling of bouncing droplets. **Droplets 2019**, Durham, United Kingdom.
- 2019 [P] Tadrist, L., A. Iazzolino, Y. Tourtit, A. Chafaï, T. Gilet and P. Lambert, Capillary forces at the light of contact conformity : pick-and-release micro- objects. **ECIS 2019**, Leuven, Belgium.
- 2018 [T] Tadrist, L., Gilet, T., Bush, J. M. W., Resolving the fast dynamics in walker dynamics in walker tunnelling : making the unpredictable predictable. **Hydrodynamics Quatum Analogues 2018**, Providence (RI), USA.

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- 2017 [T] Tadrist, L. Saudreau, M. and de Langre, E, Mechanics of foliage in wind : from leaf to foliage, **EACWE 2017**, Liège, Belgium.
- 2017 [T] Tadrist, L. Gilet, T., Bouncing dynamics leads to the chaotic interaction of two walkers, **Workshop Quantum Foundations**, Marseilles, France.
- 2017 [T] Tadrist, L. Gilet, T., Bouncing dynamics leads to the chaotic interaction of two walkers, **Waves and particles, novel insights**, Mexico city, Mexico.
- 2016 [T] Tadrist, L. Gilet, T., Stochastic interaction of walkers, **APS - Division of Fluid Dynamics**, Portland (OR), USA.
- 2016 [T] Tadrist, L. and Darbois-Textier, B., The bounce of an inflated ball, **Physics of Sports**, Palaiseau, France.
- 2015 [T] Tadrist, L. Saudreau, M. de Langre, E., Wind induced Foliage movement, **Fluid and Elasticity**, Biarritz, France.
- 2014 [T] Tadrist, L. Saudreau, M. de Langre, E. Flutter of leaves by torsional galloping, **Wind and Trees**, Águas de São Pedro, Brazil.
- 2013 [P] Tadrist, L. Saudreau, M. de Langre, E. External loading changes Leaf Inclination Angle : Effects on direct light interception, **Functional Structural Plant Model**, Saariselkä, Finland.
- 2012 [P] Tadrist, L. Saudreau, M. de Langre, E. Leaf Flexibility and Tree Geometry : Effects on Leaf Inclination Angle Distribution, **Plant Biomechanics**, Clermont-Ferrand, France.
- 2010 Forterre, Y. Quillet, C. Mahadevan, L. Richard, D. Tadrist, L. Simple experiments in thin plates : Persistence length of curvature and Snap- buckling instability. **APS March Meeting**, 2010.

[T] : Talk; [P] : Poster

National meetings

1. Journées de la matière condensée, Marseille, 28-31 octobre 2024.

National research groups

1. GDR physique des plantes. (2016, 2018, 2022, 2024)
2. Rencontres du Non linéaire. (2017, 2018, 2019)
3. GDR Mephy (Mécanique-physique). (2013, 2014, 2015, 2023)

Research Supervision

Postdoctoral Supervision

1. Paul LACORRE, 2023-2025, *Stiffening of pressurized cellular solids, finite element study*. Funding : ANR MEXP. Top A Publications : 1 (+3 in progress).
2. Younès MAMMADI, 2021-2022, *Deformation of a pressurized cellular solid, influence of the inflation pattern*. Funding : ATER. Top A Publication : 1.
Current position : Associate professor, ENSAM, Aix-en-Provence.

Doctoral supervision

1. Louison FIORE, 2023-2026, *Mechanics of pressurized cellular tissues : Movements and resilience of the pulvinus of Mimosa Pudica*. Funding : ANR MEXP. Top A Publications : 1 (+3 in preparation). In progress. S.R : 50%.
2. Raphaël PERRIER, 2021-2025, *Contribution to the realization of an actuator mimicking the multi-scale structure of skeletal muscle*. ED Sciences du mouvement humain. Top A Publications : 2. Thesis defended on 26/05/2025. S.R : 50%.
3. Juliette LEPAGNEUL, 2021-2024, *Modeling the fatigue life of a planetary roller screw in aeronautical conditions*. CIFRE Airbus Helicopters. Top A Publications : 2. Thesis defended on 12/07/2024. S.R : 50%.
Current position : Specialist engineer, Airbus Helicopters.

S.R : Supervision rate.

PhD thesis examiner

1. Kairi Adamson PhD Thesis, opponent, University of Tartu, 24/09/2024.

PhD thesis comitee member

1. Lenaïs Mauguen, Underwater-acoustic-informed neural networks, 2024-2027.

Supervision of Master 2 students

1. Mourad OUTMANI, 03/25-07/25, *Biomimetism of nervous information transport*.
2. Lucas DUBOIS, 03/24-07/24, *KombuWood. Kombucha 3D-printing*.
3. Louison FIORE, 03/23-07/23, *Mechanical properties of pressurized cellular material*.
4. Andoni IZAGUIRRE, 09/21-06/22, *Testing bench development for impacts on neural cells*. Collaborator : Santiago RIVERA (Institut de neurophysiopathologie, UMR 7051, Marseille). Erasmus Mondragon university/Aix-Marseille University
5. Raphaël PERRIER, 03/21-07/21, *Design of an elementary actuator bio-inspired of myosine-actine bounds*. Polytech Tours/Aix-Marseille Université

6. Olivier Leblanc, 09/19-06/20, Walking droplets in stadium, Catholic University of Louvain.
7. Victòr VILARDUYA, 09/18-06/19, Dynamics of leaves under rain, University of Liège.

Supervision of Master 1 students

1. Lucie BETTINGER, 04/23-06/23, Characterisation of a muscle mock-up actuator, Aix-Marseille University.
2. Elise NICOLAS, 04/22-06/22, measurement protocol to evaluate neural cells integrity and inflammation after impact, Aix-Marseille University.
3. Amina MLAIKI, 04/20-06/20, Design of an impact test bench, Aix-Marseille University.
4. Dylan PANNIER, 04/20-06/20, Allometric laws of muscle fibers, Aix-Marseille University.
5. Benoît CALICE, 04/19-07/19, Dynamic of beams under localized impact, University of Liège.
6. Kevin JULIO, 03/14-06/14, Effect of leaf shape on flutter instability, INSA Haut de France/École polytechnique.
7. Nina WANG, 03/13-06/13, Elastic reconfiguration under snow, École polytechnique.

Supervision : others (L3, BUT, TIPE)

1. Loïc ROGNON 04/25-07/25, (BUT 3 GMP), Impact of a two-mode vibration system : experimental set-up, Aix-Marseille University.
2. Sharmaine BIN SUHERRTO, 05/25-06/25, (BUT 2 GMP), Design and fabrication of a micro-osmotic pump, Aix-Marseille University.
3. Hakimi BIN MOHAMAD NIZAM, 05/25-06/25, (BUT 2 GMP), Test bench for potato parenchyma mechanical properties varying osmosis, Aix-Marseille University.
4. Eliud FORTERRE, 05/24, (TIPE), rupture of a climbing rope.
5. Emmanuel ATTALI, 04/24-07/24, (L3 ENS Rennes), energy transfer during impact : study of a two-mode system, Aix-Marseille University.
6. Danish BIN SHAIFFIRUDIN, 04/24-06/24, (BUT 2 GMP), Timelapse of a drying Basilic, (T.E. : 100%), Aix-Marseille University.
7. Souhaila MOUMANE, 04/23-06/23, WebApp creation (BUT 2 Info)

Fundings and awards

Research funding (PI) :

- 2022–2026 ANR, MExP (Machina Ex Pulvino), 319 k€
- 2022 Carnot Star, ChocCell : shock on neural cells, ≈10 k€.
- 2022 PEPS INSIS CNRS, ChocCell : shock on neural cells, 17 k€
- 2021 PhD funding (Doctoral school 353), muscle mock-up, PhD student : R. Perrier.
- 2021 PhD funding (CIFRE, Airbus Helicopters), fatigue lifespan of planetary roller-screw, PhD student : J. Lepagneul.
- 2018 FNRS mobility, Tunelling effect for walking droplets, visiting scholar at MIT (Massachusetts USA)
- 2017 Chargé de recherche (Junior researcher) at FNRS, Duo Corpora : interaction of walking droplets, FRS-FNRS, University of Liège, Belgium. 400 k€
- 2012 PhD funding (ASN), wind-foliage interaction, LadHyX, École polytechnique, France.
- 2007–2012 Normalien student, Physics department, ENS-Cachan, France.

Research funding (Collaborator) :

- 2024–2027 ANR ResiRob, PI : Stéphane Violet (ISM, Aix-Marseille Université)
- 2023–2026 A*MIDEX Activ'Plant, PI : Yoël Forterre (IUSTI, Aix-Marseille Université)

Academic funding :

- 2024 FIP 3D-NoPrint, Plastic recycling for 3D printing, 40 k€ (FIP : 28 k€, IUT : 12 k€)
Equipment : 1 shredder, 1 air dryer, 1 winding extruder, 1 thermoformer, 1 injection press.
- 2022 FIP Inproto, Rapid prototyping room, 53 k€ (FIP : 30 k€, IUT : 23 k€)
Equipment : 1 laser cutter, 7 3D-printers, 7 CAD workstations, 1 collaborative screen, 1 video projector, 1 electronic component soldering station. Upgrading of the premises (air extraction, air conditioning and disabled access).

Awards :

- 2019 Best application paper award, *Pick and release of micro-objects : An actuation free method to change the conformity of a capillary contact.*, MARSS2019, Helsinki, Finland.
- 2018 Couverture du journal *Chaos : an Interdisciplinary journal of Nonlinear Science*, Volume 28, Number 3, part 1 of 2. FRS-FNRS, Université de Liège, Belgique.
- 2015 Prix de thèse inno-vent pour la recherche innovante en ingénierie du vent. Association de l'Ingénierie du Vent, AIV
- 2009 Prix d'action internationale ENS Cachan : "*Thermotaxis of E. Coli*", Engineering Department, University of California à San Diego, USA.

Science and society

1. *Des micro-gouttes pour la vie*, with S. van Loo, I love science Festival, 2019, Bruxelles, Belgium.
2. *Biomimétisme : comprendre les plantes avec des outils d'ingénieur*, with T. Gillet, Cycle de conférence de la haute école de la province de Liège, 2018, Liège, Belgium.
3. *plantes en vibration*, with P. Hémon and E. de Langre, Fête de la science, 2013, Palaiseau, France.

B. Principales publications

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B1. Analogie hydrodynamique à la mécanique quantique

1. **Tadrif, L.**, Shim, J. B., Gilet, T., & Schlagheck, P. (2018). Faraday instability and subthreshold Faraday waves : surface waves emitted by walkers. *Journal of Fluid Mechanics*, 848, 906-945.
2. **Tadrif, L.**, Sampara, N., Schlagheck, P., & Gilet, T. (2018). Interaction of two walkers : Perturbed vertical dynamics as a source of chaos. *Chaos : An Interdisciplinary Journal of Nonlinear Science*, 28(9).

Faraday instability and subthreshold Faraday waves: surface waves emitted by walkers

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A walker is a fluid entity comprising a bouncing droplet coupled to the waves that it generates at the surface of a vibrated bath. Thanks to this coupling, walkers exhibit a series of wave–particle features formerly thought to be exclusive to the quantum realm. In this paper, we derive a model of the Faraday surface waves generated by an impact upon a vertically vibrated liquid surface. We then particularise this theoretical framework to the case of forcing slightly below the Faraday instability threshold. Among others, this theory yields a rationale for the cosine dependence of the wave amplitude to the phase shift between impact and forcing, as well as the characteristic time scale and length scale of viscous damping. The theory is validated with experiments of bead impact on a vibrated bath. We finally discuss implications of these results for the analogy between walkers and quantum particles.

Key words: Faraday waves, parametric instability, waves/free-surface flows

1. Introduction

Michael Faraday (1831) gave a first experimental description of the instability that would be named after him. The Faraday instability occurs when a container filled with liquid is vertically vibrated with an acceleration $\Gamma g \cos(\Omega t)$, where g is the gravitational acceleration and $\Omega/2\pi$ is the shaking frequency. When the dimensionless amplitude Γ is higher than a threshold Γ_F , the forcing is sufficient to counterbalance viscous dissipation (Kumar & Tuckerman 1994), the initially flat free surface destabilises and beautiful patterns of standing waves spontaneously appear. The resonance is often subharmonic so the frequency of the standing waves is $\Omega/4\pi$. The Faraday wavelength λ_F , defined as the characteristic wavelength of the standing waves, is usually close to the wavelength prescribed by the dispersion relation of gravity–capillary waves with a frequency $\Omega/4\pi$. In its original and now most studied configuration (Benjamin & Ursell 1954), the depth and lateral extension of the liquid bath are respectively much deeper and much larger than the wavelength, which ensures that the resonance is subharmonic.

Below the Faraday instability threshold, all surface waves are damped and, without any perturbation, the liquid surface remains flat. A perturbation can be added in

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the form of a bouncing droplet. If the vertical vibration of the bath is of sufficient amplitude, this droplet can bounce on the bath periodically and indefinitely (Couder *et al.* 2005a; Gilet *et al.* 2008; Molacek & Bush 2013). Couder *et al.* (2005b) showed that such droplets may excite Faraday waves, especially if they can bounce once every two forcing periods (i.e. at frequency $\Omega/4\pi$). If the forcing amplitude is slightly below the instability threshold ($\Gamma \lesssim \Gamma_F$), these waves are slowly damped and they are still present when the droplet impacts again (Eddi *et al.* 2011). Consequently, the droplet impacts on a non-flat surface and some horizontal momentum can be transferred from the wave to the droplet. The latter starts to ‘walk’ on the vibrated liquid surface. The combination of a walking droplet and the underlying surface waves is called a walker. A typical walker involves a droplet of radius $R_d = 0.4$ mm and a bath of depth 6 mm, both made of silicone oil of density $\rho = 956$ kg m⁻³, kinematic viscosity $\nu = 20$ cS and surface tension $\sigma = 20.6$ mN m⁻¹. The bath is typically shaken at a frequency $\Omega/(2\pi) = 80$ Hz with an amplitude $\Gamma \sim 4$ (table 1).

Walkers form a unique macroscopic analogue of the pilot-wave theory imagined by de Broglie (1987) as an interpretation of quantum mechanics. In a series of configurations recently summarised by Bush (2015), the behaviour of walkers is strongly reminiscent of that of quantum particles. Quantisation of bound states is observed in a central force field (Perrard *et al.* 2014a,b; Labousse *et al.* 2014, 2016; Tambasco *et al.* 2016), in a rotating frame (Fort *et al.* 2010; Eddi *et al.* 2012; Harris & Bush 2014; Oza *et al.* 2014a,b), or with two-body interactions (Protière, Boudaoud & Couder 2006; Protière, Bohn & Couder 2008; Borghesi *et al.* 2014; Filoux, Hubert & Vandewalle 2015; Durey & Milewski 2017; Oza *et al.* 2017). Walkers interact with submerged boundaries through non-specular reflection (Pucci *et al.* 2016) and tunnelling (Eddi *et al.* 2009; Carmigniani *et al.* 2014; Nachbin, Milewski & Bush 2017). More remarkably, when they are confined in a cavity, they experience chaotic trajectories for which the corresponding statistics is close to the quantum statistics predicted by the Schrödinger equation (Harris *et al.* 2013; Gilet 2014, 2016). They also seem to be diffracted when they pass one by one above a submerged slit (Couder & Fort 2006, 2012; Dubertrand *et al.* 2016), although it is not yet clear that the underlying statistics is really similar to that of a diffracted quantum particle (Andersen *et al.* 2015). Walkers therefore represent a unique opportunity to probe the boundary between the macroscopic world and the quantum realm. They allow us to identify which specific behaviours of quantum particles could be explained or not through the deterministic coupling of a wave and a particle.

An appropriate model of the damped Faraday waves emitted by the bouncing droplet is necessary to rationalise the dynamics of walkers. The first model has been proposed by Benjamin & Ursell (1954) for an inviscid fluid. Since there is no dissipation, the instability threshold is then identically zero. The Navier–Stokes equations simplify into a Matthieu equation for the interface elevation $\zeta(x, y, t)$. In the limit of infinite depth, it reads

$$\frac{d^2\zeta}{dt^2} + \left[gk + \frac{\sigma}{\rho}k^3 - \Gamma gk \cos(\Omega t) \right] \zeta = 0, \quad (1.1)$$

where k is the wavenumber. There is a subharmonic resonance for the Faraday wavenumber $k_F = 2\pi/\lambda_F$ such that:

$$\frac{\Omega^2}{4} = gk_F + \frac{\sigma}{\rho}k_F^3. \quad (1.2)$$

General parameters		Two-dimensional vector position		$\mathbf{r} = (x, y)$	
Three-dimensional vector position	$\mathbf{r} = (x, y, z)$	Time		t	
Radial distance	r	Laplace variable		s	
Dimensionless time	$\tau = \Omega t/2$	Wavenumber		k	
Wavelength	λ				
Fields					
Pressure field	$P(\mathbf{r}, t)$	Velocity field		$\mathbf{v}(\mathbf{r}, t)$	
Vertical velocity	$v_z(\mathbf{r}, t)$	Surface wave profile		$\zeta(\mathbf{r}, t)$	
Fluid parameters					
Gravity acceleration	g	m s^2	Density	ρ	956 kg m^{-3}
Surface tension	σ	mN m^{-1}	Viscosity	ν	20 cS
Input parameters					
Shaker pulsation	Ω	$2\pi \times 80$	Driving strength	Γ	
Walking velocity	V_w	m^{-1}	reduced form, v	$2V_w/\Omega$	$43.8 \text{ }\mu\text{m}$
Impact phase	τ_i	mm s^{-1}			
Output parameters					
Faraday period	T_F	25 ms	Faraday threshold	Γ_F	4.155
Faraday wavelength	λ_F	4.749 mm	Faraday wavenumber	k_F	1323 m^{-1}
Phase shift	θ_F	$\pi/4$	Damping length	l	10.2 mm
Damping time	$1/\delta_F$	-49	dimensional form	$2/\Omega\delta_F$	195 ms
Diffusion coefficient	D	2.20 mm^2	dimensional form	$\Omega D/2$	$5.5 \text{ cm}^2 \text{ s}^{-1}$
Group velocity	c_F	0.983 mm	dimensional form	$\Omega c_F/2$	24.7 cm s^{-1}
Dimensionless parameters					
Free pulsation	ω_0	1	Free damping	γ_0	0.2771
Threshold distance	$\mathcal{M}$	10	Memory number	Me	7.65

TABLE 1. Table of notations used in the paper. Subscript ₀ (respectively _F and _i) refers to the solution for a null viscosity (respectively to the solution at the Faraday threshold and to the quantities relative to the impact).

TABLE 1. Table of notations used in the paper. Subscript ₀ (respectively _F and _i) refers to the solution for a null viscosity (respectively to the solution at the Faraday threshold and to the quantities relative to the impact).

In order to account for a finite viscosity ν , a phenomenological linear damping term is added to this Matthieu equation (e.g. Kumar & Tuckerman (1994), Eddi *et al.* (2011), Moláček & Bush (2013), Blanchette (2016)):

$$\frac{d^2\zeta}{dt^2} + 2\alpha\nu k^2 \frac{d\zeta}{dt} + \left[gk + \frac{\sigma}{\rho} k^3 - \Gamma gk \cos(\Omega t) \right] \zeta = 0, \quad (1.3)$$

where α is a dimensionless factor. A slightly different model is derived by Milewski *et al.* (2015) from the Navier–Stokes equations in the limit of small viscosity, without any additional phenomenological term (Durey & Milewski 2017):

$$\frac{d^2\zeta}{dt^2} + 4\nu k^2 \frac{d\zeta}{dt} + \left[gk + \frac{\sigma}{\rho} k^3 + 4\nu^2 k^4 - \Gamma gk \cos(\Omega t) \right] \zeta = 0. \quad (1.4)$$

The wave field generated by one impact of the bouncing droplet at position $\mathbf{r}_n$ at time t_n is then approximately (Moláček & Bush 2013; Milewski *et al.* 2015)

$$\zeta_1(\mathbf{r} - \mathbf{r}_n, t - t_n) = \frac{A \sin \Phi}{\sqrt{t - t_n}} J_0(k_F |\mathbf{r} - \mathbf{r}_n|) \exp\left(-\frac{\Omega(t - t_n)}{4\pi Me}\right) \cos\left(\frac{\Omega t}{2}\right), \quad (1.5)$$

where $Me = \Omega \Gamma_F / [(8\pi\nu k_F^2)(\Gamma_F - \Gamma)]$ is defined as the memory parameter, A is a constant amplitude that represents the strength of the impact and Φ is linked (but not strictly equal) to the impact phase relative to the shaker. In the presence of obstacles, this equation (1.5) can be replaced by a more elaborate Green's function that takes the wave reflection on boundaries into account (Dubertrand *et al.* 2016). It has also been suggested in Fort *et al.* (2010), Eddi *et al.* (2011) to add a spatial damping factor $e^{-|\mathbf{r} - \mathbf{r}_n|/l}$, where $l \sim 1.6\lambda_F$ is a characteristic damping length.

The wave field generated by a walker is assumed linear, so it can be computed by summing wave contributions from successive impacts:

$$\zeta(\mathbf{r}, t) = \sum_n \zeta_1(\mathbf{r} - \mathbf{r}_n, t - t_n). \quad (1.6)$$

A walker that does not experience any interaction with either boundaries or other walkers, moves at constant horizontal speed and impacts the bath at regular times $t_n = t_i + 4\pi n / \Omega$. This is referred as the ‘stroboscopic approximation’ and it is implemented in all currently existing models of walkers, with the recent exception of that of Oza *et al.* (2017). Some models assume an instantaneous interaction of the droplet with the bath for the sake of simplicity (Eddi *et al.* 2011; Gilet 2016; Durey & Milewski 2017), whereas more sophisticated models take into account a finite time of interaction of the droplet with the bath at each impact (Moláček & Bush 2013; Milewski *et al.* 2015). When the ‘stroboscopic approximation’ is combined with the additional assumption that the horizontal translation of the walker during one rebound is much less than the Faraday wavelength λ_F , the sum of impacts can be replaced by a time integral, which yields an integro-differential equation for the walker trajectory (Oza, Rosales & Bush 2013; Bush, Oza & Moláček 2014).

There are several issues with (1.5) and its variants. First, the elevation ζ of waves created by a single impact diverges in $t \rightarrow t_n$, which is unphysical. Second, the viscosity is taken into account through the damping factor α and the spatial damping length l , both of which are empirical corrections assumed to be valid only when

viscous damping is weak. However, for the typical experimental conditions described above, the damping time $1/(4\nu k_F^2) \sim 0.007$ s is actually smaller than the Faraday period $4\pi/\Omega = 0.025$ s. So viscous damping is definitely not weak, and one needs to check how much error is made by using these low-viscosity empirical approximations. Third, Faraday waves are assumed in phase with the forcing signal, and of constant amplitude A , without any proof or experimental evidence. The dependence of A to the phase shift between the impact and the forcing signal has never been evidenced experimentally.

Without perturbations, a walker moves horizontally along a straight line, with a perfectly periodic bouncing motion. Therefore, these three limitations of (1.5) may induce discrepancies between models and experiments as soon as the walker's bouncing is perturbed, either by the presence of boundaries, other walkers or waves that it left previously. Indeed, there is then no guarantee that the bouncing motion will remain periodic, and any perturbation of the impact phase would induce a different weight of each rebound to the wave field. This weight is also strongly dependent on temporal and spatial damping factors. The latter basically defines the distance below which a significant interaction between multiple walkers is expected. A wave field constructed with different weights for each impact will necessarily be qualitatively different, and so will the resulting walker trajectory be. The aforementioned perturbations by boundaries, other walkers and self-interferences, all correspond to configurations where quantum-like phenomena were observed. They seem to be key to the analogy between walkers and quantum particles.

The main aim of this article is to obtain a quantitatively more accurate understanding of the surface waves emitted by walkers. The Faraday instability has been the subject of much theoretical work since Benjamin & Ursell (1954). Most notably, Kumar & Tuckerman (1994) solved the full linearised Navier–Stokes equations (without any assumption on viscosity or depth) with a spectral method based on Floquet's theory. They showed, by comparing curves of marginal stability, that the damped Matthieu equation (1.3) misses an important additional dissipation in the boundary layer at the liquid/air interface that only the full model can capture. However, because of the parametric forcing, the harmonics of the Floquet series are coupled to each other. Consequently, the dispersion relation can only be obtained by solving a discouraging eigenvalue problem for a matrix of infinite dimension.

The truncation of the Floquet series of Kumar & Tuckerman (1994) to four harmonics ($\pm\Omega/2$ and $\pm3\Omega/2$) has been investigated by Kumar (1996). With this truncated model, they could show that right above the threshold ($\Gamma \gtrsim \Gamma_F$), Faraday waves are always subharmonic when both the depth h and the Faraday wavelength λ_F are much larger than the size of the boundary layer $\sqrt{2\nu/\Omega}$. This is the case for walkers since $\sqrt{2\nu/\Omega} \simeq 300$ $\mu\text{m} \ll h \simeq 6$ mm and $\lambda_F \simeq 4.7$ mm. Special wave patterns, including solitons, appear at the bicritical point where the boundary layer thickness is of the same order of magnitude as either the depth h or the wavelength λ_F (Lioubashevski, Arbell & Fineberg 1996; Wagner, Müller & Knorr 2003). An extreme truncation to only the subharmonic response $\pm\Omega/2$ has been performed by Beyer & Friedrich (1995) as well as Müller *et al.* (1997). Müller *et al.* (1997) could then derive an analytical expression for the Faraday threshold Γ_F in the limit of low viscosity (i.e. $\nu k^2/\Omega \ll 1$) and large depth.

In this paper, we follow the approach of Müller *et al.* (1997) to describe the Faraday waves emitted by walkers. In § 2, we first derive an analytical model of slightly damped Faraday waves, while keeping control on the approximations that are made (e.g. the approximation of low viscosity). We then calculate the wave field

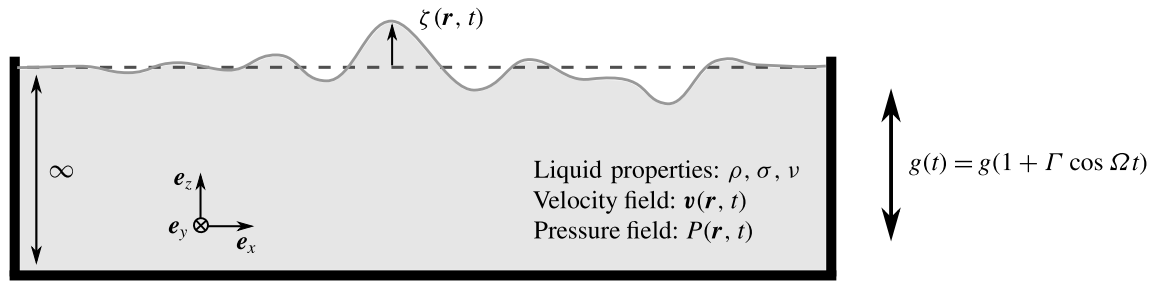


FIGURE 1. A liquid bath of infinite depth is vibrated vertically. The liquid experiences a time varying effective gravity $g(t)$. The elevation of the liquid surface is denoted $\zeta(x, y, t) = \zeta(\mathbf{r}, t)$. The velocity and pressure fields inside the liquid bath are $\mathbf{v}(\mathbf{r}, t)$ and $P(\mathbf{r}, t)$, respectively.

generated by a series of impacts. In §3, we report the results of experiments on Faraday waves excited by an impacting bead. We show that our model goes beyond the identified limitations of (1.5) currently used to describe walkers. In §4, we finally discuss the implication of this wave model for the development of the analogy between walkers and quantum particles.

2. Surface wave theory for walkers

2.1. The surface wave equation

We consider an infinitely extended and infinitely deep Newtonian liquid with density ρ , surface tension σ and viscosity ν (figure 1). This liquid is incompressible, i.e. $\nabla \cdot \mathbf{v}(\mathbf{r}, t) = 0$, where $\mathbf{v}$ is the velocity field within the liquid. It is subjected to a periodic vertical vibration of frequency Ω and acceleration amplitude Γg , where $g = 9.81 \text{ m s}^{-2}$ is the gravitational acceleration. Within the comoving frame, the liquid is then effectively experiencing a time-periodic modulation of gravity according to

$$g(t) = g[1 + \Gamma \cos(\Omega t)]. \quad (2.1)$$

The spatio-temporal evolution of the liquid is governed by the Navier–Stokes equation,

$$\frac{\partial}{\partial t} \mathbf{v}(\mathbf{r}, t) + (\mathbf{v}(\mathbf{r}, t) \cdot \nabla) \mathbf{v}(\mathbf{r}, t) = -\frac{1}{\rho} \nabla P(\mathbf{r}, t) - g(t) \mathbf{e}_z + \nu \nabla^2 \mathbf{v}(\mathbf{r}, t), \quad (2.2)$$

where $\mathbf{r} = [x, y, z]$ and $P(\mathbf{r}, t)$ denotes the pressure within the liquid relative to the atmospheric pressure. Without loss of generality, we choose a coordinate system whose origin is located at the surface of the liquid at rest, with the z -axis oriented upwards (i.e. opposite to the direction of the gravitational force). We consider small deviations from the stationary equilibrium state $\mathbf{v}^{(eq)}(\mathbf{r}, t) = 0$ and $P^{(eq)}(\mathbf{r}, t) = -\rho g(t)z$ of the liquid. Under the assumption that the amplitude of the resulting elevation ζ of the free surface is much smaller than both the thickness of the viscous boundary layer $\sqrt{2\nu/\Omega}$ and the Faraday wavelength λ , we may linearise the Navier–Stokes equation (2.2):

$$\frac{\partial}{\partial t} \mathbf{v}(\mathbf{r}, t) - \nu \nabla^2 \mathbf{v}(\mathbf{r}, t) = -\frac{1}{\rho} \nabla p(\mathbf{r}, t), \quad (2.3)$$

where $p(\mathbf{r}, t) = P(\mathbf{r}, t) + \rho g(t)z$ denotes the pressure perturbation. As discussed in appendix F, the conditions of linearisation are satisfied for experimental realisations of walkers. Indeed, the measured wave amplitude is smaller than $12 \mu\text{m}$ (Damiano *et al.* 2016), which is at least one order of magnitude smaller than both the thickness of the viscous boundary layer ($\sim 300 \mu\text{m}$) and the Faraday wavelength ($\sim 4.7 \text{ mm}$). Owing to the incompressibility of the liquid, p satisfies the Laplace equation $\nabla^2 p(\mathbf{r}, t) = 0$.

The boundary conditions at the surface $z=0$ of the liquid are also obtained under the assumption that the liquid is only weakly perturbed: the wave amplitude is assumed much smaller than both the characteristic wavelength and the boundary layer thickness. The absence of tangential shear stress at the surface can then be expressed as

$$\frac{\partial v_z}{\partial u}(x, y, 0, t) + \frac{\partial v_u}{\partial z}(x, y, 0, t) = 0, \quad (2.4)$$

with $u = x$ or y , which together with the incompressibility condition yields the boundary condition

$$\frac{\partial^2 v_z}{\partial z^2}(x, y, 0, t) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) v_z(x, y, 0, t) \quad (2.5)$$

for the vertical component of the liquid velocity. The pressure, on the other hand, satisfies the boundary condition

$$P(x, y, 0, t) = 2\nu\rho \frac{\partial v_z}{\partial z}(x, y, 0, t) - \sigma \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \zeta(x, y, t) + P^{ext}(x, y, t), \quad (2.6)$$

at the surface of the liquid. Here $\zeta(x, y, t)$ denotes the surface wave profile, i.e. the local elevation of the surface with respect to the equilibrium level $z=0$. It satisfies the kinematic condition

$$\frac{\partial}{\partial t} \zeta(x, y, t) = v_z(x, y, 0, t). \quad (2.7)$$

The pressure $P^{ext}(x, y, t)$ represents the spatio-temporal pressure profile that is induced from above the liquid, e.g. by the impact of a droplet on the surface (Milewski *et al.* 2015). This gives rise to the boundary condition

$$\begin{aligned} p(x, y, 0, t) = & 2\nu\rho \frac{\partial v_z}{\partial z}(x, y, 0, t) - \sigma \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \zeta(x, y, t) + \rho g(t)\zeta(x, y, t) \\ & + P^{ext}(x, y, t) \end{aligned} \quad (2.8)$$

for the local pressure perturbation.

Since the lateral extension of the bath is here supposed infinite, the above equations can be simplified by means of a Fourier ansatz defined through

$$X(x, y, z, t) = \frac{1}{2\pi} \int dk_x dk_y X_{\mathbf{k}}(z, t) \exp[i(k_x x + k_y y)], \quad (2.9)$$

where X is any of the variables ζ , p , v_z or P^{ext} , and where the horizontal wave vector $\mathbf{k} = (k_x, k_y)$. Defining $k = |\mathbf{k}|$, the Laplace equation for the pressure perturbation is then expressed as

$$\left(\frac{\partial^2}{\partial z^2} - k^2 \right) p_{\mathbf{k}}(z, t) = 0. \quad (2.10)$$

This equation is straightforwardly solved as $p_k(z, t) = p_k(0, t)e^{kz}$ using the boundary conditions

$$p_k(0, t) = 2\nu\rho \frac{\partial v_{z,k}}{\partial z}(0, t) + [\rho g(t) + \sigma k^2]\zeta_k(t) + P_k^{ext}(t) \quad \text{and} \quad p_k(-\infty, t) = 0. \quad (2.11a, b)$$

The Fourier transform of the linearised Navier–Stokes equation (2.3) then yields

$$\left(\frac{\partial}{\partial t} - \nu \frac{\partial^2}{\partial z^2} + \nu k^2 \right) v_{z,k}(z, t) = - \left\{ 2\nu \frac{\partial v_{z,k}}{\partial z}(0, t) + \left[g(t) + \frac{\sigma}{\rho} k^2 \right] \zeta_k(t) \right\} k e^{kz} - \frac{k}{\rho} P_k^{ext}(t) e^{kz}, \quad (2.12)$$

with boundary conditions

$$\frac{\partial}{\partial t} \zeta_k(t) = v_{z,k}(0, t) \quad (2.13)$$

and

$$\frac{\partial^2 v_{z,k}}{\partial z^2}(0, t) = -k^2 v_{z,k}(0, t). \quad (2.14)$$

We now normalise the main variables with the characteristic time scale $2/\Omega$, and we define:

$$\tau = \frac{\Omega}{2} t, \quad w_k = \frac{2}{\Omega} v_{z,k}, \quad \Pi_k = \frac{4k}{\Omega^2 \rho} P_k^{ext}, \quad \gamma_k = \frac{4\nu k^2}{\Omega} \quad \text{and} \quad \omega_k^2 = \frac{4(gk + (\sigma/\rho)k^3)}{\Omega^2}. \quad (2.15a-e)$$

With this set of notations, equations (2.12)–(2.14) become,

$$\left(\frac{\partial}{\partial \tau} - \frac{\gamma_k}{2k^2} \frac{\partial^2}{\partial z^2} + \frac{\gamma_k}{2} \right) w_k(z, \tau) = - \left\{ \frac{\gamma_k}{k} \frac{\partial w_k}{\partial z}(0, \tau) + \left[\omega_k^2 + \frac{4\Gamma gk}{\Omega^2} \cos(2\tau) \right] \zeta_k(\tau) \right\} e^{kz} - \Pi_k(\tau) e^{kz},$$

$$\frac{\partial}{\partial \tau} \zeta_k(\tau) = w_k(0, \tau) \quad \text{and} \quad \frac{\partial^2 w_k}{\partial z^2}(0, \tau) = -k^2 w_k(0, \tau). \quad (2.16)$$

We now define the Laplace transform on the dimensionless time τ with zero initial condition (bath surface at rest),

$$w_{k,s}(z) = \int_0^\infty w_k(z, \tau) e^{-s\tau} d\tau \quad (2.17)$$

and apply it to the three equations (2.16):

$$\left(s - \frac{\gamma_k}{2k^2} \frac{\partial^2}{\partial z^2} + \frac{\gamma_k}{2} \right) w_{k,s}(z) = - \left\{ \frac{\gamma_k}{k} \frac{\partial w_{k,s}}{\partial z}(0) + \omega_k^2 \zeta_{k,s} + \frac{2\Gamma gk}{\Omega^2} (\zeta_{k,s+2i} + \zeta_{k,s-2i}) \right\} e^{kz} - \Pi_{k,s} e^{kz},$$

$$s \zeta_{k,s} = w_{k,s}(0) \quad \text{and} \quad \frac{\partial^2 w_{k,s}}{\partial z^2}(0) = -k^2 w_{k,s}(0). \quad (2.18)$$

The solution to the above differential equation can be expressed as

$$w_{k,s} = [-\gamma_k e^{\sqrt{1+2s/\gamma_k} kz} + (s + \gamma_k) e^{kz}] \zeta_{k,s}, \quad (2.19)$$

provided that $\zeta_{k,s}$ satisfies the relation

$$f_k(s) \zeta_{k,s} + \frac{2\Gamma gk}{\Omega^2} (\zeta_{k,s+2i} + \zeta_{k,s-2i}) + \Pi_{k,s} = 0, \quad (2.20)$$

where we define the function

$$f_k(s) = (s + \gamma_k)^2 - \gamma_k^{3/2} \sqrt{\gamma_k + 2s} + \omega_k^2. \quad (2.21)$$

This function and all its derivatives are self-conjugated, i.e. $f_k(z^*) = (f_k(z))^*$. Similarly to Müller *et al.* (1997), we finally obtain the equation for surface waves of a shaken bath in the time domain, by back Laplace transforming the previous equation,

$$\left[\frac{\partial^2}{\partial \tau^2} + 2\gamma_k \frac{\partial}{\partial \tau} + \gamma_k^2 + \omega_k^2 \right] \zeta_k(\tau) - \frac{\gamma_k^{3/2}}{\sqrt{2\pi}} \int_{-\infty}^{\tau} \frac{e^{-\gamma_k(\tau-u)/2}}{\sqrt{\tau-u}} \left(\gamma_k \zeta_k(u) + 2 \frac{\partial \zeta_k}{\partial u}(u) \right) du + \frac{4\Gamma gk}{\Omega^2} \cos(2\tau) \zeta_k(\tau) + \Pi_k(\tau) = 0, \quad (2.22)$$

given that $\mathcal{L}^{-1}\{1/\sqrt{\gamma_k + 2s}\} = e^{-\gamma_k \tau/2}/\sqrt{2\pi \tau}$. Equation (2.22) can be integrated numerically to obtain the time dependency of each Fourier component k of the surface wave elevation. Several terms of this equation are identical to terms of the equation (1.4) obtained by Milewski *et al.* (2015) in the limit of low viscosity. However, the convolution integral in (2.22), which is an additional signature of viscous shear, is missing in (1.4).

2.1.1. Waves on an unshaken bath in the absence of external perturbation

In the absence of any external perturbation of the liquid (i.e. for $\Gamma = 0$ and $\Pi_{k,s} = 0$ for all k), the surface wave equation (2.20) becomes $f_k(s) \zeta_{k,s} = 0$. The condition of existence of waves $\zeta_{k,s} \neq 0$ leads to the dispersion relation of free viscous gravity-capillary waves:

$$f_k(s) = (s + \gamma_k)^2 - \gamma_k^{3/2} \sqrt{\gamma_k + 2s} + \omega_k^2 = 0. \quad (2.23)$$

Equation (2.23) can be solved for s in the low-viscosity limit $\gamma_k \ll 1$:

$$\text{Re}(s) = -\gamma_k \left\{ 1 - \frac{1}{2} \left(\frac{\gamma_k}{\omega_k} \right)^{1/2} - \frac{1}{8} \left(\frac{\gamma_k}{\omega_k} \right)^{3/2} - \frac{1}{64} \left(\frac{\gamma_k}{\omega_k} \right)^{5/2} + O \left(\left(\frac{\gamma_k}{\omega_k} \right)^{9/2} \right) \right\}, \quad (2.24)$$

$$\text{Im}(s) = \omega_k \left\{ 1 - \frac{1}{2} \left(\frac{\gamma_k}{\omega_k} \right)^{3/2} + \frac{1}{8} \left(\frac{\gamma_k}{\omega_k} \right)^{5/2} - \frac{1}{64} \left(\frac{\gamma_k}{\omega_k} \right)^{7/2} + \frac{1}{8} \left(\frac{\gamma_k}{\omega_k} \right)^4 + O \left(\left(\frac{\gamma_k}{\omega_k} \right)^{9/2} \right) \right\}, \quad (2.25)$$

where $\text{Re}(s)$ and $\text{Im}(s)$ represent respectively the damping and the frequency of the waves. The coefficients γ_k and ω_k therefore correspond to the first-order damping and frequency of gravity-capillary waves in the limit of low viscosity.

2.2. Faraday waves, just below the Faraday threshold

We now consider Faraday waves on a bath that is shaken just below the Faraday threshold Γ_F , i.e. in the conditions experienced by walkers. The goal of this section is to determine the damping and the phase shift of these waves. When they are subharmonic (as observed experimentally), they can be obtained by solving (2.22) with the Floquet ansatz

$$\zeta_k(t) = \sum_{l=-\infty}^{\infty} \zeta_k^{(l)} e^{(2l-1)i\tau} e^{\delta_k \tau}, \quad (2.26)$$

where δ_k is a small complex perturbation from the subharmonic resonance frequency that should vanish when the instability threshold is reached. Equivalently in the Laplace domain,

$$\zeta_{k,s} = \sum_{l=-\infty}^{\infty} \frac{\zeta_k^{(l)}}{s + i(1 - 2l) - \delta_k}. \quad (2.27)$$

Obtaining the exact solution to (2.20) from this ansatz requires solving an infinite system of algebraic equations where all the Floquet components $\zeta_k^{(l)}$ are coupled to each other (Kumar & Tuckerman 1994). As already suggested by Müller *et al.* (1997), we can simplify this ansatz by restricting it to the two most relevant Floquet components $\zeta_k^{(0)}$ and $\zeta_k^{(1)}$ of the wave, namely those that are associated with angular frequencies in the vicinity of $\pm\Omega/2$. This truncated Floquet ansatz can be written as

$$\left. \begin{aligned} \zeta_k(\tau) &\simeq (\zeta_k^{(1)} e^{i\tau} + \zeta_k^{(0)} e^{-i\tau}) e^{\delta_k \tau}, \\ \zeta_{k,s} &\simeq \frac{\zeta_k^{(1)}}{s - i - \delta_k} + \frac{\zeta_k^{(0)}}{s + i - \delta_k}. \end{aligned} \right\} \quad (2.28)$$

It is shown in appendix A that this truncation to the two first terms is valid as long as $|f_k(i)|^2/|f_k(3i)|^2 \ll 1$, which is true for wavenumbers $k \sim k_F$ in the low-viscosity limit.

In the absence of external pressure applied on the bath, $\Pi_k(\tau) = 0$, substitution of this ansatz into (2.20) and evaluation in $s = -i + \delta_k$ and $s = i + \delta_k$ yields

$$f_k(i + \delta_k) \zeta_k^{(1)} + \frac{2\Gamma g k}{\Omega^2} \zeta_k^{(0)} = 0, \quad (2.29)$$

$$f_k(-i + \delta_k) \zeta_k^{(0)} + \frac{2\Gamma g k}{\Omega^2} \zeta_k^{(1)} = 0. \quad (2.30)$$

These two equations admit non-trivial solutions if and only if

$$f_k(i + \delta_k) f_k(-i + \delta_k) - \left(\frac{2\Gamma g k}{\Omega^2} \right)^2 = 0. \quad (2.31)$$

2.2.1. Faraday instability

The special choice $\delta_k = 0$ in (2.31) yields the condition for the emergence of self-sustained Faraday waves at the subharmonic resonance. The dimensionless driving acceleration Γ_k necessary to make the wavenumber k unstable is then given by

$$\Gamma_k = \frac{\Omega^2 |f_k(i)|}{2gk} \quad (2.32)$$

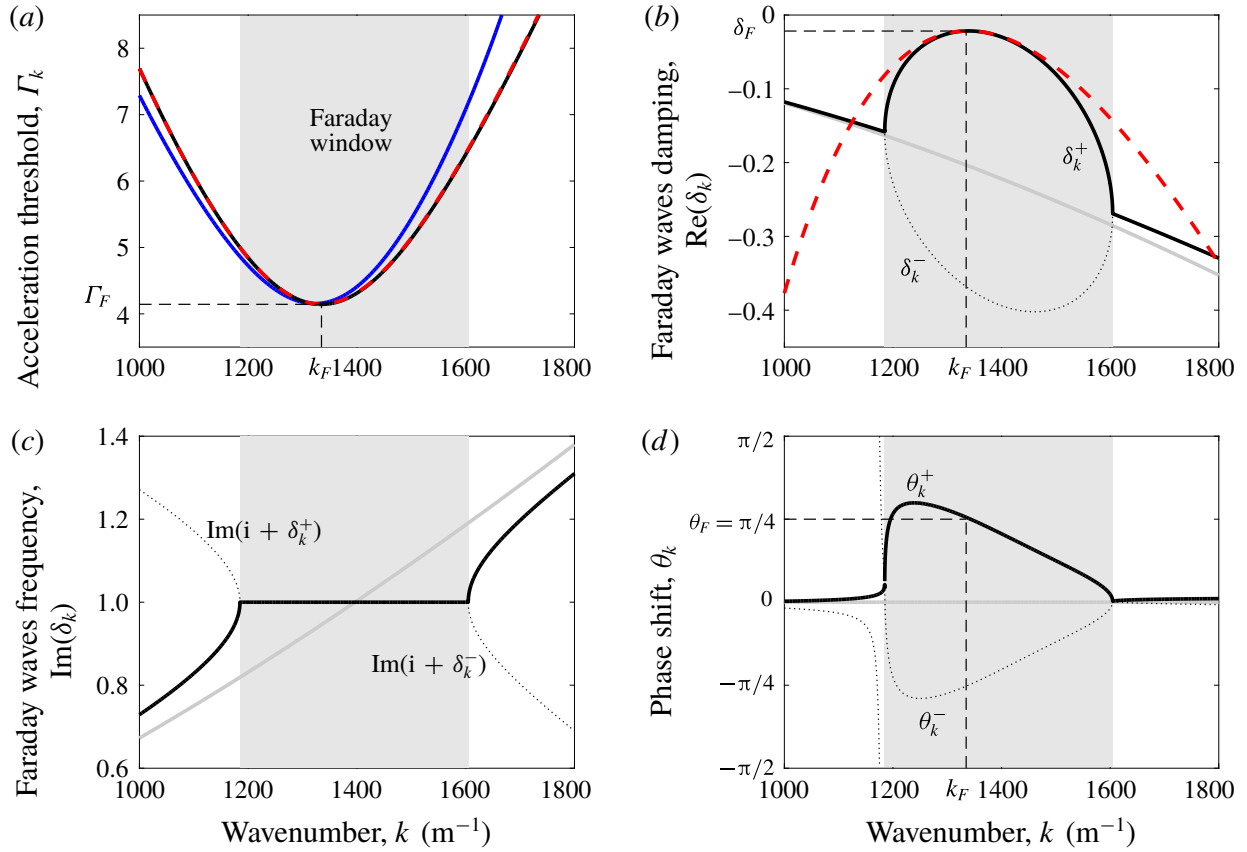


FIGURE 2. (Colour online) (a) Acceleration threshold Γ_k required to sustain waves of wavenumber k . The minimum of Γ_k gives the most unstable wavenumber, k_F . (b) Damping of the Faraday waves $\text{Re}(\delta_k)$. (c) Frequency of the Faraday wave $\text{Im}(i + \delta_k)$. (d) Phase shift θ_k of the Faraday waves with respect to the forcing signal. In (a), the blue curve represents the solution of the Floquet's ansatz with truncation to 4 harmonics (A 10), while the black curve corresponds to the truncation to 2 harmonics (2.32). In (b,c) the dotted line and the thick solid line represent equations (2.38) and (2.39), respectively. In (d), they represent (2.47). In (a,b) the red dashed curve is the low-viscosity approximation of the truncation to two harmonics (B 17) and (B 22). In (b,c) the damping and frequency of unforced waves ((2.24) and (2.25), respectively) are represented in light grey. In (a–d) all equations have been evaluated with numerical values of table 1. In particular, the distance to threshold $\mathcal{M} = (\Gamma_F - \Gamma)/\Gamma_F = 10$. The grey shade indicates the Faraday window, i.e. the range of wavenumbers k that produce waves locked to half the driving frequency.

since $f_k(s)$ is self-conjugated. This function Γ_k is represented in figure 2(a). There is always a global minimum of Γ_k that corresponds to the most unstable wavenumber, k_F :

$$\left. \frac{\partial \Gamma_k}{\partial k} \right|_{k_F} = 0. \quad (2.33)$$

The corresponding acceleration threshold is denoted $\Gamma_F = \Gamma_k(k_F)$. The effect of the truncation is seen in figure 2(a), where (2.32) is compared to the solution of the Floquet ansatz with a truncation to higher-order terms (appendix A). Close to the Faraday threshold k_F , the difference between the two truncations becomes negligible.

2.2.2. Waves just below the Faraday threshold

When the driving acceleration Γ is tuned slightly below the Faraday threshold Γ_F , some small complex detuning δ_k should arise for the wavenumbers $k \sim k_F$, as prescribed by (2.31). A good approximation of this detuning can be captured by expanding the function $f_k(\pm i + \delta_k)$ to the second order in δ_k :

$$f_k(\pm i + \delta_k) = f_k(\pm i) + \delta_k \dot{f}_k(\pm i) + \delta_k^2 \ddot{f}_k(\pm i)/2, \quad (2.34)$$

where $\dot{f}_k = \partial f_k / \partial s$. Substitution in (2.31) yields

$$\begin{aligned} & \frac{\delta_k^2}{2} (f_k(i) \ddot{f}_k(-i) + f_k(-i) \ddot{f}_k(i) + 2 \dot{f}_k(i) \dot{f}_k(-i)) \\ & + \delta_k (f_k(i) \dot{f}_k(-i) + f_k(-i) \dot{f}_k(i)) + \left(\frac{2gk}{\Omega^2} \right)^2 (\Gamma_k^2 - \Gamma^2) = 0. \end{aligned} \quad (2.35)$$

The coefficients of this second-order polynomial in δ_k are all real, because f_k and its derivatives are self-conjugated. If the corresponding discriminant is positive, i.e. if

$$\Gamma_k^2 - \Gamma^2 < \left(\frac{\Omega^2}{2gk} \right)^2 \frac{(f_k(i) \dot{f}_k(-i) + f_k(-i) \dot{f}_k(i))^2}{2(f_k(i) \ddot{f}_k(-i) + f_k(-i) \ddot{f}_k(i) + 2 \dot{f}_k(i) \dot{f}_k(-i))}, \quad (2.36)$$

then both solutions $\delta_k \in \text{Re}$ and the resulting waves (cf. (2.28)) are locked at half the driving frequency, i.e. at $\Omega/2$ (figure 2c). For a given $\Gamma \lesssim \Gamma_F$, equation (2.36) is valid for a small range of k around k_F that we call the Faraday window (shaded regions in figure 2). Outside this range, waves are travelling waves with group velocity different from 0, and their frequency is detuned from the forcing frequency since the corresponding δ_k is complex. The latter waves compound the capillary wave packet by opposition to standing Faraday waves.

Equation (2.35) admits two solutions

$$\delta_k^\pm = -\frac{b}{2a} \left(1 \mp \sqrt{1 - \frac{4ac}{b^2} (\Gamma_k^2 - \Gamma^2)} \right), \quad (2.37)$$

where coefficients a , b and c are obtained by identification of (2.35) with $a\delta_k^2 + b\delta_k + c(\Gamma_k^2 - \Gamma^2) = 0$. Since $\Gamma_k \simeq \Gamma$ inside the Faraday window, these solutions can be expanded in Taylor series for $|4ac(\Gamma_k^2 - \Gamma^2)/b^2| \ll 1$. To the first order,

$$\delta_k^- = -\frac{b}{a} \left[1 + O\left(\frac{ac(\Gamma_k^2 - \Gamma^2)}{b^2}\right) \right] \simeq -\frac{2(f_k(i) \dot{f}_k(-i) + f_k(-i) \dot{f}_k(i))}{f_k(i) \ddot{f}_k(-i) + f_k(-i) \ddot{f}_k(i) + 2 \dot{f}_k(i) \dot{f}_k(-i)}, \quad (2.38)$$

and

$$\delta_k^+ = -\frac{c(\Gamma_k^2 - \Gamma^2)}{b} \left[1 + O\left(\frac{ac(\Gamma_k^2 - \Gamma^2)}{b^2}\right) \right] \simeq -\frac{(2gk/\Omega^2)^2 (\Gamma_k^2 - \Gamma^2)}{f_k(i) \dot{f}_k(-i) + f_k(-i) \dot{f}_k(i)}. \quad (2.39)$$

The real and imaginary parts of these solutions $\delta_k^\pm$ are represented in figures 2(b) and 2(c), respectively. The imaginary part $\text{Im}(\delta_k^\pm)$ corresponds to some detuning from the Faraday frequency $\Omega/2$, and it is identically 0 in the Faraday window around k_F . The corresponding waves are standing. Outside from this Faraday window, $\text{Im}(\delta_k^\pm + i)$

increases with k , so the group velocity is positive and the solution corresponds to travelling waves. Inside the Faraday window, the real part of δ_k^- remains largely negative so it describes Faraday waves that are strongly damped and that vanish quickly. By contrast, when Γ approaches Γ_F , the real part of δ_k^+ can be arbitrarily close to 0 for $k \simeq k_F$, in the Faraday window. Consequently, δ_k^+ describes some long-lived standing Faraday waves. These latter waves are those observed to travel with the walker, and they are crucial to the description of its dynamics since their amplitude remains significant during many rebounds.

The memory Me of the waves, initially described by Eddi *et al.* (2011), is defined as the damping time of these long-lived Faraday waves, expressed in the number of rebounds of the droplet. Since the bouncing period corresponds to the Faraday period $T_F = 4\pi/\Omega$, the memory is given by

$$Me = \frac{-1}{2\pi\delta_F} \simeq \frac{1}{4\pi} \frac{f_F(i)\dot{f}_F(-i) + f_F(-i)\dot{f}_F(i)}{|f_F(i)|^2} \mathcal{M}, \quad (2.40)$$

where $\delta_F = \lim_{k \rightarrow k_F} \text{Re}(\delta_k^+)$ and where

$$\mathcal{M} = \frac{\Gamma_F}{\Gamma_F - \Gamma} \quad (2.41)$$

is called the ‘distance to the Faraday threshold’. This latter parameter has often been used as a proxy for the memory in experimental studies of walkers (Eddi *et al.* 2011).

The expression of δ_k^+ in (2.39) may be particularised to the wavenumbers around k_F , which will likely compose the major part of the walker’s wave field. For $\Gamma \lesssim \Gamma_F$ (i.e. for $\mathcal{M} \gg 1$), we can expand δ_k^+ in a Taylor series around k_F . Recalling that $\partial \Gamma_k / \partial k = 0$ in $k = k_F$, we obtain

$$\begin{aligned} \delta_k^+ = & - \left[\frac{\Gamma_F(2gk_F/\Omega^2)^2}{(f_F(i)\dot{f}_F(-i) + f_F(-i)\dot{f}_F(i))} \left(\frac{\partial^2 \Gamma_k}{\partial k^2} \right)_F (k - k_F)^2 + O((k - k_F)^3) \right] \\ & - \frac{2}{\mathcal{M}} \left[\frac{\Gamma_F^2(2gk_F/\Omega^2)^2}{f_F(i)\dot{f}_F(-i) + f_F(-i)\dot{f}_F(i)} + O(k - k_F) \right] + O(\mathcal{M}^{-2}), \end{aligned} \quad (2.42)$$

where $f_F = f_{k_F}$ and $\dot{f}_F = \dot{f}_{k_F}$. This approximation of δ_k^+ is plotted with a red dashed line in figure 2(b). We identify this expression with $\delta_k^+ = \delta_F - D(k - k_F)^2 + O((k - k_F)^3) + O(\mathcal{M}^{-1})$ and define the diffusion coefficient D as

$$D = - \frac{1}{2} \frac{\partial^2 \delta_k^+}{\partial k^2} \Big|_{k=k_F} = \frac{\Gamma_F(2gk_F/\Omega^2)^2}{(f_F(i)\dot{f}_F(-i) + f_F(-i)\dot{f}_F(i))} \left(\frac{\partial^2 \Gamma_k}{\partial k^2} \right)_F. \quad (2.43)$$

Since

$$\left(\frac{\partial^2 \Gamma_k}{\partial k^2} \right)_F = \frac{1}{2k_F^2 \Gamma_F} \frac{\partial^2 (k \Gamma_k)^2}{\partial k^2} \Big|_F - \frac{\Gamma_F}{k_F^2} \quad (2.44)$$

and $k^2 \Gamma_k^2 = \Omega^4 |f_k(i)|^2 / (2g)^2$, D can be expressed in terms of f_k as

$$D = \frac{(\partial^2 |f_k(i)|^2 / \partial k^2)_F - 2|f_F(i)|^2 / k_F^2}{2(f_F(i)\dot{f}_F(-i) + f_F(-i)\dot{f}_F(i))}. \quad (2.45)$$

The amplitude ζ_k^+ of the long-lived Faraday waves (respectively ζ_k^- for short-lived waves) is given from (2.29):

$$\zeta_k^\pm(\tau) \propto e^{\delta_k^\pm \tau} \left(e^{i\tau} - \frac{f_k(i + \delta_k^\pm)}{2\Gamma gk/\Omega^2} e^{-i\tau} \right). \quad (2.46)$$

We introduce

$$\theta_k^\pm = \arctan \left(\frac{\text{Im}\{f_k(i + \delta_k^\pm)\}}{2\Gamma gk/\Omega^2 - \text{Re}\{f_k(i + \delta_k^\pm)\}} \right), \quad (2.47)$$

in order to express the total wave field $\zeta_k(\tau)$ as

$$\zeta_k(\tau) = A_k^+ e^{\delta_k^+ \tau} \cos(\tau + \theta_k^+) + A_k^- e^{\delta_k^- \tau} \cos(\tau + \theta_k^-), \quad (2.48)$$

where A_k^+ and A_k^- stand for the relative amplitude of long-lived and short-lived waves, respectively. The angles $\theta_k^\pm$ then correspond to the phase shift between these waves and the driving acceleration. They are represented in figure 2(d). Outside the Faraday window, they are both close to 0, while inside the Faraday window they strongly vary with k in a non-trivial way. Remarkably around k_F , these phase shifts are $\theta_F^\pm \simeq \pm\pi/4$.

2.3. Single impact of a droplet

We now consider the impact of a droplet at the surface of the liquid at a time τ_i . While sophisticated theoretical frameworks have been developed in order to accurately calculate the interaction of a droplet with the surface of a liquid (Moláček & Bush 2012; Molacek & Bush 2013; Milewski *et al.* 2015), we shall, for illustration purposes, employ a rather simplified description of the droplet impact. Namely, we model its temporal profile with a delta function. The corresponding dimensionless pressure exerted by the droplet on the liquid bath is then

$$\Pi_k(\tau) = \delta(\tau - \tau_i) v_k. \quad (2.49)$$

By integrating (2.22) across the delta kick, it is shown that v_k corresponds to the negative change of velocity of ζ_k following impact. If the surface is perfectly flat and at rest before the impact, the wave profile is axisymmetric, i.e. $\zeta_k(\tau) = \zeta_k(\tau)$. The initial conditions in $\tau \rightarrow \tau_i^+$ are $\zeta_k = 0$ and $\partial\zeta_k/\partial\tau = -v_k$. They determine the coefficients $A_k^\pm$ of (2.48):

$$A_k^\pm = - \frac{2 \cos(\tau_i + \theta_k^\mp) v_k e^{-\delta_k^\pm \tau_i}}{(\delta_k^\pm - \delta_k^\mp)(\cos(2\tau_i + \theta_k^\pm + \theta_k^\mp) + \cos(\theta_k^\pm - \theta_k^\mp)) - 2 \sin(\theta_k^\pm - \theta_k^\mp)}. \quad (2.50)$$

Transforming back to the spatial domain according to (2.9) then yields the radially symmetric surface wave profile

$$\zeta_1(r, \tau) = \int_0^\infty \zeta_k(\tau) J_0(kr) k dk \quad (2.51)$$

that is generated by a delta kick, where J_0 is the Bessel function of the first kind of order zero.

At large times $\tau \gg \tau_i$ for $\Gamma \lesssim \Gamma_F$, the wave field is dominated by the contribution ζ_k^+ , so

$$\zeta_1(r, \tau) = \int_0^\infty B_k^+(\tau_i) \cos(\tau + \theta_k^+) e^{\delta_k^+(\tau - \tau_i)} J_0(kr) k dk, \quad (2.52)$$

where $B_k^+(\tau_i) = A_k^+ e^{\delta_k^+ \tau_i}$. With the expansion $\delta_k^+ = \delta_F - D(k - k_F)^2 + O((k - k_F)^3) + O(\mathcal{M}^{-1})$, we can then approximate this integral thanks to the saddle-point method in the formal limit $\tau \rightarrow \infty$. Calculations detailed in appendix C yield:

$$\zeta_1(r, \tau) \simeq B_F^+(\tau_i) k_F \sqrt{\frac{\pi}{D(\tau - \tau_i)}} \cos(\tau + \theta_F^+) J_0(k_F r) \exp \left[-\frac{\tau - \tau_i}{2\pi M e} - \frac{r^2}{4D(\tau - \tau_i)} \right], \quad (2.53)$$

which is at least valid when $k_F r \ll 1$ and $k_F r \gg 1$. Equation (2.53) is the impulse response of the Faraday waves in the long-time limit. According to Duhamel's principle, the response to any sophisticated impact signal $\Pi_k(\tau)$ can be inferred by convoluting this impulse response with $\Pi_k(\tau)$.

Equation (2.53) may be compared to the main previous model of waves generated by a single impact of a droplet (1.5), which can be rewritten as:

$$\zeta(r, t) = \frac{A(\tau_i)}{\sqrt{\tau - \tau_i}} \cos(\tau) J_0(k_F r) \exp \left(-\frac{\tau - \tau_i}{2\pi M e} \right). \quad (2.54)$$

The main additions of (2.53) to (2.54) are (i) the phase shift θ_F^+ , (ii) a diffusive spreading in the exponential decay (diffusion coefficient D) and (iii) a well-defined amplitude B_F^+ that depends on the impact phase τ_i through (2.50). Modifications similar to (ii) and (iii) can be deduced with additional calculations, starting from the theory of Moláček & Bush (2013).

2.4. Surface waves emitted by a walker

The long-lived Faraday wave generated by a walker is obtained by superimposing impulse responses corresponding to each droplet impact at time τ_n and position $\mathbf{r}_n$. In the absence of other walkers, submerged boundaries or body forces, a walker bounces periodically every Faraday period and moves in a straight line at constant speed V_w . Consequently,

$$\tau_n = \tau_i + 2\pi n, \quad n \in \mathbb{Z}, \quad \text{and} \quad \mathbf{r}_n = -(\tau - \tau_n) \mathbf{v}, \quad (2.55a, b)$$

where $v = 2V_w/\Omega$ is the normalised walking speed. The origin $\mathbf{r} = 0$ is set where the walker is at time τ . The Faraday wave field is then given by

$$\zeta_w(\mathbf{r}, \tau) = \sum_{n=-\infty}^{[(\tau - \tau_i)/2\pi]} \zeta_1(|\mathbf{r} - \mathbf{r}_n|, \tau - 2\pi n), \quad (2.56)$$

where $[(\tau - \tau_i)/2\pi]$ denotes the largest integer smaller than $(\tau - \tau_i)/2\pi$.

This sum is evaluated in appendix D. It yields

$$\begin{aligned} \zeta_w(\mathbf{r}, \tau) = & \frac{B_F^+ k_F}{2\sqrt{D}} \cos(\tau + \theta_F^+) \exp \left(-\frac{vr \cos \theta}{2D} \right) \\ & \times \int_{-\pi}^{\pi} d\varphi \frac{\exp[ik_F r \sin(\theta - \varphi)]}{\sqrt{a - ik_F v \sin \varphi}} \exp \left(-\frac{r}{\sqrt{D}} \sqrt{a - ik_F v \sin \varphi} \right), \end{aligned} \quad (2.57)$$

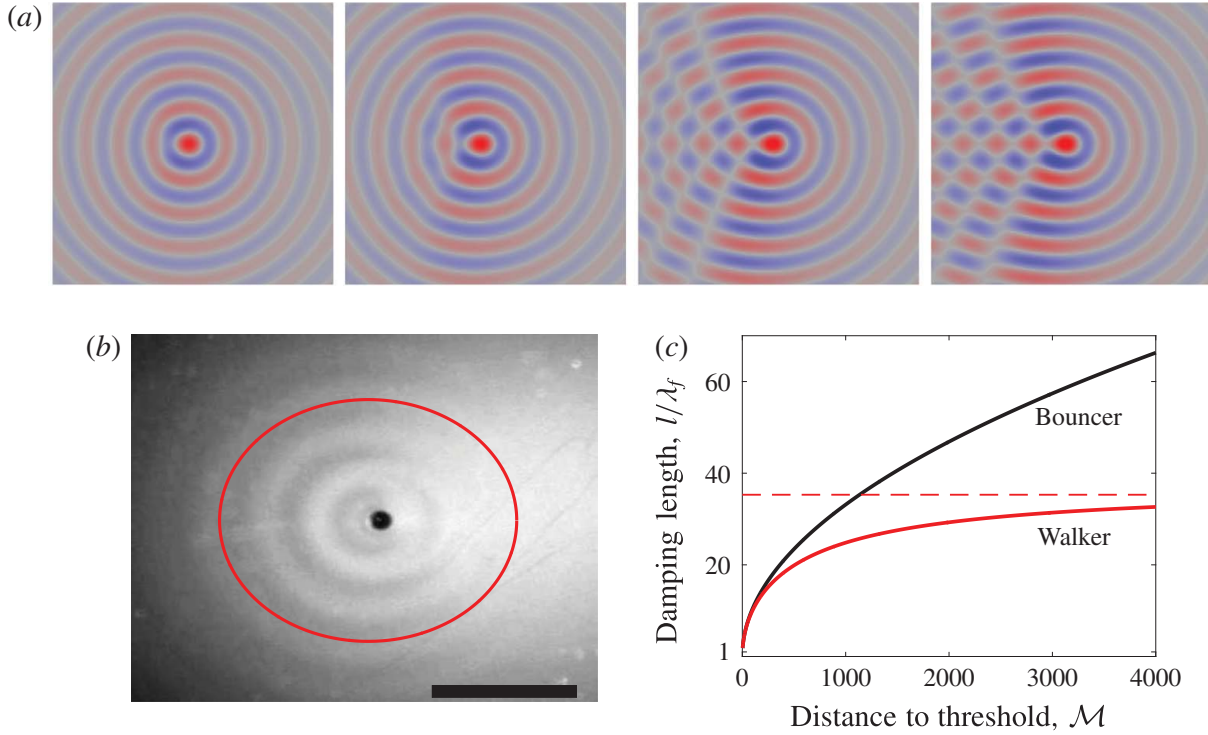


FIGURE 3. (Colour online) (a) Asymmetric wave field of a walker, calculated numerically from equation (2.57), for different values of the memory Me . From left to right, $Me = 7.7$, $Me = 16$, $Me = 82$ and $Me = 447$ respectively. The walker velocity is fixed to $V_w = 10 \text{ mm s}^{-1}$ from left to right, and other parameters are given in table 1. The field of view is $4 \times 4 \text{ cm}$. (b) Anisotropic damping length $l(\theta)$, compared to the experimental wave field of a walker. The red curve corresponds to (2.60) with $\mathcal{M} = 10$ and other numerical values from table 1. In the experiment, the walking speed is 10 mm s^{-1} and $\mathcal{M} = 10$. The scale bar is 1 cm . (c) Lateral damping length ($\theta = \pi/2$) for a bouncer (black curve, $V_w = 0$) and a walker (red curve, $V_w = 11 \text{ mm s}^{-1}$), as a function of the distance to threshold $\mathcal{M}$.

where $\cos \theta = \mathbf{v} \cdot \mathbf{r}/(vr)$ and

$$a = \frac{1}{2\pi Me} + \frac{v^2}{4D}. \quad (2.58)$$

The corresponding wave field can be calculated by numerically integrating (2.57). Some characteristic wave fields are shown in figure 3(a), for different values of the memory Me . With increasing Me , the sum includes significant contributions from an increasing number of past impacts. Some interferences appear in the wake of the walker, similarly to what is seen experimentally (Eddi *et al.* 2011).

In the limit of small walking speed and low memory ($2\pi Me k_F v \ll 1$), the integral (2.57) can be solved analytically (appendix D):

$$\zeta_w(\mathbf{r}, \tau) = \frac{B_F^+ k_F}{\sqrt{v^2 + \frac{2D}{\pi Me}}} \cos(\tau + \theta_F^+) J_0 \left(k_F \left| \mathbf{r} + \frac{r}{\sqrt{v^2 + \frac{2D}{\pi Me}}} \mathbf{v} \right| \right) \exp[-r/l(\theta)], \quad (2.59)$$

where

$$l(\theta) = \frac{2D}{v \cos \theta + \sqrt{v^2 + \frac{2D}{\pi Me}}}. \quad (2.60)$$

Equation (2.59) corresponds to a Doppler-shifted Bessel profile which is exponentially attenuated on a length scale $l(\theta)$ that varies with direction θ . The resulting wavelength is smaller in front of the walker and larger in its wake. The damping distance is smaller in front of the walker than in its wake, as illustrated in figure 3(b).

For either larger walking speed or larger memory, interferences between Bessel functions are more complex and solving the integral (2.57) is not straightforward. Nevertheless, an upper bound to the wave field can be found, as derived in appendix D:

$$\left| \frac{\zeta_w(\mathbf{r}, \tau)}{\cos(\tau + \theta_F^+)} \right| \leq \frac{\pi B_F^+ k_F}{\sqrt{Da}} \exp \left[-\frac{r}{l(\theta)} \right]. \quad (2.61)$$

This bound indicates that the Faraday waves emitted by the walkers are always subjected to the same exponential spatial damping $l(\theta)$ defined in (2.60).

In the case of a bouncer ($v = 0$), this damping length $l(\theta) = \sqrt{2\pi DMe}$ is independent of θ (axisymmetric damping) and it grows indefinitely with increasing memory. At the Faraday threshold (infinite memory), a bouncer would generate waves that are not spatially damped. By contrast, waves are always spatially damped for a walker with finite speed v . Indeed, when $Me \rightarrow \infty$, $l(\theta) \rightarrow 2D/[v(1 + \cos \theta)]$. This horizontal asymptote is illustrated in figure 3(c). Hence, equation (2.61) predicts a finite spatial damping even at infinite memory, and this for all directions except $\theta = \pi$.

2.5. Approximations specific to the experimental regime of walkers

The subthreshold Faraday wave theory developed in previous sections is now particularised to the regime of walkers that corresponds to most experiments reported up to now (Bush 2015). Typical values for each parameter are summarised in table 1. The liquid is silicone oil with density $\rho = 956 \text{ kg m}^{-3}$, surface tension $\sigma = 20.6 \times 10^{-3} \text{ N m}^{-1}$ and viscosity $\nu = 20 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$. The bath is vibrated sinusoidally at a frequency $\Omega/2\pi = 80 \text{ Hz}$.

In the limit of vanishing viscosity ν , $\gamma_k \rightarrow 0$, and $f_k(i) \rightarrow \omega_k^2 - 1$. Therefore, according to (2.32), the instability threshold $\Gamma_F \rightarrow 0$ in $k = k_0$ such that $\omega_0^2 = \omega_{k_0}^2 = 1$. For a walker, $k_0 = 1319.5 \text{ m}^{-1}$, which corresponds to a wavelength of 4.76 mm, already close to experimental observations (Eddi *et al.* 2011). The corresponding damping (at $\nu = 20 \text{ cS}$) is $\gamma_0 = \gamma_{k_0} = 0.277$. Although this value is not much smaller than 1, we can perform a Taylor expansion in γ_0 for each quantity of interest.

First, we prove in appendix B that the most unstable wavenumber k_F , defined from (2.33), can be approximated by:

$$k_F = k_0 + \frac{1}{2c_0} \gamma_0^{3/2} - \frac{2 + k_0 c_0}{2k_0 c_0^2} \gamma_0^2 + \frac{12 + k_0 c_0}{8k_0 c_0^2} \gamma_0^{5/2} + \left(\frac{1}{4k_0 c_0^2} - \frac{d_0}{8c_0^3} \right) \gamma_0^3 + O(\gamma_0^{7/2}), \quad (2.62)$$

where $d_0 = (\partial^2 \omega_k^2 / \partial k^2)(k_0) = 24\sigma k_0 / \rho \Omega^2$ and $c_0 = (\partial \omega_k / \partial k)(k_0) = (2(g + 3\sigma k_0^2 / \rho) / \Omega^2)$ is the group velocity of free waves around the wavenumber k_0 . For walkers, $c_0 = 0.97$ mm (which corresponds to a speed $\Omega c_0 / 2 = 0.24$ m s⁻¹) and the approximation (2.62) gives $k_F \simeq 1338.7$ m⁻¹ and $\lambda_f = 4.693$ mm, while a numerical computation of the exact solution of (2.33) without any approximation except the Floquet's first-order truncation would give $k_F = 1334.8$ m⁻¹ and $\lambda_f = 4.707$ mm.

From appendix B, it follows that

$$\left. \begin{aligned} \omega_F^2 &= 1 + \gamma_0^{3/2} - \frac{2 + k_0 c_0}{k_0 c_0} \gamma_0^2 + \frac{12 + k_0 c_0}{4 k_0 c_0} \gamma_0^{5/2} + O(\gamma_0^3), \\ \gamma_F &= \gamma_0 + \frac{\gamma_0^{5/2}}{k_0 c_0} + O(\gamma_0^3). \end{aligned} \right\} \quad (2.63)$$

Equation (2.32) for the corresponding instability threshold $\Gamma_F = \Gamma_k(k_F)$ is then approximated by:

$$\Gamma_F = \frac{\Omega^2}{g k_0} \gamma_0 - \frac{\Omega^2}{2 g k_0} \gamma_0^{3/2} + O(\gamma_0^{5/2}). \quad (2.64)$$

The first term of Γ_F corresponds to the threshold predicted from the damped Matthieu equation (1.3) with a phenomenological factor $\alpha = 2$ (Sampara & Gilet 2016). Since for walkers $\gamma_0 = 0.277$, a limitation to this first term would yield $\Gamma_F = 5.41$. Considering instead the first two terms yields $\Gamma_F = 3.985$. The exact solution of (2.32) obtained numerically is $\Gamma_F = 4.144$, which is very close to experimental values. The correction in $\gamma_0^{3/2}$ is therefore a considerable improvement of this theory with respect to the damped Matthieu equation.

The memory Me defined in (2.40) can be approximated by:

$$Me = \frac{\mathcal{M}}{2\pi\gamma_0} \left(1 + \frac{\gamma_0^{1/2}}{2} + \frac{\gamma_0}{4} + O(\gamma_0^{3/2}) \right). \quad (2.65)$$

It is therefore inversely proportional to the viscosity ν and an inviscid fluid would yield a theoretically infinite memory. In the case of walkers, $(1 + \gamma_0^{1/2}/2 + \gamma_0/4)/2\pi\gamma_0 \sim 0.76$, which is fairly close to unity. In previous work on walkers, Me has often been mixed up with the distance to threshold $\mathcal{M}$, without much consequence. Close to threshold ($\Gamma \lesssim \Gamma_F$), the damping factor of short-lived Faraday waves $\delta_F^- = -2\gamma_0 + O(\gamma_0^{3/2})$ is significantly more negative than the damping of long-lived waves $\delta_F^+ = -1/(2\pi Me)$.

The diffusion coefficient D associated with long-lived Faraday waves and defined in (2.45) is approximated by

$$D = \frac{c_0^2}{2\gamma_0} \left(1 + \frac{\gamma_0^{1/2}}{2} + O(\gamma_0) \right) \quad (2.66)$$

which is about $D \simeq 4.40$ mm² for a walker.

The phase shift of long-lived and short-lived Faraday waves near the Faraday threshold is:

$$\theta_F^\pm = \pm \pi/4 + O(\gamma_0), \quad (2.67)$$

and consequently,

$$A_F^\pm = v_F \cos\left(\tau_i \mp \frac{\pi}{4}\right) e^{-\delta_F^\pm \tau_i} + O(\gamma_0) \quad \text{and} \quad B_F^+ = v_F \cos\left(\tau_i - \frac{\pi}{4}\right) + O(\gamma_0). \quad (2.68a,b)$$

The long-lived impulse response (2.53) then becomes:

$$\begin{aligned} \zeta_1(r, \tau) \simeq & v_F k_F \sqrt{\frac{\pi}{D(\tau - \tau_i)}} J_0(k_F r) \cos\left(\tau_i - \frac{\pi}{4}\right) \cos\left(\tau + \frac{\pi}{4}\right) \\ & \times \exp\left[-\frac{\tau - \tau_i}{2\pi M e} - \frac{r^2}{4D(\tau - \tau_i)}\right], \end{aligned} \quad (2.69)$$

which reads in dimensional form,

$$\begin{aligned} \zeta_1(r, t) \simeq & v_F k_F \sqrt{\frac{2\pi}{D\Omega(t - t_i)}} J_0(k_F r) \cos\left(\frac{\Omega t_i}{2} - \frac{\pi}{4}\right) \cos\left(\frac{\Omega t}{2} + \frac{\pi}{4}\right) \\ & \times \exp\left[-\frac{(t - t_i)}{MeT_F} - \frac{r^2}{2D\Omega(t - t_i)}\right]. \end{aligned} \quad (2.70)$$

The upper bound (2.61) on the wave field of a walker that moves at constant normalised speed v becomes

$$\left| \frac{\zeta_w(\mathbf{r}, \tau)}{\cos(\tau + \pi/4)} \right| \leq \frac{2\pi v_F k_F \cos(\tau_i - \pi/4)}{\sqrt{v^2 + \frac{c_0^2}{\mathcal{M}}}} \exp\left[-\frac{r}{l(\theta)}\right] \quad (2.71)$$

with

$$l(\theta) \simeq \frac{c_0^2}{2\gamma_0[v \cos \theta + \sqrt{v^2 + c_0^2/\mathcal{M}}]}. \quad (2.72)$$

For distance to threshold $\mathcal{M} \gg c_0^2/v^2$ (which is approximately 490 for walkers),

$$l(0) \simeq \frac{c_0^2}{4\gamma_0 v} \simeq 1.9 \text{ cm}, \quad \text{and} \quad l(\pi) \simeq \frac{\mathcal{M}}{\gamma_0}. \quad (2.73a,b)$$

The damping length l can therefore be as large as wanted in the wake of the walker ($\theta = \pi$), but it is strongly limited in front of the walker, independently of $\mathcal{M}$. Nevertheless, $l(0)$ could be increased by decreasing γ_0 , i.e. by decreasing viscosity and associated dissipation.

At sufficiently low $\mathcal{M}$, given by (2.59) is approximated by:

$$\zeta_w(\mathbf{r}, t) \simeq \frac{v_F k_F}{\sqrt{v^2 + \frac{c_0^2}{\mathcal{M}}}} \cos\left(\tau_i - \frac{\pi}{4}\right) \cos\left(\tau + \frac{\pi}{4}\right) J_0\left[k_F \left|\mathbf{r} + \frac{r}{\sqrt{v^2 + \frac{c_0^2}{\mathcal{M}}}} \mathbf{v}\right|\right] e^{-r/l(\theta)}. \quad (2.74)$$

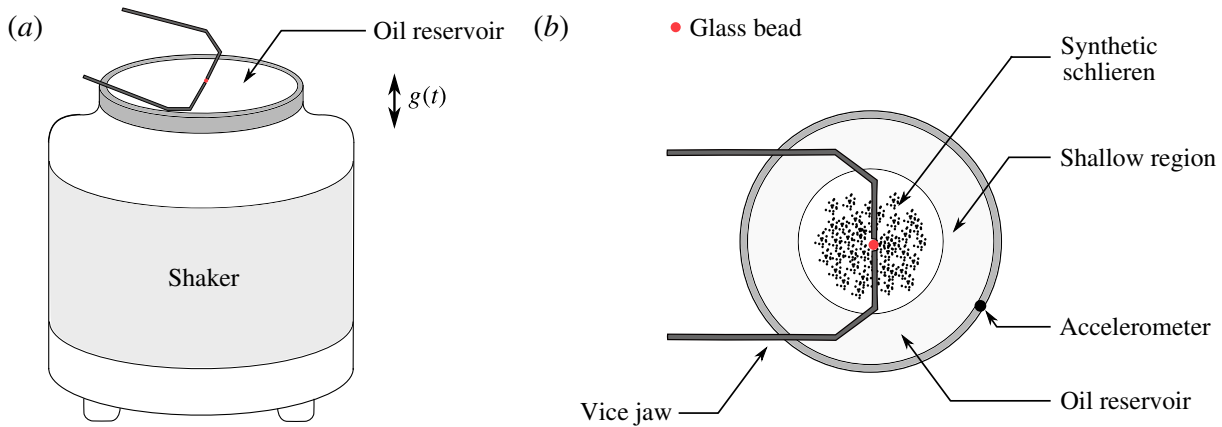


FIGURE 4. (Colour online) Tilted view (a) and top view (b) of the experimental set-up. A large transparent reservoir of oil is mounted on top of a shaker. Below the reservoir a transparent sheet is printed with a synthetic schlieren. The border of the reservoir is a shallow region where surface waves are strongly damped. A glass bead is released on demand above the reservoir with a vice jaw opened with a translation stage. The acceleration of the reservoir is measured with an accelerometer placed on the border of the reservoir.

This approximation is only valid when $k_F v/a \ll 1$ (appendix D), which here reduces to

$$\mathcal{M} \ll \left(\frac{k_F v}{\gamma_0} - \frac{v^2}{c_0^2} \right)^{-1}. \quad (2.75)$$

This limit is $\mathcal{M} \ll 4.9$ for a walker at speed $V_w = 11 \text{ mm s}^{-1}$. Furthermore, since $\mathcal{M} \ll c_0^2/v^2 \simeq 490$, the Doppler shift along the walker trajectory in (2.74) can be approximated by

$$k_{\text{Doppler}} = k_F \pm \frac{v}{c_0} \sqrt{\mathcal{M}}. \quad (2.76)$$

This expression is very similar to the one obtained numerically in Milewski *et al.* (2015).

3. Experimental observation of the surface waves emitted by a localised impact

This section aims at validating experimentally the theory of subthreshold surface waves developed in § 2. Specifically, we characterise (i) the spatial damping of the Faraday waves upon a single impact, (ii) the phase shift between these waves and the shaker position and (iii) the dependence of the wave amplitude on the phase of impact. These three points are major additions of the present theory with respect to previous models of the Faraday waves generated by walkers (Eddi *et al.* 2011; Moláček & Bush 2013).

3.1. Experimental set-up

The goal of the experiment is to finely quantify the spatio-temporal evolution of the surface waves generated by the single localised impact of a bead. The experimental set-up, depicted in figure 4, comprises a transparent cylindrical reservoir of diameter

22 cm filled with approximately 5 mm of silicone oil (Sigma-Aldrich). The reservoir is delimited by a shallow region (width 3 cm, depth <1 mm), thanks to which surface waves are strongly damped. This trick allows us to eliminate the additional waves generated by the meniscus at the boundaries of the reservoir (Bechhoefer *et al.* 1995; Nguyem Thu Lam & Caps 2011). The reservoir is also large enough ($22\text{ cm} \simeq 46\lambda_f$) to avoid confinement effects that may stabilise the bath when the bath diameter is less than $10\lambda_f$ (Nguyem Thu Lam & Caps 2011). The reservoir is placed on top of an electromagnetic shaker (V400LT, Data Physics). The shaker motion is accurately controlled via an accelerometer tightly fixed on the reservoir, a computer interface and a feedback loop, as detailed in Sampara & Gilet (2016). The resulting effective gravity acceleration is $g(t) = g(1 + \Gamma \cos \Omega t)$. The frequency is fixed to $\Omega = 2\pi 80\text{ s}^{-1}$.

The bead is made of glass and its diameter is 1 mm. A device that can release the bead on demand is placed at 1 cm above the bath. It consists in a vice with two jaws that grip the bead. It is mechanically insulated from the shaker, so it is not sensitive to its vibrations. A translation stage allows us to open the vice and release the bead. The bead does not acquire any significant velocity from the vice, so it starts falling from rest.

Measurements must be gathered over many Faraday periods, with a time resolution well below the shaker period $2\pi/\Omega$. Therefore, we here use high-speed imaging (Phantom MIRO110), with an acquisition frequency of 4000 fps (i.e. 50 frames per period of the shaker). The shaker is illuminated with 4 home-made high-power LEDs to ensure strong illumination of the waves while limiting the associated heating. Thanks to a semi-refrangent mirror placed at 45° above the shaker, optical paths of both illumination and recording are vertical near the bath surface. Thanks to a macro lens Zeiss Milvus 2/100M, the spatial resolution is brought down to $70\text{ }\mu\text{m}$ by pixel. The camera is synchronised with the shaker thanks to a DACQ (National Instruments) and a Labview interface that returns the impact phase τ_i , with respect to the Faraday period T_F . Details on this crucial synchronisation are provided in appendix E.

The free-surface synthetic schlieren technique developed by Moisy, Rabaud & Salsac (2009) is well adapted to measure wave elevations in real time. It consists in placing a transparent sheet printed with randomly spaced dots (dot diameter $100\text{ }\mu\text{m}$, 50 % density) over a white background at the bottom of the reservoir. The image of these dots moves owing to the light refraction at the free surface. The displacement of the imaged dots is therefore related to the local slope of the free surface. This technique was already successfully applied to measure the 3-D shape of the long-lived Faraday waves generated by walkers, with an acquisition frequency lower than $\Omega/(2\pi)$ (Eddi *et al.* 2011; Damiano *et al.* 2016). However, in its original form, it is not well suited to the measurements intended here. Indeed, it requires a large field of view that must not be obstructed by the device which would drop the bead. Moreover, owing to the limited amount of information (3 GB) that can be acquired by a high-speed camera in a single shot, the recording duration at full spatial resolution would be a fraction of a second, which is definitely not sufficient to capture the long-term behaviour of Faraday waves. Finally, the reconstruction algorithm of Moisy's technique would demand large computational resources. In order to circumvent these limitations, we here use an adaptation of Moisy's technique. Since the wave field is known to be axisymmetric, we restrict the field of view to a radial zone of $2 \times 15\text{ mm}$ from the impact point. This allows us to record much longer films (several seconds at 4000 fps), without any obstruction of the field of view by the bead-releasing device. The displacement of the imaged dots is calculated along a radius from impact point. The one-dimensional radial gradient

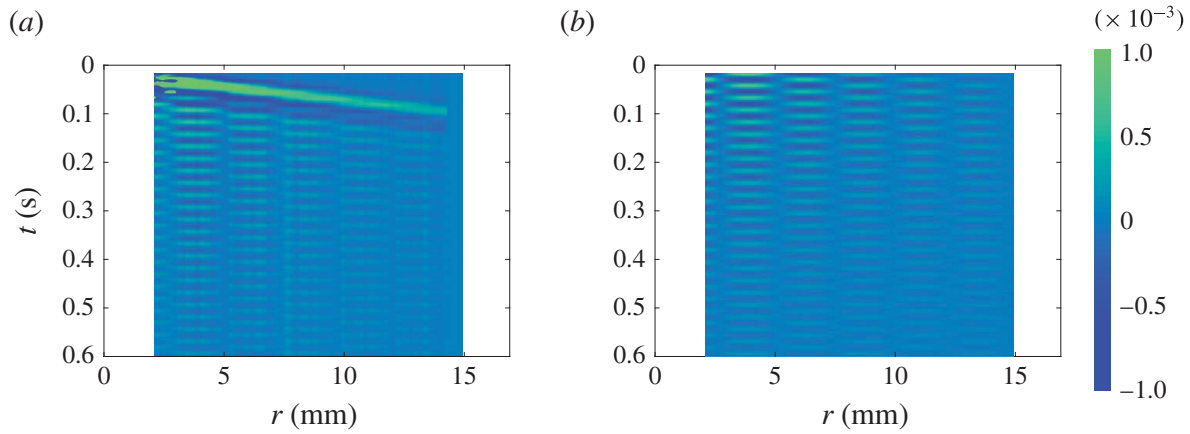


FIGURE 5. (Colour online) Qualitative comparison of the spatio-temporal evolution of the dimensionless radial wave gradient $\partial_r \zeta(r, t)$, represented here with a colour code. The impact is located in $(r, t) = (0, 0)$. (a) Experiment at $\mathcal{M} = 30$ and $\nu = 20$ cS. Experimental parameters are gathered in table 1. (b) Theoretical prediction (3.1).

of the elevation $\zeta(r, t)$ is then calculated with a 1-D adaptation of the reconstruction algorithm detailed in Moisy *et al.* (2009). The restriction to a quasi-1-D wave pattern severely decreases the computational time of the reconstruction. However, it can be shown that the 1-D algorithm does not benefit anymore from the error compensation mechanism present in the 2-D algorithm. In order to limit the measurement error, we first remove most of the noise with deshaking and median filters. We also avoid the error build-up inherent to the spatial integration of $\partial_r \zeta$ to obtain $\zeta(r, t)$. We instead directly compare the measured $\partial_r \zeta$ to the theoretical prediction

$$\frac{\partial \zeta_1}{\partial r}(r, t) = -v_F k_F \sqrt{\frac{2\pi}{D\Omega(t-t_i)}} \cos\left(\frac{\Omega t_i}{2} - \frac{\pi}{4}\right) \cos\left(\frac{\Omega t}{2} + \frac{\pi}{4}\right) \times \left[J_1(k_F r) + \frac{r J_0(k_F r)}{D\Omega(t-t_i)} \right] \exp\left[-\frac{(t-t_i)}{MeT_F} - \frac{r^2}{2D\Omega(t-t_i)} \right]. \quad (3.1)$$

Several input parameters are systematically varied, including the impact phase of the bead relative to the shaker, the shaker amplitude and corresponding distance to threshold $\mathcal{M}$, and the liquid viscosity (5 cS and 20 cS silicone oil).

3.2. Spatio-temporal evolution of the wave field

A typical evolution of the wave field is represented in the space-time diagram of figure 5(a). We observe that the impact first generates a large capillary wave packet that travels radially outwards with an almost constant velocity of 16 cm s^{-1} . The long-lived Faraday waves described in § 2 are observed in the wake of this wave packet. They have a pulsation $\Omega/2$ and a wavelength $2\pi/k_F \simeq 4.7 \text{ mm}$. The surface appears still in the region that is not yet visited by the wave packet (i.e. in the upper-right corner of figure 5a).

This experiment can be qualitatively compared to the theoretical prediction (3.1), where the amplitude of this latter is fitted on the experiments (figure 5b). Overall, this comparison is promising. However, the capillary wave packet is not present in the prediction, which is expected since only long standing Faraday waves were considered in (3.1). This latter also predicts waves of finite amplitude everywhere on the bath at

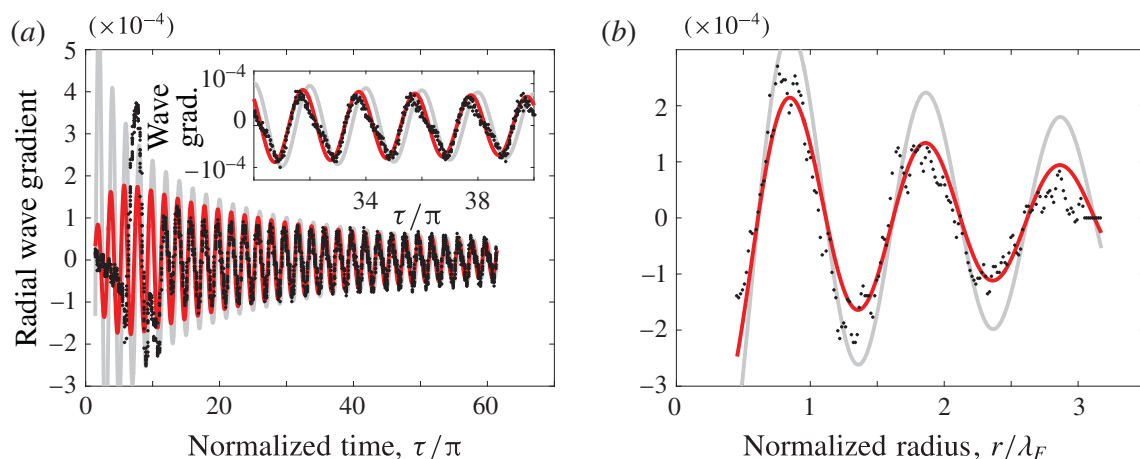


FIGURE 6. (Colour online) Radial gradient of surface waves, (a) versus time at a fixed distance $r = 1.32$ cm from the impact, and (b) versus position at a fixed time, $t = 0.233$ s after impact. The experimental data are a subset of those represented in figure 5(a). Black dots correspond to the gradient measured experimentally. The grey and red solid curves correspond to (1.5) and (3.1), respectively. The inset in (a) is a zoom of the main graph on a smaller time window. The present model (red curve) predicts a phase shift of $+\pi/4$ between the Faraday waves and the forcing signal, which is in better agreement with experiments than previous models (grey curve).

$t > t_i$, independently of the passage of the capillary wave packet, which was hard to observe experimentally. Nevertheless, their amplitude remains very small shortly after impact and far from the impact point, which may be the reason why the bath appears still this region of the experimental spatio-temporal diagram.

A more quantitative comparison is provided in figure 6(a,b), where the radial gradient of the wave is represented as a function of time (at a given position), and as a function of space (at a given time), respectively. The theoretical prediction (3.1) of long-lived Faraday waves is compared to both experiments and the previous model of Moláček & Bush (2013) (given by the gradient of (1.5)). The amplitude factors (v_F in (3.1), and $A \sin \Phi$ in (1.5)) are fitted to the experimental data in the region (r, t) that corresponds to the space–time domain after the passage of the capillary wave packet.

In figure 6(a), it is seen that both models capture well the long-term evolution of the wave at a given position. Nevertheless, a zoom on a small time window (inset of figure 6a) reveals that the factor $\cos(\Omega t/2 + \pi/4)$ of (3.1) fits experimental data better than the factor $\cos(\Omega t/2)$ of (1.5). In the short-term regime (i.e. before the passage of the capillary wave packet), the experimental wave amplitude is very small. None of the models accurately capture this behaviour, although the addition of short-lived waves in (3.1) might improve the prediction. However, equation (1.5) diverges at short times, while equation (3.1) grows from zero according to an exponential–diffusive behaviour, which is much more realistic. Moreover in (1.5), the information of impact is immediately transmitted everywhere on the bath. By contrast, the present theory predicts that after an impact the bath is first quiet and destabilises progressively owing to diffusive spreading.

The radial evolution of the wave at a given time, seen in figure 6(b), is qualitatively similar for both models and experiment. Nevertheless, the spatial damping predicted by (3.1) is closer to experiments than the one of (1.5).

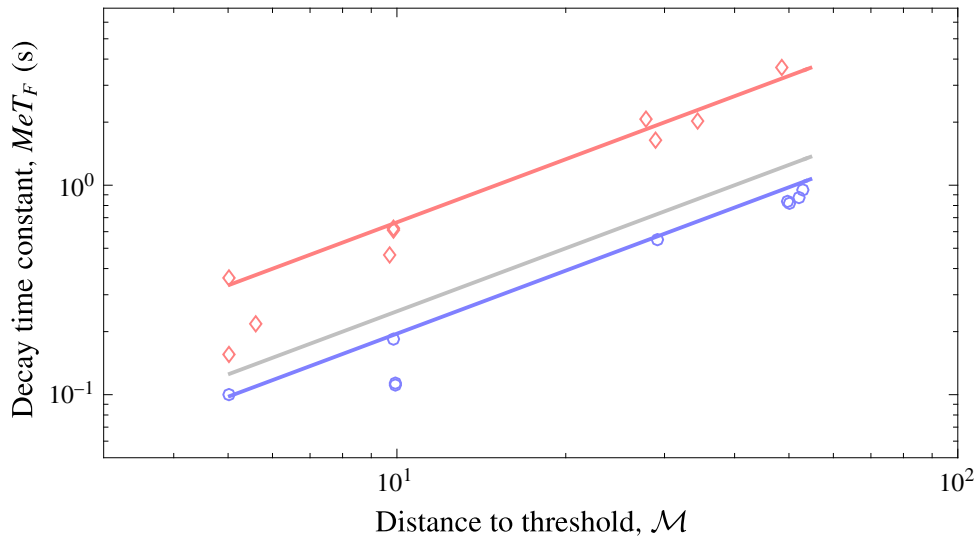


FIGURE 7. (Colour online) Temporal damping MeT_F , as a function of the distance to threshold $\mathcal{M}$. Red diamonds ($\diamond$) and blue circles ($\circ$) correspond to experiments with silicone oil of viscosity 5 cS and 20 cS, respectively. Red and blue solid lines correspond to (2.65) with the corresponding viscosity. The grey line represents $MeT_F = \mathcal{M}$.

The temporal damping of Faraday waves (envelope of the curve in figure 6a) was already observed by Eddi *et al.* (2011). It was then interpreted as the memory Me of the waves, which revealed to be a crucial parameter for the dynamics of walkers (Bush 2015). However, the definition given by Eddi *et al.* (2011) differs from (2.65), as the former is independent of viscosity while the latter is inversely proportional to viscosity in first approximation. In order to validate (2.40), we have measured the damping time MeT_F as a function of the distance to threshold $\mathcal{M}$ for two different silicone oils, of kinematic viscosity 5 cS and 20 cS respectively (figure 7). The comparison with the theoretical prediction (2.65) is excellent. In particular MeT_F is proportional to $\mathcal{M}$ and inversely proportional to viscosity.

3.3. Wave amplitude versus impact phase

The amplitude of the Faraday waves generated by an impact is reported for different phases of impact $\tau_i = t_i\Omega/2$ in figure 8. This amplitude is measured as the experimental envelope of $\partial_r\zeta(r, t)$ at a particular location r and time t . It varies sinusoidally with τ_i , so there are some impact phases for which the long-term excited wave is almost non-existent. Our model (3.1) predicts a dependence in $\cos(\tau_i - \pi/4)$ which is qualitatively similar. However, a dependence in $\cos(\tau_i + \pi/5)$ would correspond much better to the measurements.

This phase shift between experiments and theory is likely due to the fact that (3.1) represents an impulse response of surface waves, while in experiments the penetration of the bead into the bath is not instantaneous. According to Duhamel's principle, the response to the bead impact can be calculated by convolving the impulse response with the sophisticated pressure signal $\Pi_k(\tau)$ applied by the bead on the bath. This convolution is expected to approximately shift the effective impact time by half the time of penetration, which is of the order of the contact time $5\sqrt{\rho R_d^3/\sigma} \sim 12$ ms for a bead of radius $R_d = 0.5$ mm (Molacek & Bush 2013). On the other hand, the time delay corresponding to a phase shift from $-\pi/4$ to $\pi/5$ is 5.625 ms, which is very close to the 6 ms that such convolution would predict.

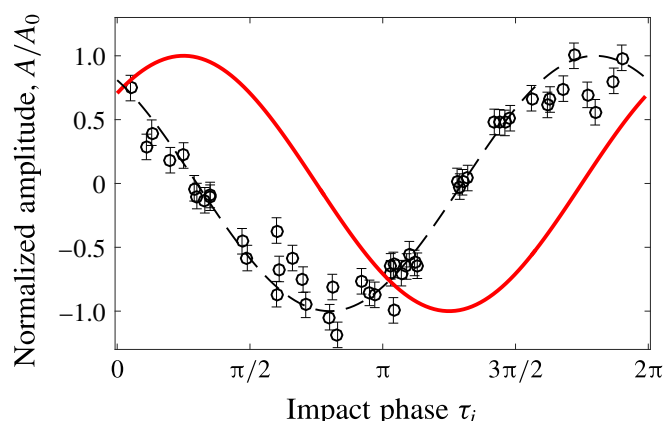


FIGURE 8. (Colour online) Amplitude of the Faraday waves gradient, normalised with $A_0 = 1.9 \times 10^{-4}$ (at fixed position $r = 1.32$ cm and time $t = 0.233$ s), as a function of impact phase τ_i . Symbols correspond to experimental data with oil viscosity $\nu = 20$ cS, $\Omega = 80 \times 2\pi$ s $^{-1}$ and $\mathcal{M} = 30$. The red curve is the theoretical prediction $\cos(\tau_i - \pi/4)$ from the impulse response (3.1) while the black dashed line is $\cos(\tau_i + \pi/5)$.

4. Discussion and conclusion

In § 2, we introduced a theory for the Faraday instability derived from Navier–Stokes equations and based on the only assumption that the surface waves are linear and purely subharmonic. This assumption allows us to truncate the Floquet series (2.26), and it is valid when $\Gamma \lesssim \Gamma_F$ (i.e. close to the Faraday threshold) for wavenumbers $k \simeq k_F$. We have then calculated the dispersion relation of Faraday waves, the Faraday threshold Γ_F and the Faraday wavenumber, k_F . We have shown that for $\Gamma \lesssim \Gamma_F$, there is a range of k in the vicinity of k_F , called the Faraday window, in which the frequency is locked to half the driving frequency (subharmonic response). Waves within this window are standing Faraday waves, while waves outside this window are dispersive travelling waves. The Faraday waves are long lived, and their damping time is proportional to the so-called distance to threshold $\mathcal{M} = \Gamma_F / (\Gamma_F - \Gamma)$, as already observed in previous studies (Eddi *et al.* 2011). The present theory further predicts (see (2.65)) that the damping time is approximately inversely proportional to the fluid viscosity.

The impulse response of the surface waves has been calculated in § 2.3. Similarly to previous models (e.g. (Eddi *et al.* 2011; Moláček & Bush 2013; Milewski *et al.* 2015)), its spatial dependence involves a Bessel function $J_0(k_F r)$ and its time evolution is a damped oscillation at frequency $\Omega/2$ (subharmonic response). Some of these previous models refer to a similar exponential damping in space. We show here that this damping is not strictly present in the impulse response. Instead, the wave amplitude is modulated by an exponential whose argument scales as $-r^2/(\tau - \tau_i)$. This modulation captures the diffusive propagation of the impulse excitation to the entire bath. A direct consequence is that two walkers do not instantly feel the impacts of each other. The time oscillation of the waves is phase shifted by $\pi/4$ compared to the shaker signal. In other terms, the wave elevation $\zeta(r, t)$ is identically zero when the shaker is at its lowest position. This phase shift, confirmed experimentally, may significantly affect the vertical bouncing dynamics of the walker. Indeed, when the droplet impacts the bath, it feels an absolute acceleration that corresponds to the sum of contributions from the shaker motion and from the overlying phase-shifted Faraday waves.

Experiment	V_w (mm s ⁻¹)	$\sqrt{2}d$ (mm)	$\mathcal{M}$	$MeT_F V_w / \sqrt{2}d$	$l / \sqrt{2}d$	R/l
Couder & Fort (2006)	18.9	20.2	5–20	0.12–0.5	0.8	2.5–4.7
Andersen <i>et al.</i> (2015)	15	14	21	0.5	1.17	2.82
Pucci <i>et al.</i> (2018)	12	28.3	500	5.3	2.18	1.15

TABLE 2. Experimental conditions used in three previous double-slit experiments with walkers (Couder & Fort 2006; Andersen *et al.* 2015; Pucci *et al.* 2018). Walking speed V_w , path length of the waves $\sqrt{2}d$, distance to threshold $\mathcal{M}$, normalised damping time $MeT_F V_w / \sqrt{2}d$, normalised damping length $l / \sqrt{2}d$ and normalised distance to boundaries R/l .

The wave field of a walker can be constructed by adding impulse responses corresponding to each impact of the droplet. This calculation has here been done in the case of a single walker moving at constant speed along a straight line. Under these assumptions, the vertical bouncing motion is perfectly periodic, and all impulse responses are given the same weight. However, the amplitude of the impulse response is modulated according to the phase of impact τ_i relative to the shaker, as predicted by this theory and confirmed experimentally. We expect the vertical dynamics of the walker to be perturbed as soon as additional Faraday waves are present. These Faraday waves may originate from the presence of other walkers, from the vicinity of a boundary, or from a previous passage of the walker at the same location (in the case of very high memory). In each of these cases, the vertical dynamics is not strictly periodic. Consequently, impact phase variations must be taken into account in the construction of the resulting wave field through an appropriately weighted summation of impulse responses. These refinements of the temporal description of Faraday waves are likely to be an additional source of chaos in the dynamics of walkers.

The wave of a regular walker is shown to be at least exponentially damped in space. The characteristic damping length $l(\theta)$ is calculated (while it was experimentally estimated in previous works, e.g. in Eddi *et al.* (2011)). It increases with $\mathcal{M}$ and reaches a finite asymptotic value for $\mathcal{M} \rightarrow \infty$. It also decreases with increasing speed or viscosity. This damping length plays a crucial role for the walker dynamics, especially in the landmark double-slit experiment initially performed with walkers by Couder & Fort (2006) and latter contested by Andersen *et al.* (2015). In this experiment, a walking droplet passing to one slit has to possibly interfere with the waves that it sends to the other slit. Such interference is only possible if the waves emitted by the droplet before it crosses the slits are still significantly present when they are received by the droplet beyond the slits. This requires the damping time to be larger than the time required to cross the slits, and the damping length should then be significantly larger than the path length that they have to travel. If the slits are separated by a distance d , the damping length should then be larger than $\sqrt{2}d$. Experimental conditions of Couder & Fort (2006), Andersen *et al.* (2015), Pucci *et al.* (2018) are reported in table 2. As a matter of fact, none of them, except the very recent one of Pucci *et al.* (2018), satisfied the two conditions for significant interferences: the damping time was always lower than the time to cross the slit, and the damping length was not much larger than d . In order to possibly observe interference patterns with single walkers, one would have to reproduce this experiment with a significantly higher memory (i.e. closer to Faraday threshold) and with a fluid

of significantly lower viscosity. Boundaries other than those that generate the expected diffraction must be placed farther than the damping length. So for example, the radius R of the reservoir must satisfy $R \gg l$, which was not the case in the work of Pucci *et al.* (2018). More generally, the damping length represents the distance at which the walker can probe its surrounding, would there be a boundary or other walkers. It is therefore a crucial parameter to the investigation of many other experiments on walkers, including those involving surreal trajectories, tunnelling or interactions between several walkers.

By contrast with walkers and Faraday waves, quantum mechanics is intrinsically non-dissipative. This feature can conceptually be approached with the walkers, by considering the limit of vanishing viscosity. In appendix F, we show that the waves generated by walkers remain linear in this limit, even though the thickness of the viscous boundary layer decreases to zero. In § 2.5, we have provided developments in series in the low-viscosity limit. If viscosity vanished (i.e. here $\gamma_0 \rightarrow 0$), the Faraday threshold Γ_F would vanish too, so the amplitude of the required driving vibration could be arbitrarily small. The memory Me would diverge and excitations from the bouncing droplets would be remembered by the wave forever. The diffusion coefficient D and the damping length l would diverge too, so each droplet impact would instantly modify the whole bath surface (with the compressibility of the liquid as a limit), i.e. each walker would instantly feel the presence of others, no matter how far away they are. Therefore, in the limit of vanishing viscosity, walkers become intrinsically non-local objects composed of a local particle coupled to a non-local wave field.

Walkers currently represent the only realisation at the macroscopic scale of a pilot-wave system similar to the one proposed by de Broglie (1987) to interpret quantum mechanics (Bush 2015). Many phenomena initially thought to be specific to the quantum realm have been observed with these walkers. The theory of Faraday waves developed in this paper brings a rigorous framework for the modelling of the wave field generated by a walker. It comprises additions to the previous Faraday wave models that are essential to the description of walkers interfering with others, with boundaries or with their past. It also provides insights on the locality of a walker, which has to be understood before experiments on the possible entanglement of two walkers could potentially be imagined. We hope that that this paper will help the building of a consistent wave theory to describe walkers similarly to Schrödinger's equation for quantum objects. It would then better reveal the limits of the analogy between walkers and the quantum world.

Acknowledgements

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Appendix A. Truncation of the Floquet ansatz

The (2.20) of surface waves in the absence of external pressure $\Pi_k = 0$ is recalled here:

$$f_k(s)\zeta_{k,s} + \frac{2\Gamma gk}{\Omega^2}(\zeta_{k,s+2i} + \zeta_{k,s-2i}) = 0. \quad (\text{A } 1)$$

It can be solved with the Floquet ansatz (2.27):

$$\zeta_{k,s} = \sum_{l=-\infty}^{\infty} \frac{\zeta_k^{(l)}}{s - (2l-1)i}, \quad (\text{A } 2)$$

which directly implies that

$$\zeta_{k,s \pm 2i} = \sum_{l=-\infty}^{\infty} \frac{\zeta_k^{(l)}}{s \pm 2i - (2l-1)i} = \sum_{l=-\infty}^{\infty} \frac{\zeta_k^{(l \mp 1)}}{s - (2l-1)i}. \quad (\text{A } 3)$$

Substitution of (A 2) and (A 3) in (A 1) yields

$$\sum_{l=-\infty}^{\infty} \frac{1}{s - (2l-1)i} \left(f_k(s) \zeta_k^{(l)} + \frac{2\Gamma gk}{\Omega^2} (\zeta_k^{(l-1)} + \zeta_k^{(l+1)}) \right) = 0. \quad (\text{A } 4)$$

The Heaviside cover-up method is then applied to obtain a linear system of infinite dimension for the Floquet coefficients $\zeta_k^{(l)}$:

$$\forall l \in \mathbb{Z}, \quad f_k((2l-1)i) \zeta_k^{(l)} + \frac{2\Gamma gk}{\Omega^2} (\zeta_k^{(l-1)} + \zeta_k^{(l+1)}) = 0, \quad (\text{A } 5)$$

or in matrix form

$$\underbrace{\begin{pmatrix} \ddots & \ddots & \ddots & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & f(-3i) & \alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & f(-i) & \alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha & f(i) & \alpha & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & f(3i) & \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & \ddots & \ddots & \ddots \end{pmatrix}}_K \begin{pmatrix} \vdots \\ \zeta^{(-1)} \\ \zeta^{(0)} \\ \zeta^{(1)} \\ \zeta^{(2)} \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \end{pmatrix}, \quad (\text{A } 6)$$

where we denoted $\alpha = 2\Gamma gk/\Omega^2$ for readability. The matrix is tridiagonal, which indicates that each Floquet mode is coupled to its neighbour modes. The condition of existence of surface waves is bound to the obtention of a non-trivial solution to this system. This can only be achieved when the determinant $\det K = 0$, which yields the threshold Γ_k associated with each wavenumber k for the non-truncated Floquet ansatz.

In the main text, we truncated the series to the two first modes, which led to (2.29) and (2.30). The approximation of this truncation may be evaluated by considering the presence of the two next modes. With a truncation enlarged to four modes, the linear system becomes

$$\begin{pmatrix} f_k(-3i) & \alpha & 0 & 0 \\ \alpha & f_k(-i) & \alpha & 0 \\ 0 & \alpha & f_k(i) & \alpha \\ 0 & 0 & \alpha & f_k(3i) \end{pmatrix} \begin{pmatrix} \zeta^{(-1)} \\ \zeta^{(0)} \\ \zeta^{(1)} \\ \zeta^{(2)} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \quad (\text{A } 7)$$

A non-trivial solution is found when the determinant is zero, i.e.

$$\frac{\alpha^4}{|f_k(3i)|^2} - \alpha^2 \left(1 + \frac{f_k(-i)}{f_k(3i)} + \frac{f_k(i)}{f_k(-3i)} \right) + |f_k(i)|^2 = 0. \quad (\text{A } 8)$$

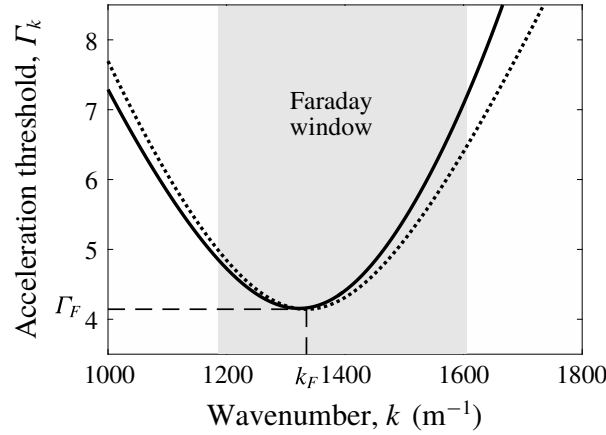


FIGURE 9. Faraday threshold Γ_k of each wavenumber k . The dashed line represents a truncation to the two first modes. The solid line corresponds to the truncation to either four or twenty modes – which are indistinguishable from each other.

The curve Γ_k is given by the smallest positive solution to this equation, namely,

$$\frac{\alpha}{|f_k(3i)|} = \frac{1}{\sqrt{2}} \sqrt{\left(1 + \frac{f_k(-i)}{f_k(3i)} + \frac{f_k(i)}{f_k(-3i)}\right) - \sqrt{\left(1 + \frac{f_k(-i)}{f_k(3i)} + \frac{f_k(i)}{f_k(-3i)}\right)^2 - 4 \frac{|f_k(i)|^2}{|f_k(3i)|^2}}}. \quad (\text{A } 9)$$

Around k_F , in the low-viscosity limit (cf. appendix B), one can show that for $k \sim k_F$, $|f_k(i)|^2 \sim 4\gamma_k^2$ and $|f_k(3i)|^2 \sim 64$. Consequently, $|f(i)|_F^2/|f(3i)|_F^2 \sim \gamma_F^2/16 \ll 1$. Equation (A 9) can then be expanded in Taylor series, which yields

$$\Gamma_k = \frac{\Omega^2 |f_k(i)|}{2gk} \left(1 - \frac{|f_k(i)|^2}{2|f_k(3i)|^2} \frac{1}{(1 + f_k(-i)/f_k(3i) + f_k(i)/f_k(-3i))^2}\right). \quad (\text{A } 10)$$

The correction to Γ_k corresponding to the addition of four terms instead of two in the Floquet ansatz is of the order of $|f(i)|^2/2|f(3i)|^2 \sim \gamma_F^2/32 < 1\%$ in the limit of small viscosity and for $k \sim k_F$.

Figure 9 represents the curve Γ_k , calculated with a truncation to 2 modes, to 4 modes and to 20 modes respectively. The first two modes already provide a very good estimation of the curve Γ_k , especially for $k \simeq k_F$. The curves Γ_k for four and twenty modes are almost indistinguishable. For walker parameters (cf. table 1), $\Gamma_F = 4.155$ with a truncation to 20 modes whereas $\Gamma_F = 4.144$ with a truncation to only 2 modes. Similarly, the Faraday wavelength is $\lambda_F = 4.749$ mm with 20 modes, instead of $\lambda_F = 4.706$ with two modes. In both cases, the relative error from the truncation to two modes is less than 1%.

Appendix B. Low-viscosity asymptotic behaviour

B.1. Preliminary calculations

The function f_k and its derivatives are first recalled:

$$f_k(s) = (s + \gamma_k)^2 + \omega_k^2 - \gamma_k^{3/2} \sqrt{\gamma_k + 2s}, \quad (\text{B } 1)$$

$$\dot{f}_k(s) = 2(s + \gamma_k) - \frac{\gamma_k^{3/2}}{\sqrt{\gamma_k + 2s}}, \quad (\text{B } 2)$$

$$\ddot{f}_k(s) = 2 + \left(\frac{\gamma_k}{\gamma_k + 2s} \right)^{3/2}. \quad (\text{B } 3)$$

They are all self-conjugated, so e.g. $f_k(s^*) = f_k(s)^*$.

In order to evaluate them in $s = \pm i$, we first calculate

$$\sqrt{\gamma_k \pm 2i} = \frac{1}{\sqrt{2}} \sqrt{\sqrt{\gamma_k^2 + 4} + \gamma_k} \pm \frac{i}{\sqrt{2}} \sqrt{\sqrt{\gamma_k^2 + 4} - \gamma_k}. \quad (\text{B } 4)$$

Only the positive real part is considered, since the vertical fluid velocity must remain finite when $z \rightarrow -\infty$ in (2.19).

Therefore, without any approximation,

$$\text{Re}(f_k(i)) = \gamma_k^2 + \omega_k^2 - 1 - \frac{\gamma_k^{3/2}}{\sqrt{2}} \sqrt{\sqrt{\gamma_k^2 + 4} + \gamma_k}, \quad (\text{B } 5)$$

$$\text{Im}(f_k(i)) = 2\gamma_k - \frac{\gamma_k^{3/2}}{\sqrt{2}} \sqrt{\sqrt{\gamma_k^2 + 4} - \gamma_k}, \quad (\text{B } 6)$$

$$\text{Re}(\dot{f}_k(i)) = 2\gamma_k - \frac{\gamma_k^{3/2}}{\sqrt{2}\sqrt{\gamma_k^2 + 4}} \sqrt{\sqrt{\gamma_k^2 + 4} + \gamma_k}, \quad (\text{B } 7)$$

$$\text{Im}(\dot{f}_k(i)) = 2 + \frac{\gamma_k^{3/2}}{\sqrt{2}\sqrt{\gamma_k^2 + 4}} \sqrt{\sqrt{\gamma_k^2 + 4} - \gamma_k}, \quad (\text{B } 8)$$

$$\text{Re}(\ddot{f}_k(i)) = 2 + \frac{\gamma_k^{3/2}}{\sqrt{2}(\gamma_k^2 + 4)^{3/2}} \left(\gamma_k \sqrt{\sqrt{\gamma_k^2 + 4} + \gamma_k} - 2\sqrt{\sqrt{\gamma_k^2 + 4} - \gamma_k} \right), \quad (\text{B } 9)$$

$$\text{Im}(\ddot{f}_k(i)) = -\frac{\gamma_k^{3/2}}{\sqrt{2}(\gamma_k^2 + 4)^{3/2}} \left(\gamma_k \sqrt{\sqrt{\gamma_k^2 + 4} - \gamma_k} + 2\sqrt{\sqrt{\gamma_k^2 + 4} + \gamma_k} \right). \quad (\text{B } 10)$$

When the viscosity is sufficiently small, $\gamma_k \ll 1$, and both $f_k(i)$ and its derivatives can be approximated with a Taylor series in γ_k :

$$\text{Re}(f_k(i)) = \omega_k^2 - 1 - \gamma_k^{3/2} + \gamma_k^2 - \frac{1}{4}\gamma_k^{5/2} + O(\gamma_k^{7/2}), \quad (\text{B } 11)$$

$$\text{Im}(f_k(i)) = 2\gamma_k - \gamma_k^{3/2} + \frac{1}{4}\gamma_k^{5/2} + O(\gamma_k^{7/2}), \quad (\text{B } 12)$$

$$\text{Re}(\dot{f}_k(i)) = 2\gamma_k - \frac{\gamma_k^{3/2}}{2} - \frac{\gamma_k^{5/2}}{8} + O(\gamma_k^{7/2}), \quad (\text{B } 13)$$

$$\text{Im}(\dot{f}_k(i)) = 2 + \frac{\gamma_k^{3/2}}{2} - \frac{\gamma_k^{5/2}}{8} + O(\gamma_k^{7/2}), \quad (\text{B } 14)$$

$$\text{Re}(\ddot{f}_k(i)) = 2 - \frac{\gamma_k^{3/2}}{4} + \frac{3}{16}\gamma_k^{5/2} + O(\gamma_k^{7/2}), \quad (\text{B } 15)$$

$$\text{Im}(\ddot{f}_k(i)) = -\frac{\gamma_k^{3/2}}{4} - \frac{3}{16}\gamma_k^{5/2} + O(\gamma_k^{7/2}). \quad (\text{B } 16)$$

The following combinations of $f_k(i)$ and its derivatives are regularly called in § 2:

$$|f_k(i)|^2 = (\omega_k^2 - 1)^2 - 2(\omega_k^2 - 1)\gamma_k^{3/2} + 2(\omega_k^2 + 1)\gamma_k^2 - \left(4 + \frac{\omega_k^2 - 1}{2}\right)\gamma_k^{5/2} + 2\gamma_k^3 + O(\gamma_k^{7/2}), \quad (\text{B } 17)$$

$$f_k(i)\dot{f}_k(-i) + f_k(-i)\dot{f}_k(i) = 4\gamma_k(\omega_k^2 + 1) - (\omega_k^2 + 3)\gamma_k^{3/2} - \left(\frac{\omega_k^2 + 3}{4}\right)\gamma_k^{5/2} + O(\gamma_k^3), \quad (\text{B } 18)$$

$$\frac{f_k(i)\ddot{f}_k(-i) + f_k(-i)\ddot{f}_k(i)}{2} + \dot{f}_k(i)\dot{f}_k(-i) = 2(\omega_k^2 + 1) - \frac{\omega_k^2 - 1}{4}\gamma_k^{3/2} + 6\gamma_k^2 + \frac{3\omega_k^2 - 59}{16}\gamma_k^{5/2} + O(\gamma_k^{7/2}). \quad (\text{B } 19)$$

Finally, the two first k -derivative of $|f_k(i)|^2$, useful for the calculation of the Faraday threshold and the diffusion coefficient, are expanded as:

$$\frac{k}{2} \frac{\partial |f_k(i)|^2}{\partial k} = 2k(\omega_k^2 - 1)\omega_k c_k - (2k\omega_k c_k + 3(\omega_k^2 - 1))\gamma_k^{3/2} + (2k\omega_k c_k + 4(\omega_k^2 + 1))\gamma_k^2 - \left[\frac{k\omega_k c_k}{2} + \frac{5}{2}\left(4 + \frac{\omega_k^2 - 1}{2}\right)\right]\gamma_k^{5/2} + 6\gamma_k^3 + O(\gamma_k^{7/2}), \quad (\text{B } 20)$$

$$\frac{\partial^2 |f_k(i)|^2}{\partial k^2} = 8\omega_k^2 c_k^2 + 4(\omega_k^2 - 1)(c_k^2 + \omega_k \dot{c}_k) - \left(4(c_k^2 + \omega_k \dot{c}_k) + \frac{12}{k^2}(\omega_k^2 - 1) + \frac{24\omega_k c_k}{k}\right)\gamma_k^{3/2} + O(\gamma_k^2), \quad (\text{B } 21)$$

where $c_k = \partial\omega_k/\partial k$ is the dimensionless group velocity, and $\dot{c}_k = \partial c_k/\partial k$.

The low-viscosity approximation of the damping of Faraday waves reads

$$\delta_k^+ = - \frac{\left((\omega_k^2 - 1)^2 - 2(\omega_k^2 - 1)\gamma_k^{3/2} + 2(\omega_k^2 + 1)\gamma_k^2 - \left(4 + \frac{\omega_k^2 - 1}{2}\right)\gamma_k^{5/2} - (2\Gamma gk/\Omega^2)^2\right)}{4(\omega_k^2 + 1)\gamma_k - (\omega_k^2 + 3)\gamma_k^{3/2} - (\omega_k^2 + 3)\gamma_k^{5/2}/4} + O(\gamma_k^3). \quad (\text{B } 22)$$

B.2. Determination of k_F , ω_F and c_F

The most unstable wavenumber k_F is defined as the solution of

$$\frac{k}{2} \frac{\partial |f_k(i)|^2}{\partial k} = |f_k(i)|^2. \quad (\text{B } 23)$$

According to the approximations (B 17) and (B 20), it satisfies

$$\begin{aligned} & (\omega_F^2 - 1)(\omega_F^2 - 1 - 2k_F\omega_F c_F) + (\omega_F^2 - 1 + 2k_F\omega_F c_F)\gamma_F^{3/2} \\ & - (2(\omega_F^2 + 1) + 2k_F\omega_F c_F)\gamma_F^2 + \left(6 + \frac{3}{4}(\omega_F^2 - 1) + \frac{k_F\omega_F c_F}{2}\right)\gamma_F^{5/2} \\ & - 4\gamma_F^3 + O(\gamma_F^{7/2}) = 0, \end{aligned} \quad (\text{B } 24)$$

where $\omega_F = \omega_{k_F}$ and $c_F = c_{k_F}$.

When the viscosity ν is identically zero, $\gamma_F = 0$ and (B 24) is simplified

$$(\omega_F^2 - 1)(\omega_F^2 - 1 - 2k_F\omega_F c_F) = 0. \quad (\text{B } 25)$$

The second factor is strictly negative for $k_F > 0$, so k_F must satisfy $\omega_F^2 = 1$. We define

$$k_0 = \lim_{\nu \rightarrow 0} k_F \quad (\text{B } 26)$$

for which $\omega_0 = \omega_{k_0} = 1$, $c_0 = c_{k_0}$ and $\gamma_0 = \gamma_{k_0}$.

The damping factor $\gamma_0 = 4\nu k_0^2 / \Omega$ is a better basis for an asymptotic expansion at low viscosity, since it does not directly depend on the Faraday wavenumber k_F , which is still unknown. Equation (B 24) can be rewritten in terms of γ_0 since $\gamma_F = k_F^2 \gamma_0 / k_0^2$:

$$\begin{aligned} & (\omega_F^2 - 1)(\omega_F^2 - 1 - 2k_F\omega_F c_F) + (\omega_F^2 - 1 + 2k_F\omega_F c_F) \left(\frac{k_F}{k_0}\right)^3 \gamma_0^{3/2} \\ & - (2(\omega_F^2 + 1) + 2k_F\omega_F c_F) \left(\frac{k_F}{k_0}\right)^4 \gamma_0^2 + \left(6 + \frac{3}{4}(\omega_F^2 - 1) + \frac{k_F\omega_F c_F}{2}\right) \left(\frac{k_F}{k_0}\right)^5 \gamma_0^{5/2} \\ & - 4 \left(\frac{k_F}{k_0}\right)^6 \gamma_0^3 + O(\gamma_0^{7/2}) = 0. \end{aligned} \quad (\text{B } 27)$$

We formulate the ansatz

$$k_F = k_0 + \epsilon \gamma_0^\alpha + o(\gamma_0^\alpha), \quad (\text{B } 28)$$

where $\alpha > 0$, from which we directly infer that

$$\omega_F^2 = 1 + 2c_0 \epsilon \gamma_0^\alpha + o(\gamma_0^\alpha) \quad (\text{B } 29)$$

and

$$2\omega_F c_F = 2c_0 + \frac{24\sigma}{\rho \Omega^2} k_0 \epsilon \gamma_0^\alpha + o(\gamma_0^\alpha). \quad (\text{B } 30)$$

Inserting this ansatz in (B 27) yields

$$2c_0 k_0 (\gamma_0^{3/2} - 2c_0 \epsilon \gamma_0^\alpha) + o(\gamma_0^\alpha) + o(\gamma_0^{3/2}) = 0 \quad (\text{B } 31)$$

from which we deduce $\alpha = 3/2$ and $\epsilon = 1/(2c_0)$, so $k_F = k_0 + (1/2c_0)\gamma_0^{3/2} + o(\gamma_0^{3/2})$. We can iterate this procedure with a new ansatz $k_F = k_0 + \gamma_0^{3/2}/(2c_0) + \epsilon \gamma_0^\alpha + o(\gamma_0^\alpha)$ in order to find higher-order corrections. This yields:

$$\left. \begin{aligned} k_F &= k_0 + \frac{1}{2c_0} \gamma_0^{3/2} - \frac{2 + k_0 c_0}{2k_0 c_0^2} \gamma_0^2 + \frac{12 + k_0 c_0}{8k_0 c_0^2} \gamma_0^{5/2} + \left(\frac{1}{4k_0 c_0^2} - \frac{d_0}{8c_0^3}\right) \gamma_0^3 + O(\gamma_0^{7/2}), \\ \omega_F^2 &= 1 + \gamma_0^{3/2} - \frac{2 + k_0 c_0}{k_0 c_0} \gamma_0^2 + \frac{12 + k_0 c_0}{4k_0 c_0} \gamma_0^{5/2} + O(\gamma_0^3), \\ \gamma_F &= \gamma_0 + \frac{\gamma_0^{5/2}}{k_0 c_0} + O(\gamma_0^3), \end{aligned} \right\} \quad (\text{B } 32)$$

where $d_0 = (\partial^2 \omega_k^2 / \partial k^2)(k_0)$.

From there, we evaluate:

$$\left. \begin{aligned} \operatorname{Re}(f_F(i)) &= \frac{-2}{k_0 c_0} \gamma_0^2 + \frac{3}{k_0 c_0} \gamma_0^{5/2} + O(\gamma_0^3), \\ \operatorname{Im}(f_F(i)) &= 2\gamma_0 - \gamma_0^{3/2} + \frac{8 + k_0 c_0}{4k_0 c_0} \gamma_0^{5/2} + O(\gamma_0^3), \\ f_F(i)\dot{f}_F(-i) + f_F(-i)\dot{f}_F(i) &= 8\gamma_0 - 4\gamma_0^{3/2} + O(\gamma_0^2), \\ \frac{f_F(i)\ddot{f}_F(-i) + f_F(-i)\ddot{f}_F(i)}{2} + \dot{f}_F(i)\dot{f}_F(-i) &= 2\gamma_0^{3/2} + 4\left(1 - \frac{1}{k_0 c_0}\right)\gamma_0^2 \\ &\quad + \frac{12 + 9k_0 c_0}{2k_0 c_0} \gamma_0^{5/2} + O(\gamma_0^3), \\ \left. \frac{\partial^2 |f_k(i)|^2}{\partial k^2} \right|_F &= 8c_0^2 + O(\gamma_0^{3/2}). \end{aligned} \right\} \quad (\text{B } 33)$$

Appendix C. Inverse Hankel transform

In this appendix, we evaluate the integral (2.52), namely

$$\zeta_1(r, \tau) = \int_0^\infty B_k^+ \cos(\tau + \theta_k^+) e^{\delta_k^+(\tau - \tau_i)} J_0(kr) k \, dk, \quad (\text{C } 1)$$

with the saddle-node method. First, δ_k^+ is replaced by its Taylor series around k_F :

$$\delta_k^+ \simeq \delta_F - D(k - k_F)^2. \quad (\text{C } 2)$$

Substitution in the integral yields

$$\zeta_1(r, \tau) = e^{\delta_F(\tau - \tau_i)} \int_0^\infty B_k^+ \cos(\tau + \theta_k^+) e^{-D(\tau - \tau_i)(k - k_F)^2} J_0(kr) k \, dk. \quad (\text{C } 3)$$

In the limit $k_F r \ll 1$, the Bessel function does not oscillate significantly. Then for $\tau \gg \tau_i$,

$$\begin{aligned} \zeta_1(r, \tau) &\simeq e^{\delta_F(\tau - \tau_i)} B_F^+ \cos(\tau + \theta_F^+) J_0(k_F r) k_F \int_0^\infty e^{-D(\tau - \tau_i)(k - k_F)^2} \, dk \\ &= B_F^+ k_F e^{\delta_F(\tau - \tau_i)} \cos(\tau + \theta_F^+) \sqrt{\frac{\pi}{D(\tau - \tau_i)}} J_0(k_F r). \end{aligned} \quad (\text{C } 4)$$

For $k_F r \gg 1$, the approximation of J_0 by $\operatorname{Re}\{e^{ikr + i\pi/4}/\sqrt{\pi kr/2}\}$ yields

$$\begin{aligned} \zeta_1(r, \tau) &\simeq e^{\delta_F(\tau - \tau_i)} \int_0^\infty B_k^+ \cos(\tau + \theta_k^+) e^{-D(\tau - \tau_i)(k - k_F)^2} e^{ikr + i\pi/4} \sqrt{\frac{2}{\pi kr}} k \, dk \\ &\simeq e^{\delta_F(\tau - \tau_i)} B_F^+ \cos(\tau + \theta_F^+) J_0(k_F r) k_F \int_0^\infty e^{-D(\tau - \tau_i)(k - k_F)^2} e^{i(k - k_F)r} \, dk. \end{aligned} \quad (\text{C } 5)$$

The remaining integral is calculated as

$$\begin{aligned} \int_0^\infty e^{-D(\tau - \tau_i)(k - k_F)^2} e^{i(k - k_F)r} \, dk &= \exp\left[-\frac{r^2}{4D(\tau - \tau_i)}\right] \int_0^\infty e^{-D(\tau - \tau_i)[k - k_F - (ir)/(2D(\tau - \tau_i))]^2} \, dk \\ &= \exp\left[-\frac{r^2}{4D(\tau - \tau_i)}\right] \sqrt{\frac{\pi}{D(\tau - \tau_i)}}. \end{aligned} \quad (\text{C } 6)$$

Therefore,

$$\zeta_1(r, \tau) \simeq B_F^+ k_F e^{\delta_F(\tau - \tau_i)} \cos(\tau + \theta_F^+) \sqrt{\frac{\pi}{D(\tau - \tau_i)}} J_0(k_F r) \exp\left[-\frac{r^2}{4D(\tau - \tau_i)}\right]. \quad (\text{C } 7)$$

Coincidentally, this asymptotic behaviour at $k_F r \gg 1$ is also valid for $k_F r \ll 1$. Indeed, equations (C4) and (C7) are identical as soon as $\tau - \tau_i \gg r^2/(4D)$. This condition is necessarily satisfied as soon as $\tau - \tau_i \gg 1/(4Dk_F^2)$ for $k_F r \ll 1$. For walkers, with numerical values in table 1, $1/4Dk_F^2 \sim 0.08$ so (C7) is equivalently valid at $k_F r \gg 1$ and $k_F r \ll 1$.

Appendix D. Surface waves emitted by a walker

The Faraday wave profile resulting from the previous bounces of the walker at the positions $\mathbf{r}_n = -(\tau - \tau_n)\mathbf{v}$ and the times $\tau_n = \tau_i + 2\pi n$ ($n \in \mathbb{Z}$) reads

$$\begin{aligned} \zeta_w(\mathbf{r}, \tau) &= \sum_{n=-\infty}^{[(\tau - \tau_i)/2\pi]} \zeta_1(|\mathbf{r} - \mathbf{r}_n|, \tau - \tau_n) \\ &= \sum_{n=-\infty}^{[(\tau - \tau_i)/2\pi]} B_F^+ k_F \sqrt{\frac{\pi}{D(\tau - \tau_n)}} J_0(k_F |\mathbf{r} - \mathbf{r}_n|) \cos(\tau + \theta_F^+) \\ &\quad \times \exp\left(-\frac{\tau - \tau_n}{2\pi Me} - \frac{|\mathbf{r} - \mathbf{r}_n|^2}{4D(\tau - \tau_n)}\right), \end{aligned} \quad (\text{D } 1)$$

where $[(\tau - \tau_i)/2\pi]$ denotes the largest integer smaller than $(\tau - \tau_i)/2\pi$, and where the origin $\mathbf{r} = 0$ is placed at the position of the walker at time τ . We replace the sum in the previous equation by an integration over the delay time $u = \tau - \tau_n$ going from 0 to ∞ . This is justified for $Mer^2 \gg 2\pi D$, since the sum can then be seen as an approximate Riemann integral where the interpolated function does not significantly vary over one bin.

The Graf's theorem for zeroth-order Bessel function reads

$$J_0(k_F |\mathbf{r} - \mathbf{r}_n|) = \sum_{n=-\infty}^{\infty} J_n(k_F r) J_n(k_F r_n) e^{-in\theta} \quad (\text{D } 2)$$

with

$$\theta = \arccos\left(\frac{\mathbf{v} \cdot \mathbf{r}}{vr}\right). \quad (\text{D } 3)$$

Since Bessel functions are defined as

$$J_n(k_F r) = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\varphi e^{ik_F r \sin \varphi} e^{-in\varphi} \quad (\text{D } 4)$$

and since $\sum [e^{i(\theta - \varphi - \varphi')}]^n = \delta(\theta - \varphi - \varphi')$,

$$J_0(k_F |\mathbf{r} - \mathbf{r}_n|) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp[ik_F r \sin(\theta - \varphi)] \exp(-ik_F r_n \sin \varphi) d\varphi. \quad (\text{D } 5)$$

Substitution in (D 1) yields

$$\zeta_w(\mathbf{r}, \tau) = \frac{B_F^+ k_F}{2\sqrt{\pi D}} \cos(\tau + \theta_F^+) \int_{-\pi}^{\pi} d\varphi \exp[ik_F r \sin(\theta - \varphi)] \\ \times \int_0^\infty \frac{1}{\sqrt{u}} \exp\left(-\frac{u}{2\pi Me} - \frac{r^2}{4Du} - \frac{vr \cos \theta}{2D} - \frac{v^2 u}{4D} + ik_F uv \sin \varphi\right) du. \quad (\text{D } 6)$$

Using the identity

$$\int_0^\infty \frac{1}{\sqrt{u}} \exp(-\alpha u - \beta/u) du = \sqrt{\frac{\pi}{\alpha}} \exp(-2\sqrt{\alpha\beta}) \quad (\text{D } 7)$$

for $\text{Re}(\alpha) > 0$ and $\beta > 0$, we obtain

$$\zeta_w(\mathbf{r}, \tau) = \frac{B_F^+ k_F}{2\sqrt{D}} \cos(\tau + \theta_F^+) \exp\left(-\frac{vr \cos \theta}{2D}\right) \\ \times \int_{-\pi}^{\pi} d\varphi \frac{\exp[ik_F r \sin(\theta - \varphi)]}{\sqrt{a - ik_F v \sin \varphi}} \exp\left(-\frac{r}{\sqrt{D}} \sqrt{a - ik_F v \sin \varphi}\right), \quad (\text{D } 8)$$

where

$$a = \frac{1}{2\pi Me} + \frac{v^2}{4D}. \quad (\text{D } 9)$$

For large a such that $k_F v/a \ll 1$ (for walkers, with numerical values in table 1, this corresponds to a distance to threshold $\mathcal{M} < 4.8$), we can justify the approximation

$$\sqrt{a - ik_F v \sin \varphi} \simeq \sqrt{a} - i \frac{k_F v}{2\sqrt{a}} \sin \varphi. \quad (\text{D } 10)$$

A reverse application of Graf's theorem (D 5) yields

$$\zeta_w(\mathbf{r}, \tau) = \frac{B_F^+ k_F}{2\sqrt{Da}} \cos(\tau + \theta_F^+) J_0\left(k_F \left|\mathbf{r} + \frac{r}{2\sqrt{Da}} \mathbf{v}\right|\right) \\ \times \exp\left[-\left(\sqrt{\frac{a}{D}} + \frac{v \cos \theta}{2D}\right) r\right], \quad (\text{D } 11)$$

i.e. we obtain a Doppler-shifted Bessel profile which is exponentially attenuated in space. The damping length scale, defined as $l(\theta)$, depends on the orientation θ with respect to the walking direction $\mathbf{v}/v$:

$$l(\theta) = \frac{2D}{v \cos \theta + \sqrt{v^2 + 2D/\pi Me}}. \quad (\text{D } 12)$$

In the special case of a stationary bouncer with $v = 0$, we obtain the spatial damping length $l = \sqrt{2\pi Me D}$ which diverges in the limit $Me \rightarrow \infty$.

An accurate analytical evaluation of (2.57) in the more general case $k_F v/a \gtrsim 1$ is more involved and beyond the scope of this work. However, we can straightforwardly provide an upper limit for the wave amplitude which holds in any case, namely through the inequality

$$\left| \frac{\zeta_w(\mathbf{r}, \tau)}{\cos(\tau + \theta_F^+)} \right| \leq \frac{\pi B_F^+ k_F}{\sqrt{Da}} \exp\left[-\frac{r}{l(\theta)}\right], \quad (\text{D } 13)$$

obtained by substituting $|J_0(k_F |\mathbf{r} - \mathbf{r}_n|)| \leq 1$ within (D 1).

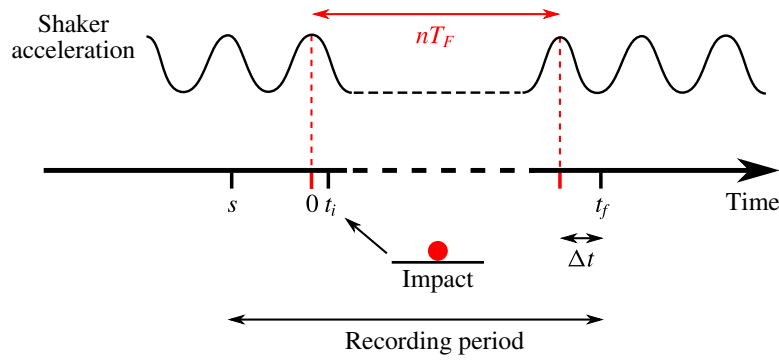


FIGURE 10. (Colour online) Measurement of the impact time t_i , from the end time of the recording t_f and the synchronisation delay Δt . Definitions of t_i , t_f and Δt are provided in the main text.

Appendix E. Experimental measurement of the impact phase

Measuring the impact phase requires a good synchronisation of the high-speed camera with the vibration $g(t)$ of the shaker. For this purpose, the trigger of the camera that marks the end of the recording is also sent to the DACQ. Both signals are simultaneously processed with a Labview interface that outputs the time delay $\Delta t \in [0, T_F/2]$ from the last recorded maximum of $g(t)$ to the end of the recording.

We then define the absolute origin of time $t=0$ as the last maximum of $g(t)$ before impact such that the end time of the recording is given by

$$t_f = \Delta t + nT_F, \quad n \in \mathbb{Z} \quad (\text{E } 1)$$

in this reference time (figure 10). The impact time $t_i \in [0, T_F]$ is then calculated as

$$t_i = \Delta t + nT_F - (t_f - t_i) = (t_i - t_f + \Delta t)[T_F], \quad (\text{E } 2)$$

where $X[T_F]$ means X modulo T_F .

The impact phase τ_i is then

$$\tau_i = \frac{2\pi t_i}{T_F} = \frac{(\Delta t + (t_i - t_f))[T_F]}{T_F}. \quad (\text{E } 3)$$

Practically, $t_f - t_i$ is computed as a number of frames divided by the acquisition frequency.

The digital output of the DACQ is refreshed at 8 kHz, so the error on Δt is less than 0.125 ms. The impact is detected with an accuracy of (± 1 frames), which corresponds to an error on $t_f - t_i$ less than 0.25 ms. The absolute error on t_i (respectively τ_i) is then less than 0.375 ms (respectively 1.5 %).

Appendix F. Validity of the linear Navier–Stokes equations for walkers

The linear theory of Faraday waves for walkers developed in this manuscript is valid as long as the amplitude of the waves is significantly smaller than both the Faraday wavelength λ_F and the thickness of the viscous boundary layer. This latter, here denoted δ_v , is of the order of $\sqrt{2\nu/\Omega}$. The amplitude of the wave field generated by classical walkers (oil viscosity 20 cSt, forcing frequency 80 Hz) is always less than 12 μm in the vicinity of a single walker, shortly after the impact of the droplet on

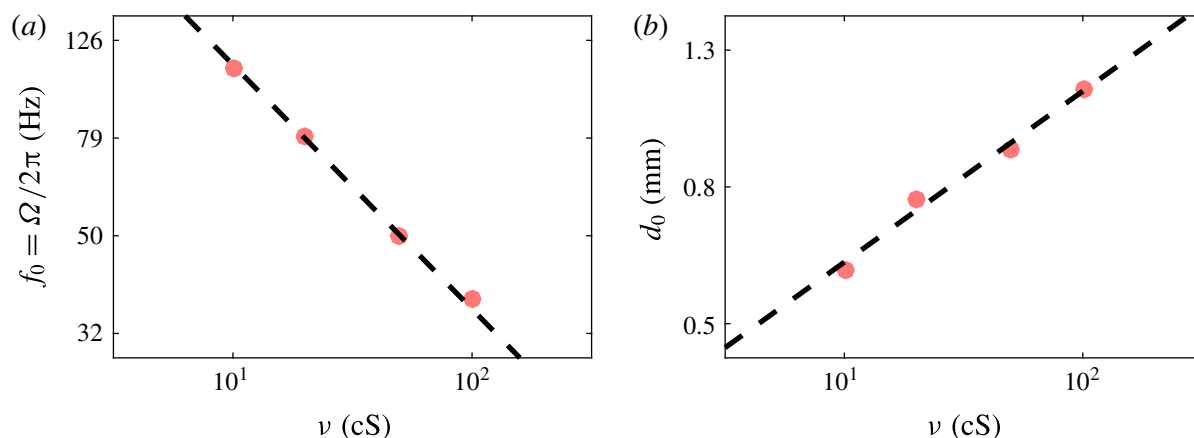


FIGURE 11. (Colour online) Data from Eddi (2011). (a) Log–log scaling of the evolution of the optimal forcing frequency f_0 of walkers with kinematic viscosity ν . The dashed line corresponds to $f_0 \nu^{1/2} = 3.55 \text{ m s}^{-3/2}$ ($f_0 \propto \nu^{-1/2}$). (b) Log–log scaling of the evolution of the optimal diameter d_0 of walking droplets with kinematic viscosity ν . The dashed line corresponds to $d_0/\nu^{1/4} = 3.46 \times 10^{-3} \text{ m}^{1/2} \text{ s}^{1/4}$ ($d_0 \propto \nu^{1/4}$).

the bath (Damiano *et al.* 2016). This amplitude is therefore more than one order of magnitude smaller than both (i) the considered wavelength $\lambda_F \simeq 4.7 \text{ mm}$ and (ii) the viscous boundary layer $\delta_v \simeq 300 \text{ }\mu\text{m}$. Since walkers are commonly studied below the Faraday threshold, the amplitude of their wave decreases with time, despite the forcing, so the measured $12 \text{ }\mu\text{m}$ can be seen as an upper bound for the wave amplitude.

The amplitude of the waves right below the impacting droplet might be larger than the measured $12 \text{ }\mu\text{m}$ on its sides. This region is challenging to observe by optical means. Nevertheless, a theoretical bound for the deflection of the bath right after impact can be obtained. This deflection mostly originates from the kinetic energy of the falling droplet. At impact, this energy is temporarily converted into surface energy, mostly associated with the bath deflection. A part of this stored energy is then restituted to the translation of the droplet (Molacek & Bush 2013), another part generates the surface wave accompanying the walker, and a last part is dissipated by viscous flows (within both the droplet and the bath) during impact. An upper bound H on the bath deflection (and on the subsequent surface waves) can be found by balancing the surface energy of the bath deflection and the kinetic energy of the incoming droplet. According to experiments (e.g. Protiere *et al.* 2006; Molacek & Bush 2013), the free fall of the droplet lasts approximately $2/3$ of the bouncing period $2/f$ (where $f = \Omega/\pi$ is the forcing frequency). Consequently, the impact speed of the droplet is $v = 2g/(3f)$. To the leading order, the crater is assumed to be a spherical cap whose curvature matches half the curvature of the droplet (Gopinath & Koch 2001). The excess of surface of this spherical cap is approximately $2\pi H^2$, independently of the droplet radius. The energy balance then yields

$$H \simeq \sqrt{\frac{\rho d^3 v^2}{24\sigma}} \simeq \sqrt{\frac{\rho d^3 g^2}{54\sigma f^2}} \simeq 82 \text{ }\mu\text{m}, \quad (\text{F } 1)$$

with a droplet diameter $d \simeq 0.8 \text{ mm}$. This pessimistic upper bound H on the bath deflection, which did not take into account any dissipation or partial transfer of energy, is nevertheless again smaller than both the wavelength λ_F and the boundary layer thickness δ_v .

In the discussion, we suggested that quantum-like phenomena would be likely to appear for large damping length l , i.e. for vanishing viscosity. However, the boundary layer thickness δ_v would then decrease and could possibly become smaller than the wave amplitude, which would invalidate a description of walkers with linear waves. There have been very few experimental attempts at generating walkers with viscosity different from 20 cSt or 50 cSt (Protiere *et al.* 2005; Eddi 2011). For each viscosity in the range $\nu \in [10, 100]$ cSt, there is a narrow window of forcing frequency $f_0(\nu) \propto \nu^{-1/2}$ and droplet size $d_0(\nu) \propto \nu^{1/4}$ in which walkers could be observed (see figure 11). Substituting f_0 and d_0 in (F 1) yields $H \propto \nu^{7/8}$. On the other hand, the boundary layer thickness δ_v would scale proportionally to $\sqrt{\nu/f_0} \propto \nu^{3/4}$. Consequently, we obtain the scaling $H/\delta \propto \nu^{1/8}$, i.e. the wave amplitude H decreases faster than the boundary layer thickness δ when the viscosity is decreased. Therefore, as far as walkers less viscous than $\nu = 20$ cSt are considered, $H \ll \delta_v$ and the associated Faraday waves can be described with a linear theory.

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Interaction of two walkers: Perturbed vertical dynamics as a source of chaos

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Interaction of two walkers: Perturbed vertical dynamics as a source of chaos

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Walkers are dual objects comprising a bouncing droplet dynamically coupled to an underlying Faraday wave at the surface of a vibrated bath. In this paper, we study the wave-mediated interaction of two walkers launched at one another, both experimentally and theoretically. Different outcomes are observed in which either the walkers scatter or they bind to each other in orbits or promenade-like motions. The outcome is highly sensitive to initial conditions, which is a signature of chaos, though the time during which perturbations are amplified is finite. The vertical bouncing dynamics, periodic for a single walker, is also strongly perturbed during the interaction, owing to the superposition of the wave contributions of each droplet. Thanks to a model based on inelastic balls coupled to the Faraday waves, we show that this perturbed vertical dynamics is the source of horizontal chaos in such a system. *Published by AIP Publishing.* <https://doi.org/10.1063/1.5031888>

A walker is a dual macroscopic object that consists of a bouncing droplet coupled to a local Faraday wave on a vibrated bath. If the bath is unconfined, a single walker will move horizontally in a straight line at constant speed. Many recent studies revealed that, in the presence of horizontal confinement, the horizontal trajectory of the walker may become chaotic. The corresponding statistical description is sometimes surprisingly close to what quantum mechanics would predict for a single quantum particle in similar confinement conditions. However, since the droplet is millimeter-sized, it is not a quantum particle: it must obey the deterministic laws of Newton, while quantum mechanics is conventionally seen as probabilistic. Could chaos bridge the gap between these two conceptions? Where does this chaos originate? In this work, we show that the presence of a second walker may generate chaos through a deep perturbation of the internal clock of the walkers: their vertical bouncing dynamics (Fig. 1). Even if the interaction of the walkers is of finite duration, the outcome is incredibly sensitive to initial conditions.

I. INTRODUCTION

In 2005, Couder *et al.*⁸ discovered by serendipity that a millimeter-sized droplet can be tightly coupled to waves at the surface of a bath vertically vibrated slightly below the Faraday instability threshold. Thanks to this vibration, the droplet bounces indefinitely without ever coalescing with the bath. Each droplet's impact generates some local Faraday waves at the bath surface. Because these waves are still present when the droplet impacts again, some horizontal momentum can be transferred from the waves to the droplet, which starts moving horizontally on the bath.³⁹

This association of a wave and a particle in the realm of Newton's laws yields behaviors that are reminiscent

of quantum particles.⁵ In a central force field, a walker experiences orbits with quantized radius and angular momentum.^{27,28,37} A similar quantization is seen when the walker moves in a rotating frame.^{17,23,34} Interactions of the walkers with bath boundaries include non-specular reflexion⁴¹ and tunneling.^{13,33} Confinement of the bath to a small cavity yields chaotic walker trajectories for which the corresponding statistics is similar to that of a quantum particle in a potential well.^{19,20,25,42} The early experiment on Young's slit diffraction of the walkers⁷ has been recently revisited^{1,40} without consensus yet.

Some efforts to rationalize the dynamics of the walkers were focused on the hydrodynamic interaction between the droplet and the waves. When a single droplet walks far from any boundary, the vertical motion of the droplet is frequency-locked to half the vibration frequency of the bath.^{31,39} Several generations of hydrodynamic models^{14,30,32,39} have been proposed to explain how the waves induce a horizontal motion superimposed to this vertical bouncing. Most of them refer to the *stroboscopic* approximation, which assumes that droplet bouncing remains locked on the bath vibration at all time. Under this assumption, two coupled equations describe the walker's dynamics. First, the horizontal momentum of the droplet responds to the local slope of the wave field. And second, the wave field is enriched by the periodic droplet impacts, often assumed to occur at discrete time and space.^{17,20} Since the horizontal displacement of the droplet between successive impacts is much less than the wavelength of the emitted Faraday waves, this sum of impacts can be replaced by an integral, which yields a single integro-differential equation for the walker trajectory.^{6,35} The motion of a single unconfined walker is then shown to be stable (resp. neutrally stable) to perturbations along (resp. perpendicular to) the direction of motion.³⁵

Chaotic horizontal trajectories may appear as soon as the walker is confined, either through external forces^{23,37} or through bath boundaries.^{25,42} The walker is then forced to

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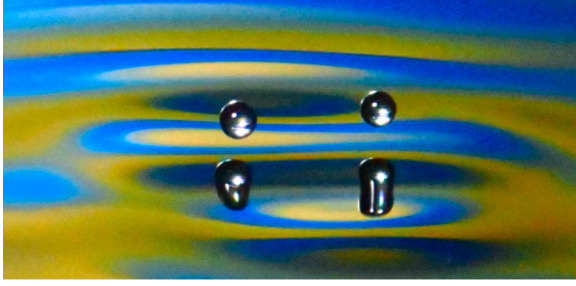


FIG. 1. Walkers are bouncing droplets coupled to the Faraday waves on a vibrated liquid bath. Two walkers sufficiently close to each other will interact through the superposition of their waves.

revisit locations where it previously left standing waves. Eddi *et al.*¹⁴ defined the memory of the wave field as the ratio between the damping time of the waves and the bouncing period. It, therefore, corresponds to the number of past droplet impacts for which the wave contribution is still significantly present in the current wave field. In other words, this is the number of previous impact positions that are recorded in the wave field. The damping time increases when the Faraday instability threshold is approached from below;²⁶ it can, therefore, be tuned by varying the forcing amplitude of the bath. Most of the quantum-like phenomena described above were observed for a memory larger than 20. The mechanisms of transition to chaos for the confined walkers were rationalized in Shirokoff,⁴⁴ Gilet,¹⁹ and Tambasco *et al.*⁴⁵

The wave-mediated interaction of multiple walkers has received significantly less attention. Protière *et al.*³⁹ reported that two identical walkers launched at one another could either scatter or bind to each other. Several bound states were observed, including orbits^{36,39} and *promenade* modes where both droplets walk either side by side^{2,4} or one behind the other.¹⁵ A difference of mass of the two droplets may yield exotic orbits.³⁸ At low memory, larger groups of walkers tend to form crystals,^{16,47} through which phonons can be excited.¹²

In this paper, we revisit the wave-mediated interaction of two walkers launched at one another, with the aim of seeing if it can be chaotic and why. Indeed, in this configuration, the walkers are not confined. So, they are not invited to revisit places where they left waves (with the exception of orbits). Moreover, in the case of scattered trajectories, the interaction time is finite and maybe not sufficient to significantly amplify perturbations of initial conditions. Therefore, the presence of chaos cannot be presumed in this configuration.

Firstly (Secs. II and III), we describe experiments of the walkers launched at one another with various initial conditions, and we characterize the long-timescale horizontal trajectories. We then focus on the short-timescale vertical bouncing dynamics of the droplets during the wave-mediated interaction. Specifically, we aim to check if the *stroboscopic* assumption does still stand though the motion of each walker is not invariant anymore to horizontal translation.

Secondly (Secs. IV and V), we propose a simple model of this interaction based on the coupling of the wave field with two inelastic balls. The purpose of this model is to check to which extent potential perturbations of the vertical dynamics could be the origin of some key features observed

TABLE I. Main parameters and variables. Subscript indices 1 and 2 refer, respectively, to walkers 1 and 2. Subscript index 0 refers to initial conditions. Subscript index i refers to the i th impact.

Parameter	Symbol	Experiment	Model	Units
<i>Variables</i>				
Horizontal position	(x, y) or $\underline{r}$			
Vertical position	z			
Time	t			
<i>Experimental parameters</i>				
Shaker frequency	f	80	80	Hz
Faraday wavelength	λ_F	4.75	4.75	mm
Shaker amplitude	a	156	192	μm
Shaker acceleration	Γ	4	5.215	
Faraday threshold	Γ_F	4.15	5.30	
Proximity to threshold	$\mathcal{M}$	30	30	
<i>Model parameters</i>				
Impact coefficient	$\alpha_1; \alpha_2$	...	[0.1, 1.1]	$\mu\text{m s}$
Velocity coefficient	B	...	[18, 200]	mm s^{-1}
<i>Initial conditions</i>				
Impact parameter	b	[0, $2\lambda_F$]	[0, $6\lambda_F$]	
Initial walker velocity	V_0	[5, 12]	[1.5, 5]	mm s^{-1}
Incident angle	θ_0	$< 5^\circ$	0°	
Initial phase difference	$\Delta\phi_0$	{0, π }	0	
<i>Output variables</i>				
Wave field elevation	$H(\underline{r}, t)$			
Scattering angle	θ			
Walker position	(x_i, y_i) or $\underline{r}_i$			
Impact time	t_i			
Impact phase	ϕ_i			
Vertical impact speed	v_i			
Next take-off time	t_i^0			

experimentally. Consequently, the model is designed in order to either let the vertical dynamics free to evolve or to force this dynamics to comply to the stroboscopic assumption. The model is solved numerically for different sets of parameters and initial conditions. The model predictions are then compared to experimental observations. In Sec. VI, the origin of chaos in wave-mediated interactions is discussed in the light of the present experiments and the model.

The main parameters and variables used through this study are summarized in Table I.

II. EXPERIMENTAL SET-UP

The experimental set-up is sketched in Fig. 2(a). A cylindrical reservoir (depth 1.0 cm, inner diameter 28.0 cm) is filled with 6 mm of silicone oil (*Sigma Aldrich*, density 956 kg m^{-3} , kinematic viscosity $\nu = 20 \text{ cS}$, and surface tension 20.9 kg s^{-2}). It is tightly screwed on top of an electromagnetic shaker (*V400LT*, *Data Physics*) that vibrates vertically. The shaker is connected to an amplifier (*DSA5- 2k*, *Data Physics*) and controlled via a Labview interface through a data acquisition device (*DAQ-National instruments USB 6212*). The elevation of the shaker is $a \cos(2\pi ft)$, where a and f are the amplitude and the frequency of the vibration, respectively. The corresponding maximal acceleration, normalized by gravity g , is $\Gamma = 4\pi^2 f^2 a/g$. The frequency is fixed to $f = 80 \text{ Hz}$, as in most experiments on the walkers.⁵ It results in a Faraday wavelength of $\lambda_F = 4.75 \text{ mm}$ and a

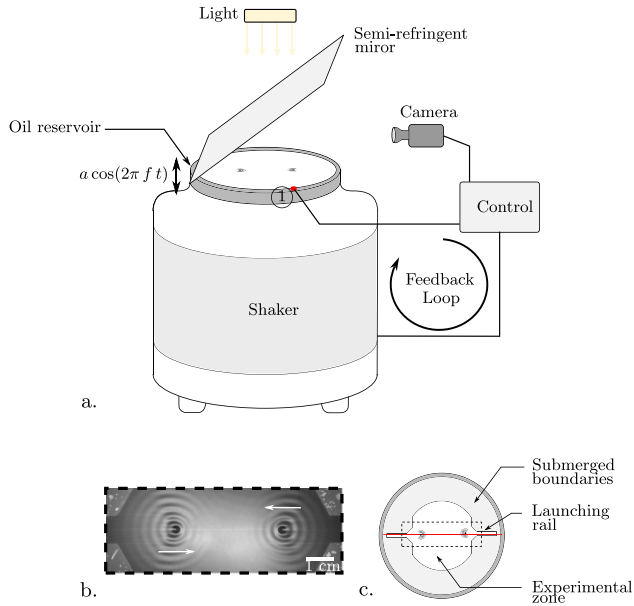


FIG. 2. (a) Experimental set-up. (b) Top view of the experiment as seen by the camera. Both droplets and their surrounding wave field are easily identified. (c) Submerged boundaries, designed to launch droplets towards each other.

Faraday threshold acceleration of $\Gamma_F = 4.15$. An accelerometer (PCB, 352C65) is fixed to the reservoir and connected to the Labview interface. The vibration amplitude is forced to $\Gamma = 4$, which corresponds to $a = 156 \mu\text{m}$, thanks to a feedback loop.⁴³ The proximity to the threshold, defined as $\mathcal{M} = \Gamma_F/(\Gamma_F - \Gamma)$, is then fixed to 30.

The same silicone oil is supplied to home-made piezoelectric dispensers^{24,46} that produce droplets of diameter between 700 and 800 μm . The droplets are synchronously released about 1 cm above the oil bath. As soon as they bounce on the bath, they quickly converge to the walking motion [Fig. 2(b)]. The droplet size is hard to measure with sufficient accuracy. However, it strongly influences the walker speed,³² which can be easily determined by image processing with a relative error less than 1%. In constant forcing conditions, Moláček and Bush³² showed that the walker velocity strictly increases with droplet mass. Thus, any observed difference in the walker speed observed in our experiment can be related to a difference in droplet mass.

Boundaries are placed inside the bath in order to confine and guide the walkers [Fig. 2(c)]. They are made of a laser-cut plastic sheet of thickness 4 mm at the bottom of the reservoir. They are thus submerged and, consequently, they do not radiate any parasitic Faraday wave.³ The oil layer above these boundaries is sufficiently thin to damp the Faraday waves emitted by the walkers.¹³ Two parallel rails of width $2\lambda_F$ are dug into these submerged boundaries in order to launch the droplets towards each other.¹⁶ The rails are either exactly aligned or separated by a distance of about $2\lambda_F$. When the walkers leave the rails, they enter into an experimental zone of diameter 17 cm $\sim 36\lambda_F$ (i.e., much larger than λ_F), where they behave as if they were not confined. They then walk in a straight line until they start interacting with the other walker, i.e., in practice, when the walkers are separated by a couple of λ_F .

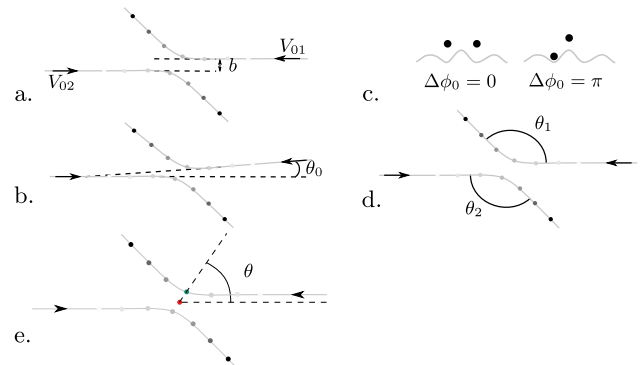


FIG. 3. Parameters that characterize each launch. (a) Impact parameter b and the walker speed (V_{01} , V_{02}); (b) incidence angle, θ_0 ; (c) initial impact phase shift $\Delta\phi_0$; (d) scattering angles θ_1 and θ_2 (in case of a scattering event); and (e) definition of the limping angle θ . The reference point is the center of mass (red dot) and the limping position is the green point (green dot).

The oil bath is both lighted and filmed from above, thanks to a semi-refractive mirror. Droplet trajectories and associated wave fields are recorded with a camera (Basler acA2040-180km) controlled in Labview, at a frequency $f/4$ or $f/2$, with an acquisition phase locked to the shaker vibration. The droplet position at each time is accurately measured in *Matlab 2015b* with a pattern matching technique. About 3000 launches are recorded successively, thanks to an automated routine in Labview. The Faraday threshold Γ_F is measured every 5 consecutive launches (roughly every 20 min) in order to compensate for any slight variations of ambient temperature and keep $\mathcal{M}$ constant. In practice, an absolute error of 0.002g on the forcing acceleration yields a relative error of about 13% on $\mathcal{M}$. Each launch consists of (1) progressively increasing the shaker acceleration to the desired value $\Gamma = (\mathcal{M} - 1)\Gamma_F/\mathcal{M}$, (2) generating droplets, (3) recording their interaction, and (4) decreasing the acceleration down to the point where the droplets coalesce with the bath.

Several input parameters are measured *a posteriori* for each launch (Fig. 3): the initial horizontal speed V_{01} and V_{02} of each walker, the impact parameter b , the incident angle θ_0 , and the initial impact phase shift $\Delta\phi_0$ between the walkers (0 if rebounds in phase or π if rebounds in opposition of phase). The impact parameter b is defined as the minimal distance that there would be between both the walkers if they were walking at constant speed without mutual interaction. Similarly, the kind of interaction (scattering, orbits, and promenades, as defined in Fig. 5) and the scattering angles (θ_1 , θ_2) are measured as output parameters.

There is almost no friction between the droplet and the bath, so the droplet trajectory is very sensitive to parasitic air currents. The experimental zone here is protected from

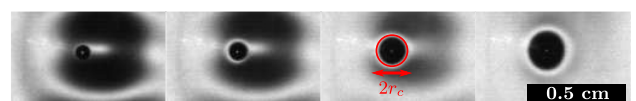


FIG. 4. Top view of the impact of a droplet on the bath and subsequent expansion of the capillary wave. Circles of radius r_c (red circle) are fitted onto the wave crest (bright circle) and the impact time t_i is defined by linear extrapolation to $r_c = 0$. Time between successive images is 1.22 ms.

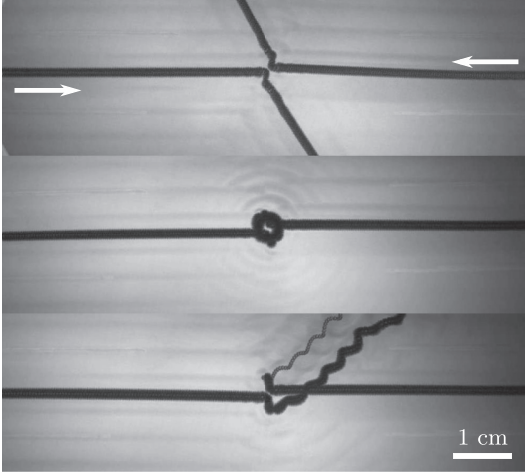


FIG. 5. Time superposition of top views, revealing the walker trajectories. Top, middle, and bottom lines correspond to scattering, orbit, and promenade events, respectively.

the environment, thanks to a cubic box of width 60 cm that surrounds the mirror, the oil reservoir, and the droplet dispensers. However, this efficient protection is not infallible. On rare occasions, the walker trajectories are seen to deviate from straight lines, even when the walkers are neither close to boundaries nor interacting with each other. The launch is discarded when the unexpected curvature $C > 0.1 \text{ m}^{-1}$, which corresponds to a lateral deviation of $\Delta x = 0.05 \lambda_F$ over a walking distance $L = 10 \lambda_F$. The drag force F_D exerted by air flows on a droplet of mass m must be almost balanced by the inertial force normal to the droplet trajectory, $m\Delta x V_0^2/L^2$. Indeed, the friction between the droplet and the bath is small, and the walkers are neutrally stable to perturbations in the direction orthogonal to the motion.³⁵ This drag force, which is then of the order of mCV_0^2 , can be compared to the force exerted by the wave field,⁵ which is estimated as $F_w \sim mgH/\lambda_F$, where H is the characteristic amplitude of the wave field. The ratio $F_D/F_w \sim CV_0^2\lambda_F/(gH)$ is of the order of 5×10^{-4} when a lower bound $H \sim 10 \mu\text{m}$ is considered.⁹ Consequently, the air drag is of negligible influence to the horizontal walker trajectory when the trajectory curvature of isolated walkers is less than 0.1 m^{-1} .

A zoom on the interaction has been recorded with a high-speed camera (*Phantom MIRO110*) and a macro lens (*Zeiss Milvus 2/100M*) at 8000 fps, i.e., at 200 frames per bouncing period. The impact time t_i and the position $\underline{r}_i$ are then inferred from the radial motion of the capillary wave emitted by the droplet impact. The measured radius $r_c(t)$ of the circular wave crest is linearly extrapolated to $r_c(t_i) = 0$ (Fig. 4), and its center corresponds to the impact point.

III. EXPERIMENTAL RESULTS

A. Interaction of two walkers

When the walkers are sufficiently close to each other, they interact through their waves, since the wave field felt by each is the sum of all the waves previously generated by both. This wave-mediated interaction leads to three main trajectory outputs: two bound states (orbits and promenades) and

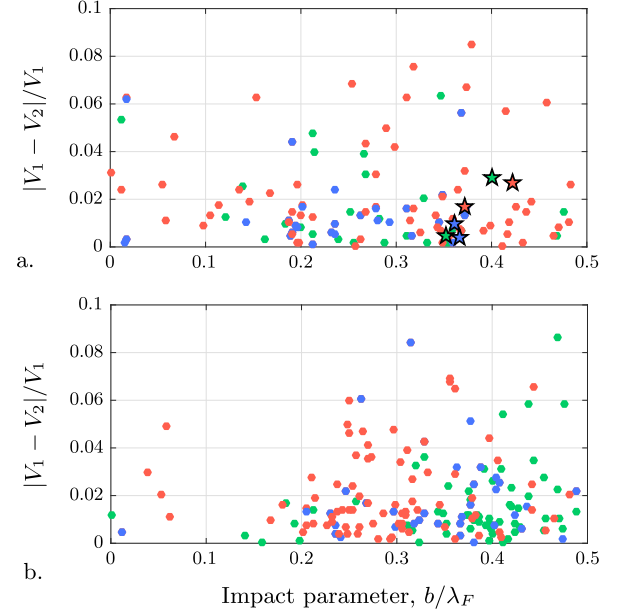


FIG. 6. Experimental outcome of the interaction of two walkers, as a function of the impact parameter b and the relative difference of walking speed $|V_1 - V_2|/V_1$: scattering (red dot), orbiting (green dot), and promenade (blue dot). (a) Droplets initially bounce with the same impact phase, i.e., $\Delta\phi_0 = 0$. (b) Droplets initially bounce with the opposite impact phase, i.e., $\Delta\phi_0 = \pi$. The stars correspond to the 6 trajectories represented in Fig. 7.

one free state (scattering). These main outcomes are depicted in Fig. 5. Orbits are usually circular trajectories, where the walkers orbit around their center of mass. The orbital radius is quantified, with successive values approximately separated by half a Faraday wavelength,³⁹ $\lambda_F/2$. Orbits of increasing radius are increasingly less stable.³⁶ In promenade trajectories, the walkers move together along parallel lines while wobbling in the transverse direction, also approximately separated by a multiple of half a Faraday wavelength,^{2,4} $\lambda_F/2$. In a scattering event, the interaction time of the walkers is finite. Eventually, the walkers spread apart and stop influencing each other. Their trajectory then becomes a straight line again, whose orientation has been shifted by θ_1 and θ_2 , respectively (Fig. 3). Most of the time, scattered angles comply with momentum conservation, which would impose $\theta_1 = \theta_2$ for the walkers of identical mass and speed.

Among the 3000 launches, we could select some for which the memory was $\mathcal{M} = 30$, the mean walker velocity was within $[9, 13] \text{ mm/s}$, the incident angle was smaller than 3° , and the curvature of the approach trajectory was less than 0.1 m^{-1} . The outcome of these launches is represented in Fig. 6 for both initial phase differences of $\Delta\phi_0 = 0$ and $\Delta\phi_0 = \pi$. Outcomes seem completely mixed on the full range of $b/\lambda_F \in [0, 0.5]$, and no zone therein is dominated by a single outcome. Nevertheless, this mixing might be artificially induced by the pooling of walker trajectories with a slightly different mean speed and incident angle.

B. Same initial conditions, different outcomes

We could further select 6 trajectories with extremely similar input parameters: identical impact phase $\Delta\phi_0 = 0$, small

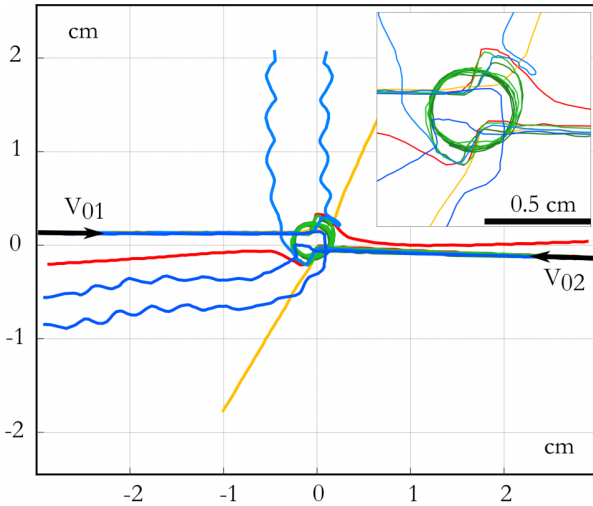


FIG. 7. Superposition of 6 experiments with approximately the same initial conditions and markedly different outcomes: scattering (red and yellow), orbiting (light and dark green), and promenades (light and dark blue).

differences in initial velocity ($|V_{01} - V_{02}|/V_{01} < 0.03$), initial velocity ($11.2 \text{ mm/s} \pm 0.5 \text{ mm/s}$), impact parameter ($b = 2 \text{ mm} \pm 0.45 \text{ mm}$), and incident angle ($\theta_0 = 2^\circ \pm 0.3^\circ$). These 6 launches are plotted in Fig. 7. Remarkably, all three different outcomes are observed in this very narrow range of input parameters. Moreover, several different scattering and promenade angles are produced. The interaction output is here strongly sensitive to the initial conditions.

C. Strong perturbations of the vertical dynamics

In order to understand why this deterministic system is so sensitive to initial conditions, we examine the sequence of impact time and position recorded with the high-speed camera, from which the speed V and phase of impact ϕ_i of each walker can be deduced. This phase is measured relative to the shaker motion, with 2π corresponding to one Faraday period $2/f$. A typical interaction is represented in Fig. 8. When the walkers are launched towards each other, they first proceed in a straight line, at constant speed and impact phase. As they approach each other, they start feeling the other's wave field. Both their velocity and their impact phase fluctuate, and their trajectory deviates from the initial straight line. When the walkers are close to each other, their added contributions to the wave field may result locally in a significantly larger wave amplitude that may be sufficient to destabilize the vertical dynamics of the droplets. Each droplet may then jump lower than expected and already impact the bath after one half of the Faraday period. Both walkers are often destabilized almost at the same time. This event, here called *limping*, corresponds to a strongly premature impact, for which the flight time is decreased by about $1/f$ (Fig. 9). The appearance of limping can be seen as a grazing bifurcation¹⁰ since it corresponds to the emergence of a new contact event between the droplet and the underlying bath. From there, the vertical bouncing dynamics is completely perturbed, which is seen from the large oscillations in both the impact phase and subsequent walker speed. One droplet experiences five

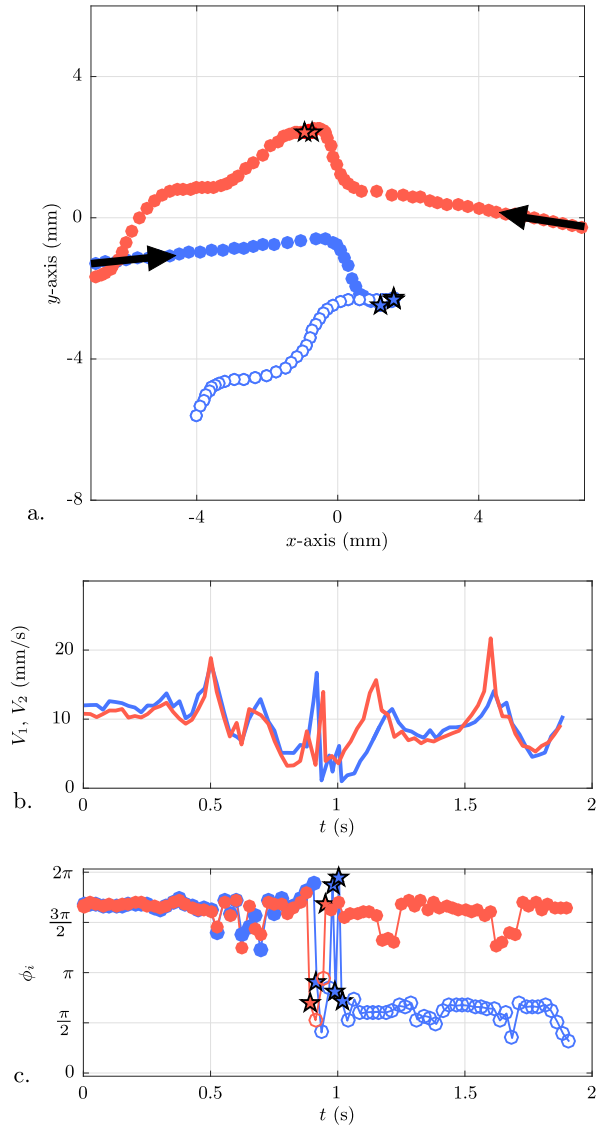


FIG. 8. Example of interaction between two walkers that ends up in a promenade. Data from each droplet are represented in blue and red, respectively. (a) Horizontal trajectory in the (x, y) plane. Each dot corresponds to one droplet impact. Stars represent limping events, when the impact phase ϕ_i is approximately shifted by π . (b) Horizontal instant speed $V_1(t)$ and $V_2(t)$ of each walker. (c) Impact phase ϕ_i of each walker.

major phase changes, while the other experiences two. Ultimately, the vertical dynamics is stabilized again and both walkers converge to their final fate, which is a promenade in the example of Fig. 8. However, sometimes one droplet does not converge back to its initial impact phase, but rather to the opposite phase. As far as it could be observed experimentally, the appearance of promenade always involves one droplet ultimately switching to the opposite phase. It can be checked in Fig. 5, where the thin undulated trace corresponds to one of the outgoing walkers being strobed during flight instead of at impact. Figure 8 reveals how strongly the vertical dynamics is affected during the interaction of two walkers, and it is almost surprising that early stroboscopic models (e.g., Ref. 39) could qualitatively reproduce some of these outcomes.

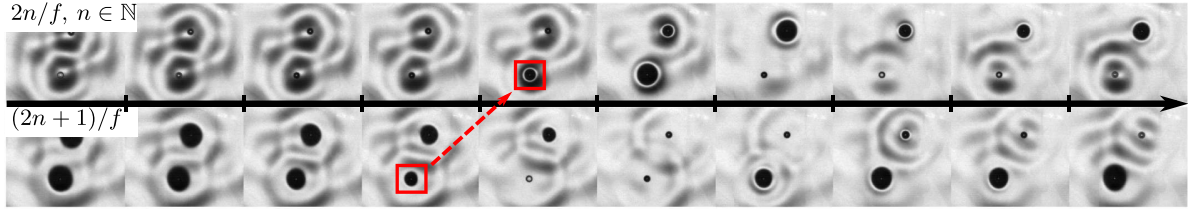


FIG. 9. Chrono-photography of a phase change event. The sequence is read from top to bottom, then from left to right. Successive pictures of the same row are separated by $1/f = 12.5$ ms and successive pictures of the same line are separated by $2/f = 25$ ms. The first limping event is highlighted within the red box. The limping events each correspond to two impacts on the bath separated by less than $3/(2f)$.

Another example of interaction is shown in Fig. 10. Both the particles are launched with speeds $V_{01} = 11.4$ mm/s and $V_{02} = 11.9$ mm/s, with $\Delta\phi_0 = 0$, and with an angle $\theta_0 = 17^\circ$. Their interaction is complex, and it involves a strong perturbation of the vertical dynamics, especially persistent for one of the walkers. Ultimately, the walkers are scattered away with angles $\theta_1 = 101.8^\circ$ and $\theta_2 = 38.1^\circ$, and they both recover their initial speed. The horizontal momentum of the droplets is, therefore, far from conserved through the interaction.

Limping is not systematically observed for any initial conditions. Nor does it occur at any location. The droplet position during limping events is reported in Fig. 11. It is defined with the distance $|r_1 - r_2|$ between both walkers and the angular position θ . This latter corresponds to the angle between the limping position and the initial position of the droplet, observed from the initial center of mass of both droplets [so $\theta = 0$ when limping is aligned with the initial trajectory, see Fig. 3(e)]. Three preferential positions of limping can be identified: the angular position $\pi/2$, the distance between the walkers $|r_1 - r_2| \simeq 0.7\lambda_F$, and the angular position π . These preferred limping positions seem relatively independent of the outcome. There might be some constructive interference

of the wave contributions of each droplet at these specific locations. A closer look at the wave field, using a schlieren technique for instance, would certainly provide additional clues on the actual conditions necessary to destabilize the walker and induce limping.

IV. A SIMPLE MODEL OF WALKERS THAT INCORPORATES THE VERTICAL DYNAMICS

In our experiments, we observed that (i) the interaction of two walkers is strongly sensitive to initial conditions and (ii) it often involves strong perturbations of the vertical bouncing dynamics (and so of the impact phase). In order to establish if (ii) is the cause of (i), we will compare the walker trajectories issued from models where the vertical dynamics of each droplet is either forced to a periodic motion or free to vary in response to the underlying waves.

Most previous models of the walkers (e.g., Refs. 14, 20, and 35) are strobed, which means that they precisely assume this perfectly periodic vertical motion of the walker, so they do not take into account possible variations of the impact phase. This stroboscopic assumption was possibly convincing for a single walker, but our observations show that it is unrealistic during the interaction of two walkers.

Conversely, the most accurate model of this vertical dynamics, based on a non-linear spring, was developed by Moláček and Bush³¹ and implemented together with an appropriate model of Faraday waves in Milewski³⁰ and Durey and Milewski¹¹. Such a complete model involves many additional

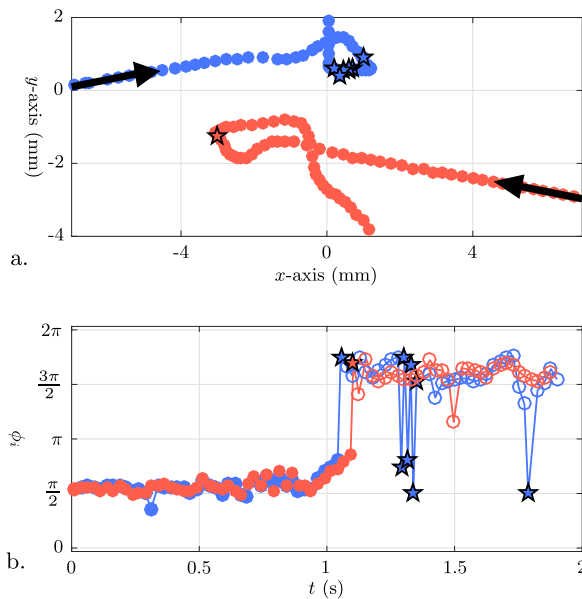


FIG. 10. Complex interaction of two walkers that results in asymmetric scattering. Data from each walker are represented in blue and red, respectively. Stars represent limping events, when the impact phase ϕ_i is approximately shifted by π . (a) Horizontal trajectory. Each droplet's impact corresponds to a colored dot. (b) Impact phase ϕ_i of each walker.

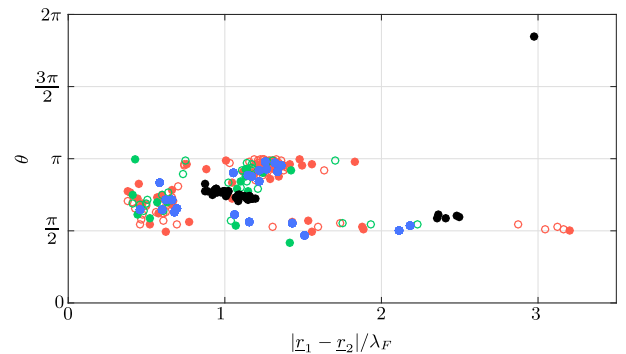


FIG. 11. Horizontal droplet positions at which the first limping event was observed: distance $|r_1 - r_2|$ between both walkers vs. angular position θ from the center of mass of the walkers. Colored dots correspond to experimental outcomes: (red) scattering, (green) orbits, (blue) promenades, and black dots correspond to numerical simulations with $\alpha = 11 \times 10^{-7}$ and $\Delta\alpha = 0.05$. Filled (resp. empty) symbols represent experiments and numerical simulations with $\Delta\phi_0 = 0$ (resp. $\Delta\phi_0 = \pi$).

ingredients in comparison to the strobed models, and it would be difficult to identify which one (if any) is responsible for the strong sensitivity to initial conditions observed with interacting walkers. A simpler bouncing model has been proposed by Galeano-Rios *et al.*¹⁸ considering the droplet as a solid sphere bouncing on the bath. However, for all these models, the resulting computation comes at a cost that makes large sets of trajectories out of reach, would they be for some systematic parameter screening or for some statistical analysis.

We here propose to start from a strobed model similar to that of Eddi *et al.*'s,¹⁴ and then to add the simplest realistic model of vertical dynamics. This latter assumes that each droplet behaves as a completely inelastic ball.²² Both the inelastic ball and the strobed versions of our model satisfy the same evolution equations for the wave field (Sec. IV A). Only the droplet dynamics is free to vary in the former (Sec. IV B) while it is forced to periodicity in the latter (Sec. IV C). We will then investigate the effect of this vertical droplet dynamics on the sensitivity of walker interactions.

A. Wave field

Each droplet's impact on the bath excites some Faraday waves. In a recent paper, we provided a theoretical expression for these waves based on a careful resolution of Navier-Stokes equations.²⁶ The wave field corresponding to one droplet impact at position $\underline{r}_i$ is

$$h(r, t) = A \sqrt{\frac{\pi}{Dt}} J_0 \left(2\pi \frac{r}{\lambda_F} \right) \exp \left(-\frac{t}{\tau} \right) \exp \left(-\frac{r^2}{4Dt} \right) \cos \left(\pi f t + \frac{\pi}{4} \right), \quad (1)$$

where $r = |\underline{r} - \underline{r}_i|$ is the distance to the impact point, J_0 is the Bessel function of the first kind, and other parameters are defined and discussed here below. This wave is modulated in time with a factor $\cos(\pi f t + \pi/4)$ that accounts for oscillations of the surface waves at the Faraday frequency $f/2$.

The forcing frequency $f = 80$ Hz, the Faraday wavelength $\lambda_F = 4.75$ mm, and the diffusion coefficient of the spreading wave $D = 5.5 \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$ were kept identical to the experimental values.²⁶ In the limit of small viscosity, the characteristic damping time τ of the waves is approximated by²⁶

$$\tau \simeq \frac{\mathcal{M}}{\pi \gamma_0 f} \left[1 + \frac{\gamma_0^{1/2}}{2} + \frac{\gamma_0}{4} + \mathcal{O}(\gamma_0^{3/2}) \right] \quad (2)$$

with $\gamma_0 \simeq \frac{8\pi v}{\lambda_F^2 f}$. Since the proximity to the Faraday threshold is $\mathcal{M} = 30$ in the experiments, we deduce $\tau \simeq 0.574$ s, which corresponds to 23 rebounds in average.

The amplitude A of the wave generated by one droplet impact is modeled as

$$A = \alpha v_i \cos \left(\phi_i + \frac{\pi}{5} \right), \quad (3)$$

where α is a constant coefficient with dimension (m s), v_i is the impact velocity of the droplet, and $\phi_i = \text{mod}(\pi f t_i, 2\pi)$ is the impact phase (with respect to the forced vibration of the shaker) associated to the impact time t_i . The proportionality

between amplitude and impact speed results from the conversion of the vertical kinetic energy of the droplet into surface energy.²⁶

The waves generated by the walkers are in the linear regime, where the corresponding amplitude (around $10 \mu\text{m}$) is much smaller than both the thickness of the boundary layer ($300 \mu\text{m}$) and the Faraday wavelength ($\lambda_F = 4.75$ mm).²⁶ Therefore, the total wave field $H(\underline{r}, t)$ is simply obtained by summing waves generated at each impact i of droplet j at position $\underline{r}_{ij}$ and impact time t_{ij} :

$$H(\underline{r}, t) = \sum_{j \in \{1, 2\}} \sum_{i=-\infty}^n h(|\underline{r} - \underline{r}_{ij}|, t - t_{ij}). \quad (4)$$

The waves created at each droplet's impact are added to the wave field once the droplet has taken off. In future work, more accurate models of the impact dynamics (e.g., Ref. 18) could be implemented to better capture the wave-droplet interaction. These models may suggest alternative hypotheses for the present low-level model of impact dynamics, as for example, updating the wave field when the droplet lands instead of when it takes off.

B. Droplet dynamics: The inelastic ball model

In this model, we assume that the droplet's impact on the bath is perfectly inelastic.²² At impact, all the kinetic energy (relative to the bath) is lost, and the droplet then sticks to the bath surface. When the absolute acceleration (pointing downwards) becomes smaller than $-g$, the droplet takes off from the bath. It inherits from the instant velocity of the bath at take off. Since air drag is negligible,³² the droplet is then in free fall until the next impact.

Experimentally, the coefficient of restitution of walking droplets is small but not zero,³¹ so the droplet conserves some of its initial momentum during the impact, i.e., it slightly slides on the bath surface. Moreover, the droplet interacts with the bath for a finite duration, so it inherits from the momentum of the wave averaged over the whole contact duration.³² We here neglect both of these effects in order to keep our inelastic ball model simple and as close as possible to the strobed model, while being able to qualitatively capture the walker's dynamics.

1. Vertical dynamics

The shaker elevation follows a purely sinusoidal excitation of amplitude a and frequency f . The absolute elevation of the bath surface is then

$$z_b(\underline{r}, t) = a \cos(2\pi f t) + H(\underline{r}, t). \quad (5)$$

In the regime of large memory, the damping time τ of the wave envelope is much larger than the Faraday period $2/f$. Consequently, the time derivatives of $H(\underline{r}, t)$ are dominated by the sinusoidal variation of $H(\underline{r}, t)$ at frequency $f/2$. Therefore, the local speed and acceleration of the bath surface are

$$\begin{aligned} \dot{z}_b(\underline{r}, t) &\simeq -\pi f \left[2a \sin(2\pi f t) + H(\underline{r}, t) \tan \left(\pi f t + \frac{\pi}{4} \right) \right], \\ \ddot{z}_b(\underline{r}, t) &\simeq -\pi^2 f^2 [4a \cos(2\pi f t) + H(\underline{r}, t)]. \end{aligned} \quad (6)$$

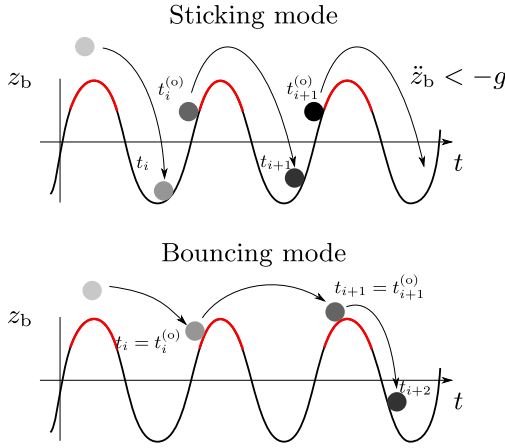


FIG. 12. Vertical dynamics of the perfectly inelastic ball. The curve represents the local elevation of the bath surface. Its red parts correspond to times at which the bath's acceleration satisfies $\ddot{z}_b < -g$. Times of impact t_i and take-off $t_i^{(o)}$ of the droplet are indicated. They are distinct in case of sticking, while they are identical in case of bouncing.

After the impact, the droplet will then move along vertically with the bath, while its horizontal position will remain in r_i . So its vertical position will be $z_d = z_b(r_i, t)$.

At time $t_i^{(o)}$ defined by $\ddot{z}_b(r_i, t_i^{(o)}) = -g$, the droplet takes off from the bath and inherits its vertical velocity $\dot{z}_b$. Its free fall trajectory then obeys

$$z_d(t) = z_b(r_i, t_i^{(o)}) + \dot{z}_b(r_i, t_i^{(o)})(t - t_i^{(o)}) - \frac{g}{2}(t - t_i^{(o)})^2. \quad (7)$$

The droplet impacts the bath again at time t_{i+1} defined as the first time after $t_i^{(o)}$ that satisfies

$$z_d(t_{i+1}) = z_b[r_i + \dot{r}_i(t_{i+1} - t_i^{(o)}), t_{i+1}]. \quad (8)$$

The impact velocity is then $v_{i+1} = \dot{z}_b(r_i, t_{i+1}) - \dot{z}_d(t_{i+1})$. If the acceleration $\ddot{z}_b(r_{i+1}, t_{i+1}^{(o)}) > -g$, the droplet sticks again on the bath surface until $t_{i+1}^{(o)} > t_{i+1}$. Otherwise, the droplet bounces, i.e., it directly takes off at $t_{i+1}^{(o)} = t_{i+1}$. Both scenarios are illustrated in Fig. 12. The sticking mode is a fair approximation of the walker's vertical dynamics,³¹ where the sticking time is a substitute for the experimental contact time.

2. Horizontal dynamics

At each take-off, the droplet inherits its vertical velocity from the velocity of the bath. In the horizontal direction, this speed is assumed to be proportional to the slope of the wave field, at the position of impact r_i and at the time of take-off $t_i^{(o)}$, i.e.,

$$\dot{r}_i = -B \nabla H(r_i, t_i^{(o)}). \quad (9)$$

A similar assumption is found in most strobed models (e.g., Refs. 14 and 20). The coefficient B , the dimension of which is (m s^{-1}) , could possibly depend on the impact velocity v_i . However, we neglect this dependence and consider B as constant for the sake of keeping the model simple and focusing on modifications of the vertical dynamics only.

Equations (1), (4), and (9) contain the core of the horizontal wave-particle coupling. They are present to some extent in

almost all existing models of the walkers. The two formers capture how each droplet deforms the global wave field, and the latter indicates how each droplet moves in response to the local wave field. The parameter B can therefore be seen as the strength of the coupling between the wave and the droplet horizontal dynamics.

C. Droplet dynamics: The strobed model

In the strobed model, the vertical dynamics of the droplet is forced to a periodic regime. The impact velocity v_i , the impact phase ϕ_i , the impact time t_i , and the take-off time $t_i^{(o)}$ are, therefore, forced to be constant. For each simulation and set of parameters (α, B) , these four constants $(v_i, \phi_i, t_i, t_i^{(o)})$ are chosen identical to the values produced by the inelastic ball model for a single unconfined walker, for which the vertical dynamics converges to a periodic mode. Consequently, as far as isolated walkers are concerned, the inelastic ball and strobed models provide identical predictions. Comparing both models when two walkers interact will, therefore, shed a focused light on the contribution of the vertical dynamics to the interaction.

D. Parameters and initial conditions

The amplitude of the shaker vibration is fixed to $a = 0.156$ mm in experiments, which determines $\mathcal{M} = 30$ since the Faraday threshold is $\Gamma_F \simeq 4.15$. However, with such an amplitude, the inelastic impact model cannot reproduce the walking regime with one impact every two forcing periods. This discrepancy with experiments precisely originates from the perfectly inelastic assumption. The period doubling bifurcation (from one impact every period to one impact every two periods) is generically found in all bouncing ball systems. The forcing amplitude corresponding to this bifurcation increases when the coefficient of restitution decreases.²⁹ While the bifurcation occurs at $\Gamma \simeq 2.5$ in experiments³², it only occurs at $\Gamma \simeq 4.5$ in our inelastic model, which corresponds to $a = 0.175$ mm. Fortunately, a can be fixed independently of $\mathcal{M}$ in our model, while both are actually linked through the Faraday threshold Γ_F in experiments. We have, therefore, chosen $a = 0.192$ mm (i.e., $\Gamma = 5.125$) for the simulations, which yields a vertical dynamics beyond the period doubling bifurcation, in qualitative agreement with experiments.

Simulations are initialized by *preparing* walkers individually, i.e., as if they were isolated and unconfined on the bath. The inelastic ball model is run for 800 impacts (i.e., more than 30 times the memory) with each walker alone. The walker converges to a constant horizontal speed and a periodic vertical dynamics (from which v_i , ϕ_i , t_i , and $t_i^{(o)}$ are determined for the strobed model). The wave field also reaches a steady state in the framework of the droplet.

These initialized walkers (droplets and associated waves) are then artificially positioned together on the bath, at more than $50\lambda_F$ from each other. They are also reoriented to move towards each other on parallel trajectories ($\theta_0 = 0$) separated by the impact parameter b . They are synchronized in order to initially bounce in phase, i.e., $\Delta\phi_0 = 0$ [Fig. 3(c)]. Similar results and conclusions were observed for $\Delta\phi_0 = \pi$, so they are not reported here for the sake of conciseness.

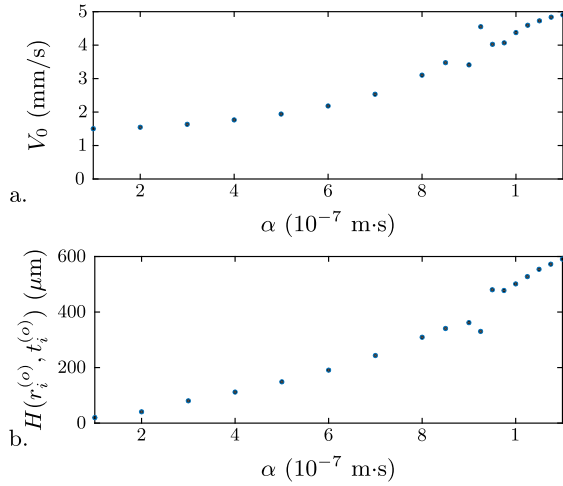


FIG. 13. (a) Horizontal velocity of the walker V_0 and (b) amplitude of the wave H below the walker at take-off, both as functions of α . The product $\alpha B = 2 \times 10^{-8} \text{ m}^2$ is kept constant.

Both models have been extensively tested numerically, mostly for different values of α and B . In previous strobed models (e.g., Ref. 20), the horizontal dynamics of the walker did only depend on the product $A \cdot B$ of the individual wave amplitude A and the horizontal speed coefficient B . So each coefficient could be fixed arbitrarily as long as their product was controlled. In the strobed and inelastic models proposed here, A is not fixed anymore, since it depends on both v_i and ϕ_i . Instead, we focused on keeping the product $\alpha \cdot B = 2 \times 10^{-8} \text{ m}^2$ constant while varying α in the range $[1, 11] \times 10^{-7} \text{ ms}$. The horizontal speed V_0 associated to a single unconfined walker and the wave amplitude $H(r_i^{(o)}, t_i^{(o)})$ right below the walker when it takes off are both reported as functions of α in Fig. 13. They both increase with α over the considered range, though the increase of H is comparatively much more pronounced than that of V_0 .

Both walkers are not necessarily identical. Any difference in mass between the walkers would result in a difference in walking speed ($V_{01} \neq V_{02}$), which can here be captured with a different α for each walker, i.e., $\alpha_1 \neq \alpha_2$. We, therefore define an asymmetry coefficient $\Delta\alpha = (\alpha_2 - \alpha_1)/\alpha_1$.

V. MODEL COMPARISON: THE ADDED VALUE OF THE VERTICAL DYNAMICS

A. Trajectories

Typical walker trajectories produced by the inelastic ball model are illustrated in Fig. 14(a). They are qualitatively similar to those observed experimentally. In particular, the computed outputs can be either scattering, orbits, or promenades. The four trajectories represented in Fig. 14(a) originate from extremely similar initial conditions and input parameters. In one of them, the walkers scatter, but their total momentum, in the sense of individual particles, is not conserved since the output angles θ_1 and θ_2 strongly differ. Similar trajectories were observed experimentally in Fig. 10. The corresponding impact phase [Fig. 14(b)] always fluctuates during the interaction, which reveals a perturbed vertical dynamics. It then converges to a smaller value for bound states (orbit and

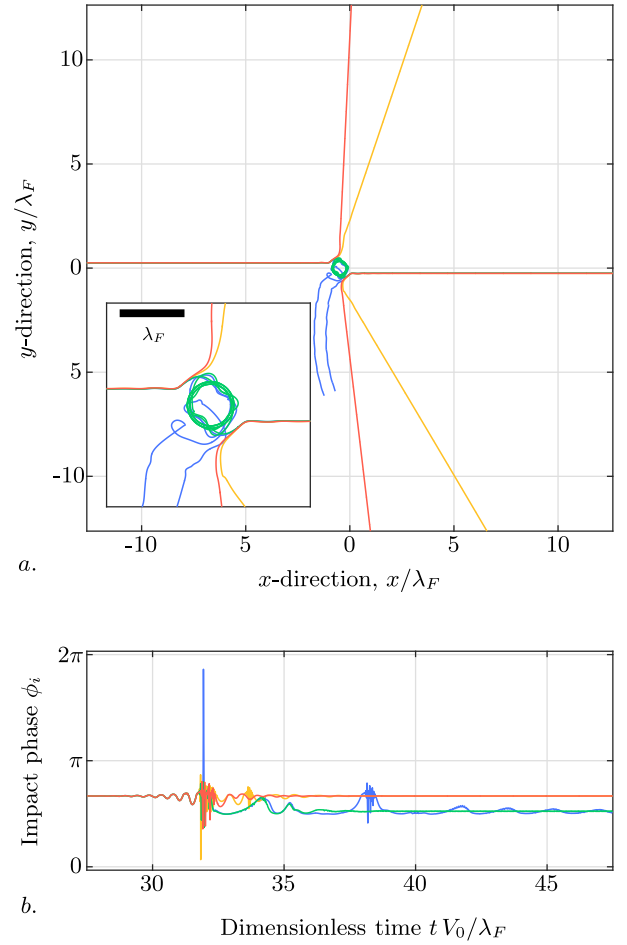


FIG. 14. (a) Trajectories computed with the inelastic ball model for $\alpha_1 = 1.1 \times 10^{-6} \text{ ms}$ and slightly different values of $\Delta\alpha$ and b (red). Scattering for $b/\lambda_F = 2.397 \times 10^{-3}$ and $\Delta\alpha = 2.717 \times 10^{-2}$ (green). Orbit for $b/\lambda_F = 2.345 \times 10^{-3}$ and $\Delta\alpha = 2.717 \times 10^{-2}$ (blue). Promenade for $b/\lambda_F = 2.345 \times 10^{-3}$ and $\Delta\alpha = 2.749 \times 10^{-2}$ (yellow). Complex asymmetric scattering for $b/\lambda_F = 2.397 \times 10^{-3}$ and $\Delta\alpha = 2.749 \times 10^{-2}$. (b) Evolution of the impact phase ϕ_i for these four trajectories.

promenade) than for free states (scattering), consistently with observations from Oza *et al.*³⁶ and Arbelaiz *et al.*²

Limping is observed when $\alpha > 10 \times 10^{-7} \text{ ms}$. The position of first limping is represented in Fig. 11 for the arbitrary choice of parameters $\alpha = 11 \times 10^{-7} \text{ ms}$ and $\Delta\alpha = 0.05$. Two preferred zones of limping are identified. The first is at angular position $\theta \simeq \pi/2$, which is very similar to experiments. The second is when the distance between the walkers is around $|r_1 - r_2| \simeq \lambda_F$, which is larger than the experimental observation $|r_1 - r_2| \simeq 0.7\lambda_F$.

B. A sensitive outcome

The type of outcome (orbits, promenade, simple, and complex scattering) represents a way to assess the sensitivity of the interaction between the walkers to initial conditions and parameters. In the maps of Fig. 15, the outcome is represented as a function of the impact parameter b/λ_F and the relative difference $\Delta\alpha$ between both walkers.

All types of outcome are found in each model configuration. Scattering always dominates at large b . Orbits are present

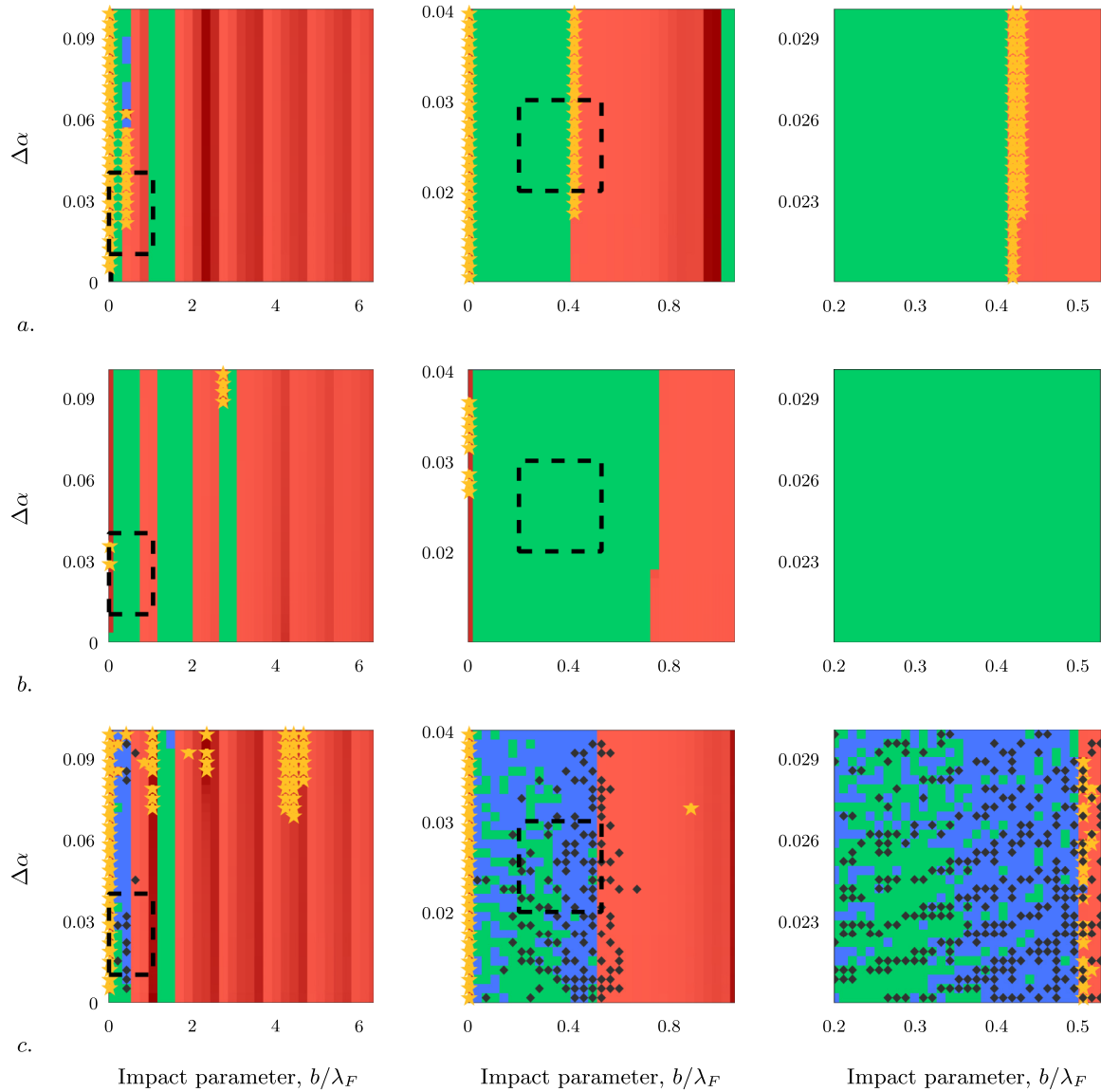


FIG. 15. Interaction outcome (colored) as a function of the impact parameter b/λ_F and the relative difference $\Delta\alpha = (\alpha_2 - \alpha_1)/\alpha_1$ between both walkers. (Green squares) Orbits, (blue squares) promenades, (red to brown squares) scattering, from small to large scattered angle θ , (yellow star) complex asymmetric scattering, for which $|\theta_1 - \theta_2| > 10^\circ$. The occurrence of limping is indicated with an overlying black diamond. (a) The strobed model with $\alpha = 11 \times 10^{-7}$ m.s. (b) The inelastic ball model with $\alpha = 3 \times 10^{-7}$ m.s. (c) The inelastic ball model with $\alpha = 11 \times 10^{-7}$ m.s. Second and third columns indicate progressive zooms on a smaller range of b/λ_F and $\Delta\alpha$.

only in certain well-defined intervals of b/λ_F that correspond to the quantification of orbital radius already observed in Protière *et al.*³⁹ and Oza *et al.*³⁶ The third interval (around $b/\lambda_F \simeq 3$) is only present for small α , which corroborates the fact that larger orbits tend to be less stable than smaller ones. Promenade modes and complex scattering are only present when $\alpha_1 \neq \alpha_2$, so they only exist when the symmetry between the walkers is broken.

The basin of attraction of each outcome is relatively regular for the strobed model, as well as for the inelastic ball model at small α . However, for the inelastic ball model at large α , these basins of attraction appear much less defined. A progressive zoom to the range $b/\lambda_F \in [0.2, 0.5]$ and $\Delta\alpha \in [0.02, 0.03]$ reveals that the basins of attraction are apparently

fractal, with a complete interpenetration of orbits and promenades. Moreover, this region of the maps is also frequently subjected to limping events, which again indicates that the vertical dynamics has been strongly perturbed during the interaction.

Figure 16 illustrates how a small region of the $(b/\lambda_F, \Delta\alpha)$ outcome map evolves as α is increased from 3×10^{-7} to 11×10^{-7} m.s. Each map is a 10×10 matrix M_{ij} corresponding to $b_i/\lambda_F \in [0.3, 0.4]$ and $\Delta\alpha_j \in [0.026, 0.029]$. The interlacing of the basins of attraction of orbits and promenades clearly appears for $\alpha > 10^{-6}$ m.s. In order to quantify this heterogeneity of the maps, we define the b -gradient matrix X with elements $X_{ij} = 1$ if $M_{i+1,j} \neq M_{ij}$ and $X_{ij} = 0$ otherwise. Similarly, we define the $\Delta\alpha$ -gradient matrix Y with elements

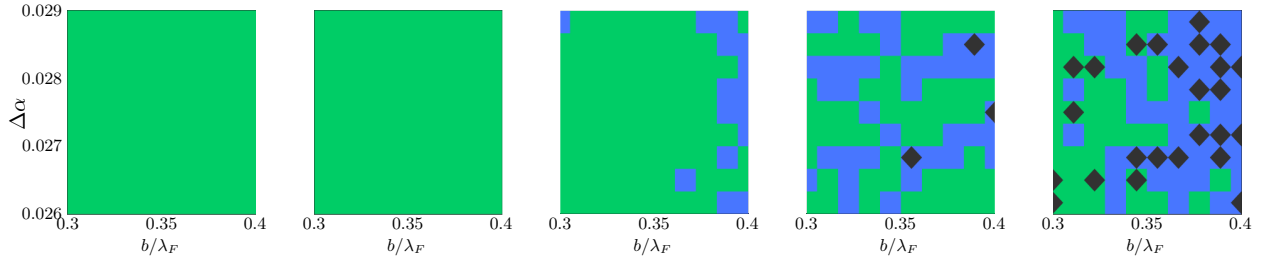


FIG. 16. Evolution of the outcome sensitivity with increasing α . From left to right, $\alpha = 3 \times 10^{-7}$ m s, $\alpha = 8 \times 10^{-7}$ m s, $\alpha = 10.25 \times 10^{-7}$ m s, $\alpha = 10.75 \times 10^{-7}$ m s, and $\alpha = 11 \times 10^{-7}$ m s.

$Y_{ij} = 1$ if $M_{i+1,j} \neq M_{ij}$ and $Y_{ij} = 0$ otherwise. We then define the outcome sensitivity as

$$S_o = \text{Tr}(XX^T) + \text{Tr}(YY^T) \quad (10)$$

and calculate it for each α [Fig. 17]. As already suggested by these maps, this outcome sensitivity S_o significantly increases for $\alpha > 10^{-6}$ m s (Fig. 17), which is approximately when limping appears.

C. Sensitivity during interaction

Similar to experiments, the output of the model is strongly sensitive to initial conditions, which is highly reminiscent of chaos. Nevertheless, the amplification time during which perturbations could grow exponentially should be finite here, since output trajectories are stable and non-chaotic on the long term. Therefore, the definition of the Lyapunov exponent must be adapted to consider the divergence of trajectories during some finite amplification time.

This amplification time is not trivially defined. Indeed, it cannot be the interaction time (during which the walkers would be closer than a threshold distance), since this latter is infinite for stable orbits and promenades. We here define the amplification time according to the fluctuations of horizontal walker speed $V(t)$ [Fig. 18(a)]. These fluctuations are more important when the walkers start interacting and choose between different outcomes. Their amplitude also oscillates with a characteristic frequency of the order of the inverse of the time needed by an isolated walker to travel one Faraday wavelength, i.e., V_0/λ_F . Practically, $\bar{V}(t)$ is computed as the difference of maximal to minimal speed within a time window of width λ_F/V_0 centered on t . The amplification time t_a is defined as the time during which $\bar{V}$ is larger than 50% of its maximal value [Fig. 18(a)]. Thanks to this definition,

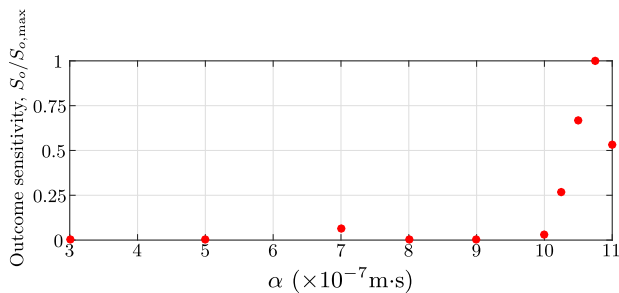


FIG. 17. Variation of the outcome sensitivity S_o , normalized by its maximum value $S_{o,\max} = 5.4 \times 10^8$, with α .

t_a remains finite and focused on the early interaction when a choice of outcome is made, even for stable orbits and promenades where the walker speed may still slightly fluctuate on the long term.

The amplification of perturbations is computed by considering how the trajectories of both walkers $r_1(t)$ and $r_2(t)$ deviate for two different launches A and B for which only the impact parameter b differs by $1 \mu\text{m} = \lambda_F/4750$. The divergence is defined as

$$\delta(t) = \sqrt{|r_1^A - r_1^B|^2 + |r_2^A - r_2^B|^2} \quad (11)$$

and its time variation is illustrated in Fig. 18(b). Before both walkers start interacting, δ is almost constant, set to $1 \mu\text{m}$. Within the amplification time window, $\delta(t)$ oscillates, again at a frequency of the order of V_0/λ_F . The envelope of $\delta(t)$ grows almost exponentially with time within this window, which is a signature of local chaos. Finally, outside the amplification time window, the exponential growth of δ stops. For orbits, δ reaches a plateau corresponding to the orbital radius, while for promenade and scattering, δ keeps growing, but now linearly with time (because the scattering angles θ_1 and θ_2 slightly differ from A to B). We define the Lyapunov gain as

$$G = \frac{V_0}{\lambda_F} \int_{-\lambda_F/(2V_0)}^{\lambda_F/(2V_0)} \log_{10} \frac{\delta(t_0 + t_a + u)}{\delta(t_0 + u)} du, \quad (12)$$

where t_0 denotes the beginning of the amplification window [Fig. 18(b)]. It corresponds to a total amplification 10^G of perturbations during the interaction time t_a .

The variation of G with the impact parameter b is represented in Fig. 18(c) for the inelastic ball model with identical $\alpha = 3.0 \times 10^{-7}$ s (i.e., waves of amplitude about $80 \mu\text{m}$) for both walkers. This Lyapunov gain is always positive, and it evolves smoothly with b . Peaks of G are nevertheless observed at the transition between outcomes (e.g., here between orbits and scattering). The sensitivity parameter S is defined as the average of G on a homogeneously distributed set of $b/\lambda_F \in [0, 1.05]$:

$$S = \frac{1}{1.05\lambda_F} \int_0^{1.05\lambda_F} G(b) db. \quad (13)$$

It expresses the factor by which trajectories separate for a given α , averaged over the impact parameter b . It is aimed to resume the chaotic potential of the interaction in a single number. It is represented as a function of α in Fig. 18(d) for both inelastic ball and strobed models. For the strobed model, S does not vary significantly with α , so the slight divergence

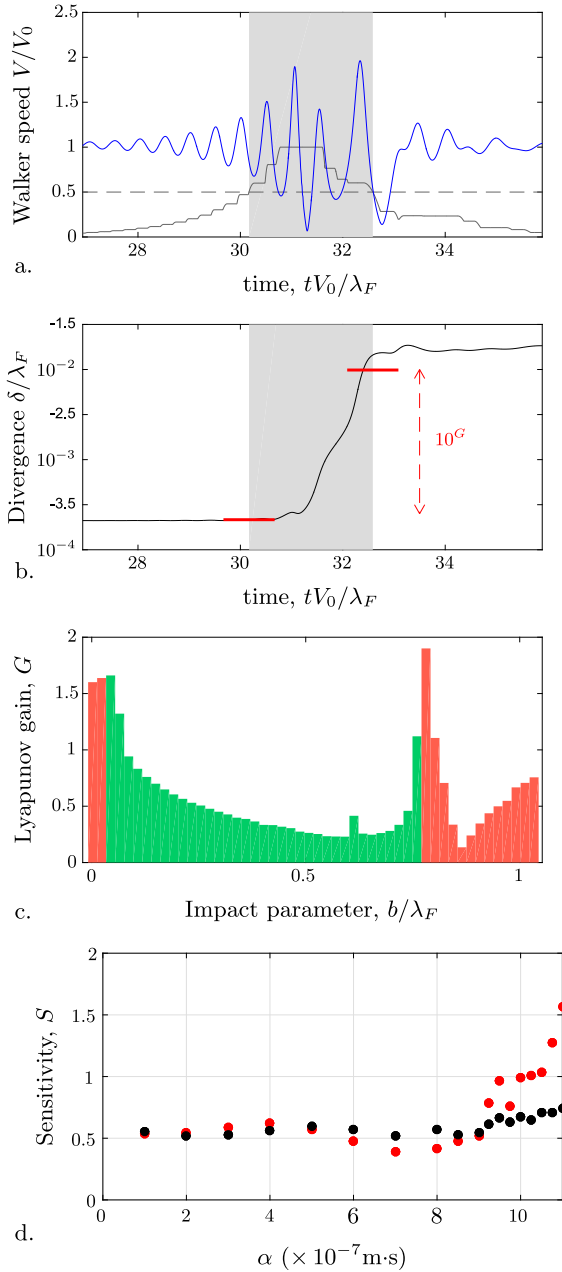


FIG. 18. (a) Amplification window (blue line). Normalized horizontal speed V/V_0 of the walker (black line). Speed fluctuations $\bar{V}/V_0$, low-pass filtered with a cut-off of frequency V_0/λ_F (grey area). Amplification window of duration t_a , where $\bar{V} > 0.5 \max(\bar{V})$. (b) Divergence $\delta(t)/\lambda_F$ of two initially similar trajectories. The Lyapunov gain G defined in Eq. (12) is the vertical red line that indicates the amplification during time t_a . (c) Lyapunov gain G as a function of the impact parameter b/λ_F for $\alpha = 3 \times 10^{-7}$ m·s. The color corresponds to the outcome of the interaction: (red) scattering or (green) orbits. (d) Evolution of the sensitivity parameter S as a function of α for (red dot) the inelastic ball model and (black dot) the strobed model. Dark edged red bullets correspond to points where limping has been observed.

of trajectories during the interaction does not really depend on the wave amplitude. By contrast, for the inelastic ball model, S increases with α for $\alpha > 10^{-6}$ m·s, meaning that at high wave amplitude, neighbor trajectories strongly diverge during the interaction. For small α , both models yield the same value of S which confirms that, on average, the Lyapunov gain G is insensitive to the vertical dynamics in this range.

Both S_o and S start increasing for $\alpha \gtrsim 10^{-6}$ m·s, which is also when limping appears. It corresponds to the wave amplitudes larger than about $450 \mu\text{m}$, that are sufficient to strongly disturb the vertical dynamics of the perfectly inelastic balls modelled here. However, the vertical dynamics might already be perturbed, to a lesser extent, for $\alpha < 10^{-6}$ m·s. In order to analyze this perturbation, we focus on the fluctuations of the impact phase, and we define the standard deviation σ_ϕ ,

$$\sigma_\phi = \sqrt{\frac{1}{N-1} \sum_{i=1}^N [\text{mod}(\phi_i, \pi) - \langle \phi \rangle]^2}, \quad (14)$$

where N is the number of impacts during the amplification window t_a defined previously, and $\langle \phi \rangle = \sum \text{mod}(\phi_i, \pi)/N$ is the mean value of the impact phase. The modulo π avoids an overweight of permanent phase changes. The impact phase fluctuation σ_ϕ , averaged over all experiments at a given α , is plotted Fig. 19. It is almost zero for $\alpha < 5 \times 10^{-7}$ m·s, and it increases steadily for larger values of α , without any discontinuity at $\alpha \simeq 10^{-6}$ m·s. Therefore, we conclude that the occurrence of limping, as a non-smooth consequence of the perturbation of the vertical dynamics, increases strongly the sensitivity to initial conditions.

VI. DISCUSSION ON THE ORIGIN OF CHAOS

In Sec. III, we observed experimentally that the interaction of two walkers can be chaotic (high sensitivity of the outcome to initial conditions and parameters) and that the associated vertical bouncing dynamics was greatly perturbed during the interaction. This perturbation culminated with the occurrence of limping events, during which a droplet prematurely hits the bath. However, experiments did not reveal if the perturbed vertical dynamics was the cause of this chaos (and thus a necessary condition). Several other phenomena might have participated in the amplification of perturbations. For example, the traveling wave emitted from each impact (on top of the Faraday waves) slightly modifies the wave field, which then appears as a succession of traveling fronts.⁹ Those traveling waves might have been sufficiently strong to destabilize the walkers when they were sufficiently close to each other. The air currents, not totally suppressed here, might have also contributed to the apparent stochasticity of the outcome. However, this latter cause is unlikely, since it was shown in Sec. II that the absence of any significant curvature in the trajectory of isolated walkers was a proof of negligible air drag.

In Secs. IV and V, we developed a simple model of a wave coupled to a perfectly inelastic particle. We have then

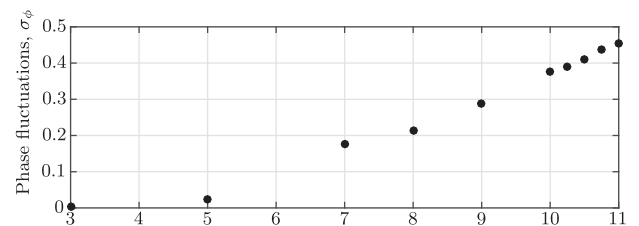


FIG. 19. Fluctuation of the vertical dynamics σ_ϕ as a function of α . Limping is observed when $\alpha \geq 10^{-6}$ m·s.

shown that this model can reproduce qualitatively the zoology of walker trajectories observed experimentally and that it exhibits sensitivity to initial conditions only when limping occurs. The latter may, therefore, be the cause of the former. A similar horizontal chaos induced by strong perturbations of the vertical dynamics was already observed in experiments with the walkers forced with multifrequency signals.⁴³ Limping in itself is a grazing bifurcation, generic to most fly-and-impact systems whose dynamics is piecewise-smooth.¹⁰ It has been observed for partially elastic balls,²⁹ for bouncing droplets on soap films for which the contact time is finite,²¹ and more importantly in the present experiments on the walkers. Limping is, therefore, not an artefact introduced by the hypotheses of zero contact time and zero coefficient of restitution, and the simplification to a perfectly inelastic ball is not responsible for artificially inducing chaos.

This inelastic assumption strongly decreases the computational time and makes the model simpler to understand. However, it comes at a cost: there is no more quantitative agreement with experiments. In order to reproduce qualitatively similar walking regimes, we need to increase the forcing acceleration to $\Gamma = 5.125$ (instead of $\Gamma = 4$ in experiments) and to select parameters (α, B) to reach a wave amplitude under the impacting droplet of up to $500\ \mu\text{m}$ (instead of an estimated value between $10\ \mu\text{m}$ and $80\ \mu\text{m}$ in experiments^{9,14,26}). With such parameters, the walking velocity (about $5\ \text{mm/s}$) is also twice smaller than in experiments. Nevertheless, the qualitative agreement between the simplified model and experiments is then excellent, in terms of both trajectory outcomes and occurrence of limping. More importantly, it allows to correlate limping to a higher sensitivity to initial conditions. The difference of the wave amplitude between our model and experiments may have different sources: (i) the perfectly inelastic hypothesis (which induces an additional dissipation of energy and subsequent stabilization), (ii) the idealisation of a complex interaction of the droplet and the bath during a non-instantaneous impact, and (iii) a difference in the impact phase and its consequence on the amplitude of the wave contribution from each impact. Fully understanding this would require a more in-depth analysis that we think is beyond the scope of this paper. Future work could include the generalization of our study to more realistic models of the droplet impact³⁰ and the wave generation²⁶ for which there would not be an arbitrary choice of parameters (α, B) .

VII. CONCLUSION

Walkers are fascinating physical systems in which a particle and a wave are coupled at the macroscopic scale. Although the walkers fall within the realm of deterministic Newton's laws, they often exhibit quantum-like behaviors. The apparent stochasticity of the walkers is induced by their chaotic dynamics. For confined individual walkers, chaos mostly originates from the memory of the wave field; this latter stores information on the past trajectory of the droplet.¹⁹

In this work, we have observed experimentally that the interaction of two walkers is also highly sensitive to initial parameters. Different outcomes are possible, including

scattered trajectories (symmetrical and asymmetric), orbits, and promenades. However, the origin of chaos cannot be the same as for confined walkers. Indeed, two interacting walkers do not necessarily explore again regions where they had already left waves. We have seen that horizontal chaos is here concomitant to limping events, which are strong perturbations of the vertical dynamics.

In order to identify the role of the vertical dynamics in generating horizontal chaos, we have proposed a simple model of walkers in which each droplet is resumed to a perfectly inelastic ball. In this model, the vertical dynamics can either be free to obey Newton's laws or it can be forced into a periodic regime (stroboscopic approximation). We have compared these two vertical dynamics, all other parameters and assumptions being equal. When allowed to, the vertical dynamics was strongly perturbed during the interaction. Limping was observed as soon as the added waves of both walkers were sufficiently high. The model could qualitatively reproduce all the observed outcomes of the interaction. Moreover, the high sensitivity to initial parameters was observed only when limping occurred. Therefore, our work has shown that limping is largely responsible for the chaotic wave-mediated interaction of two walkers.

This second origin of chaotic walker dynamics nicely complements the wave memory mechanism, especially in configurations where the walkers do not have the opportunity to revisit places where they had already left standing waves. Among others, the mysterious apparent stochasticity of a single walker passing through slits (diffraction and interference experiments^{1,7,40}) might find an explanation with some additional chaos from a perturbed vertical dynamics.

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B2. Biomécanique des feuilles et membranes pressurisées

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2. Lacorre, P., Fiore, L., Linares, J. M., & **Tadrist, L.** (2024). Pressurized membranes between walls : Thermodynamic process changes force and stiffness. *Journal of the Mechanics and Physics of Solids*, 192, 105798.



Are leaves optimally designed for self-support? An investigation on giant monocots



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HIGHLIGHTS

- Large leaves are optimally designed to self-support and intercept light efficiently contrary to small leaves.
- The optimal design of leaf with constraints on biomass and mechanics is theoretically inspected.
- Measurements on palms have been conducted.
- Leaf design for light interception is discussed.

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ABSTRACT

Leaves are the organs that intercept light and create photosynthesis. Efficient light interception is provided by leaves oriented orthogonal to most of the sun rays. Except in the polar regions, this means orthogonal to the direction of acceleration due to gravity, or simply horizontal. The leaves of almost all terrestrial plants grow in a gravity field that tends to bend them downward and therefore may counteract light interception. Plants thus allocate biomass for self-support in order to maintain their leaves horizontal. To compete with other species (inter-species competition), as well as other individuals within the same species (intra-species competition), self-support must be achieved with the least biomass produced. This study examines to what extent leaves are designed to self-support. We show here that a basic mechanical model provides the optimal dimensions of a leaf for light interception and self-support. These results are compared to measurements made on leaves of various giant monocot species, especially palm trees and banana trees. The comparison between experiments and model predictions shows that the longer palms are optimally designed for self-support whereas shorter leaves are shaped predominantly by other parameters of selection.

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1. Introduction

Leaves are responsible for intercepting light and creating sugars from photosynthesis (Vogel, 2012). The ability of a plant to produce biomass, to grow and reproduce (in other words to compete for survival) depends on the efficiency of photosynthesis. Photosynthesis yield depends on various parameters, mainly gas exchange, leaf temperature and light interception (Farquhar et al., 1980). Different factors may have a role on those key parameters: leaf perspiration, stomata aperture and ability to flutter may alter

heat and gas exchange at the level of the leaf (Roden and Pearcy, 1993; Roden, 2003).

Leaf orientation towards the sun's rays plays a key role for light interception (Tadrist et al., 2014) and different strategies have been adopted by the vegetal kingdom. In the first strategy, leaf orientation has no preferred direction to collect diffuse light that comes from every direction. In the second strategy, leaves have to be properly oriented to collect direct sun light. More complex strategies to improve the rate of photosynthesis also exist, giving rise to rather dynamical plant behaviour depending on environmental parameters. One example is the time-dependent orientation of leaves to follow the sun's position in the sky, but they can be even more complex. For instance, during a drought period, the inability to access water from the soil prevents the plant from perspiring. The leaf-refreshing effect of transpiration is cancelled, leaf temperature increases and the rate of photosynthesis drop to zero. To

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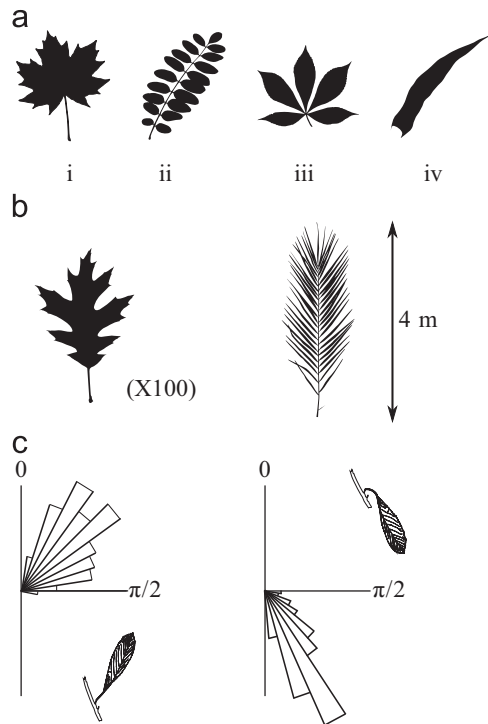


Fig. 1. (a) Variety of shapes of leaves: (i) simple leaf, (ii) palmate leaf, (iii) pinnate leaf, and (iv) sessile leaf, without petiole. (b) Variety of sizes: the first leaf is an oak leaf which has typical dimensions on the order of few centimetres, compared with phoenix palm that has a typical dimension of a few meters. In proportion, the oak leaf size has been magnified by a factor 100. The largest leaf was measured on *Raphia regalis* with a length of 25.11 m Hallé (2005). (c) Variety of leaf orientations in terms of leaf inclination angle distribution. The first distribution depicts erected leaves whereas for the second one (*Trema aspera*) most of the leaves are hanging down.

avoid such dramatic loss, leaves' orientations are changed to intercept less light, reduce leaf temperature and keep leaves active for photosynthesis (Vogel, 2012; Gonzalez-Rodriguez et al., 2015).

Because of these reasons, one may think that leaf orientation, leaf shape and leaf size would be parameters subjected to strong pressure of selection because of their role in light interception. The design of the leaf (thickness, length, width, petiole, midvein, secondary veins, etc.) must be optimal to obtain the larger rates of photosynthesis without threatening the plant's life. The highest photosynthetic rates of plants can reach up to 30% with a mean rate around 3% (Raven et al., 2003). In this context, it is surprising to observe such a variability in leaf shape, sizes and orientations (see Fig. 1). Discussions about plant mechanical optimality make a long story. The first authors to introduce the concept of plant design constrained by mechanics were McMahon and Kronauer (1976), who proposed that tree height and tree width are bound features. A few discussions of plant optimality concern branches (Wei et al., 2012) and leaves (Niklas, 1992, 1993; Niklas and Spatz, 2012), and Jensen and Zwieniecki (2013) have shown that leaf size is limited by optimal sap flow in tall trees.

Simple calculation of light interception with geometrical arguments shows that the optimal position to collect direct sun light is orthogonal to the local gravity field; see Appendix A and Tadrist et al. (2014). On terrestrial plants, leaves grow on branches and must support their own weight in order not to hang down. Some bio-material used to make the leaf must be dedicated to create surface area for light interception, but the rest of the bio-material should be used for mechanical self-support. On the one hand, if too little bio-material is used for self-support, the leaf will hang down and despite the large amount of bio-material used to create surface area, it will not intercept much light. On the other

hand, if too much bio-material is used for self-support, the leaf will be properly oriented to intercept light but will have no surface area to collect light. An optimal mass allocation trade-off exists between those two extrema. What would be the shape of a leaf that has optimally allocated biomass? Note that when no self-support is needed, all the biomass would be used to create surface area for light interception. This is the case for water lilies that occupy the interface between air and water.

Answering questions considering optimality in nature is not easy because of the large number of functions performed by an organ and because of the still larger number of environmental parameters to take into account. Those functions may naturally encourage antagonistic shapes; for instance, the optimal leaf would be thin enough to have a large surface area-to-volume ratio to enhance gas and heat exchange, but also thick enough for efficient transport of water and sap in xylem and phloem. Abiotic and biotic stresses are also shaping factors for the leaf. For example, a leaf may be designed to flutter to increase photosynthesis rate (Roden, 2003), to expel water drops and prevent fungus attacks or simply to remove herbivory insects (Yamazaki, 2011). Leaves have also developed mechanical tricks to be stiffer for the same amount of bio-material and thus to reduce the amount of biomass needed to support their own weight. Those tricks are (i) the inhomogeneity of the leaf tissues (Xylem and phloem vessels are much more lignified – and thus stiffer – tissues than mesenchymatous cells), (ii) the anisotropic placement of the tissues and (iii) the shape of the leaf itself. For instance, the shape of the leaf could lead to a stiffer U-shaped petiole (Ennos et al., 2000) or stiffer lamina through curvature-induced rigidity (Barois et al., 2014).

In this paper, we focus on the trade-off between self-support and creation of surface area for light interception. Our approach is based on a basic mechanical modelling of the leaf which neglects the different stresses or parameters of selection that apply on leaves, nor on the complex mechanical tricks developed to enhance leaf rigidity while minimizing biomass use.

Optimal leaf shape is examined for the giant monocots leaves, especially palms of palm trees and banana trees. Palm trees belong to the large family of *Arecaceae* (more than 2600 species) within the monocots clade. In this family, plants exhibit leaves of different sizes and different shapes. We choose here to study plants with consistent, simple leaf geometry: one short petiole and one long lamina with one major vein. In the first part of this paper we take advantage of this simple geometry to describe theoretically what would be the optimal leaf of a palm tree. In the second part of this paper, we describe the measurements done on actual palm trees and banana trees. Finally, we compare the theoretical results with the measurements on monocot trees. We show that the shape of larger leaves is close to the predicted optimal shape whereas the smallest palm shapes differ strongly from prediction. We predict a minimal size for which our model applies. For smaller leaves, self-support does not appear to be the strongest factor of selection.

2. Model

We aim at modelling what would be the optimal shape of a leaf under mechanical self-support constraints. We develop here a simple model based on mechanical considerations. For the sake of clarity, mechanical and geometrical assumptions are made for the considered leaf.

2.1. Model assumptions

We detail the assumptions as follow: first, the geometry of the considered leaf is chosen as the geometry of a palm of *Phoenix*

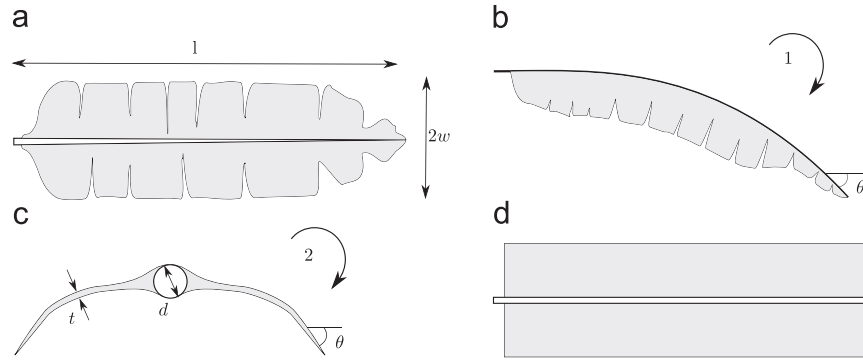


Fig. 2. Schematic views of a giant monocot leaf. (a) Top view. (b) Lateral view. The leaf hangs down because of its own weight. θ is the largest inclination angle of the leaf. This bending direction is noted 1. (c) Cross section view of the leaf. Again, the leaf hangs down because of its own weight and θ is the largest inclination angle of the leaf in this bending direction. This bending direction is noted 2. (d) Top view of the idealized leaf used for the model.

canariensis. Such a palm is composed of a short petiole that quickly becomes the midvein of the palm. The diameter of the midvein at the base of the lamina is denoted d . The lamina has a length l , a width w and a thickness t . The leaf geometrical properties are sketched in Fig. 2. Second, we consider that all the bio-material used to create the leaf is isotropic, has the same Young's modulus E and the same density ρ . Its production needs the same amount of bio-energy. Under this strong assumption, we neglect the inhomogeneity of the different tissues that comprise the leaf (xylem/phloem vessels, mesenchymatous cells, cuticle, etc.) Those tissues have non-similar mechanical properties. This will be discussed in the last part of the paper as well as the growth process. Third, we also neglect all mechanical tricks to strengthen the midvein such as U-shaped midvein (Ennos et al., 2000), turgidity-dependent rigidity (Nilsson et al., 1958; Faisal et al., 2010) and curvature induced rigidity of the lamina (Barois et al., 2014). Fourth, for the sake of simplicity, we consider that the petiole is clamped perpendicular to the gravity. Finally, we assume that the midvein weight is negligible compared to the lamina weight. This assumption can be easily released and optimal shapes can be computed numerically but it does not permit to access analytical solutions any more.

2.2. Optimal leaf

We consider a leaf supporting its own weight M within a gravitational acceleration g for a given volume of biomass V available to build the structure (Table 1). The leaf bends in the two directions indicated in Fig. 2b and c. The first direction is along the midvein and the second is perpendicular to the midvein. We note θ the final angle of the leaf. The maximal angle of a beam horizontally clamped at its one end under constant gravity loading G is

$$\theta = \frac{GL^3}{6EI}, \quad (1)$$

where I is the second momentum of area ($I = \pi d^4/64$ for a circular beam in bending). This equation is adapted for bending in direction 1

$$\frac{2\rho g w t l^3}{6E(\pi d^4/64)} = \theta, \quad (2)$$

and in direction 2

$$\frac{\rho g l t w^3}{6E(t^3 l/12)} = \theta. \quad (3)$$

These two equations are the only mechanics-derived expressions in this model. Volume, V , and surface area, S , of the leaf can

Table 1
Table of parameters.

Symbols	Parameters
θ	Maximal angle of deflection on the palm
V	Available volume of biomass
S	Leaf surface
M	Total mass of the leaf
M_e	Extra mass attached to the leaf free-end
l	Length of the leaf
w	Width of the lamina
t	Thickness of the lamina
d	Diameter of the petiole/midvein
g	Acceleration of gravity
ρ	Leaf mean density
G	Gravity loading per unit of length
C_0	Palm curvature without external loading
C	Palm curvature with external loading
l_x	Horizontal projection of palm length
E	Young's modulus of the leaf
I	Second moment of area of the leaf
$\mathcal{E}$	Daily collected solar energy
ϕ	Solar inclination angle
λ	Lagrange multiplier

be represented by

$$V = 2w t l + \pi l d^2/4 \quad \text{and} \quad S = l w. \quad (4)$$

Maximization of the surface of the leaf S under the constraint of fixed volume of biomass V and adequacy with mechanical self-support, Eqs. (2) and (3), provide four equations that give l , w , d and t . Calculation details are presented in Appendix B. Finally, we obtain

$$l = \left(\frac{23^2\theta}{511\pi} \right)^{1/4} \left(\frac{E}{\rho g} \right)^{1/4} V^{1/4} \quad (5)$$

$$w = \left(\frac{5^5\pi\theta}{2^73^311^3} \right)^{1/10} \left(\frac{E}{\rho g} \right)^{1/10} V^{3/10} \quad (6)$$

$$t = \left(\frac{\pi^35^{15}}{3^92^{11}11^9\theta^7} \right)^{1/20} \left(\frac{E}{\rho g} \right)^{-7/20} V^{9/20} \quad (7)$$

$$d = \left(\frac{2^{11}35}{11^3\pi^3\theta} \right)^{1/8} \left(\frac{E}{\rho g} \right)^{-1/8} V^{3/8} \quad (8)$$

Considering a different leaf geometry, with a cone-shaped midvein for instance (see Wei et al. (2012)) would lead to similar results but with slightly different pre-factors. To bear in mind some numerical values, a giant leaf with a volume $V = 1.0 \times$

10^{-3} m^3 , a typical Young's modulus of $E = 1.0 \times 10^8 \text{ Pa}$ and a density of $\rho = 1.3 \times 10^3 \text{ kg m}^{-3}$ the model gives a length of 80 cm, a width of 16 cm, a midvein diameter of 2.8 cm and a thickness of 0.9 mm.

3. Material and methods

To assess the hypothesis that leaves are optimally designed for self-support applies in nature, we measured leaf characteristics of palm trees and banana trees.

3.1. Samples

The palms of various species of monocots have been collected in the garden of the Rayol domain (N 43° 9' 22" E 6° 28' 54") and in private gardens in La Ciotat (N 43° 10' 25" E 5° 36' 18") and in Éguilles (N 43° 32' 39" E 5° 20' 22") all located in the south of France. The species and the characteristics of the palms used in our experiments are listed in Fig. 3. The palms selected for the study were cut from palm trees of different species living in different habitats and thus have different characteristics. Some are drought resistant such as *Sica* whereas some require wet environment like the ferns *Dicksonia antartica* and *Cyathea cooperi*. Most of the species chosen in the study were grown in south of France but originate from all over the world (South America: *Butia*, *Syagrus romanzoffiana*; Asia: *Banana tree*, *Sica*; Africa: *Phoenix canariensis*; Oceania: *Rhopalostylis sapida*, *Cyathea cooperi*, *Macrozamia communis*).

3.2. Measurements

We have measured geometrical and mechanical parameters of the leaves, their dimensions, l , w , d and t , their volume V , their mass M , their density ρ and their Young's modulus E . Immediately after being cut from the tree, fresh palms were clamped horizontally from their bases and tested with extra masses M_e attached to their free end. We took side pictures of this system, as shown in Fig. 4a, in order to estimate the Young's modulus E of the palms. After that, all the dimensions of the palm (length l , width w , petiole diameter d and thickness t) were measured. Palms used in our experiments vary from 48 to 378 cm in length and from 12 to 100 cm in width. Also, we determined the total mass M of the palm with a precision scale and reported a variation between 13 and 2434 g for the considered palms. Finally, the volume V of the palm has been estimated by measuring its displacement of water when submerged in a tank. In our experiments, the volume of a

palm varied between 21 and 2280 cm^3 . The measurements made on palms of different species are gathered in Fig. 3.

4. Results

4.1. Density

The deduction of the density $\rho = M/V$ of palms are listed in column 10 of Fig. 3. The density of palms considered in our experiments varies from 0.78×10^3 to $1.58 \times 10^3 \text{ kg m}^{-3}$. This observation means that palms of various species of monocots are denser than water and must sink. Such a counter-intuitive prediction for vegetable materials has been verified in a pool.

4.2. Young's modulus

The Young's moduli E of the different palms considered in our experiments have been deduced from their shape when clamped with an extra-load $M_e g$ applied to their free end, as shown in Fig. 4a. In such a situation, the extra load imposes a torque at the base of the palm which is expressed as $M_e g l_x$, where l_x is the length of the palm projected on the horizontal direction. Assuming that the palm behaves as an elastic beam, the previous torque is compensated by the elastic torque $El(C - C_0)$ at its base, where C is the curvature of the palm just adjacent to the clamp and C_0 the natural curvature of the palm at the same position when $M_e = 0$. Finally the torque equilibrium at the base of the palm provides

$$El(C - C_0) = M_e g l_x. \tag{9}$$

Experimentally, we measured the two quantities $C - C_0$ and l_x for various extra masses M_e with image analysis. Fig. 4b shows the

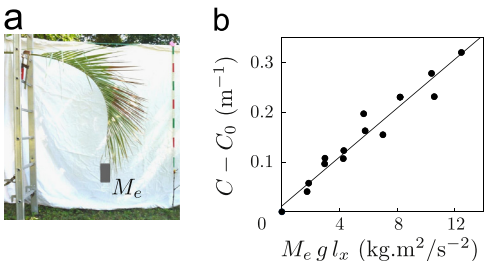


Fig. 4. (a) Palm of a *Buttia* palm tree (Exp. 2) clamped at its base. A mass of $M_e = 0.4 \text{ kg}$ is attached to its free end. (b) Plot of the relative curvature $C - C_0$ of the base of a palm of a *Syagrus romanzoffiana* tree, see experiment number 10 in Fig. 3, as a function of the torque Mgl_x imposed at the free end. The slope gives an estimation of El in $\text{kg m}^3/\text{s}^2$.

Exp.	Species	Location	l (cm)	$2w$ (cm)	d (mm)	t (μm)	V (cm^3)	M (g)	ρ (g/cm^3)	E (Pa)
1	<i>Banana tree</i>	Éguilles	49	12.2	5.5	210	21	13	0.62	8.3×10^6
2	<i>Sica</i>	Éguilles	84	9.2	11.2	720	95	91	0.96	4.5×10^{10}
3	<i>Phoenix</i>	Éguilles	124	17.9	20.1	438	315	332	1.05	2.7×10^8
4	<i>Macromazia Communis</i>	Rayol	134	26.3	9.0	591	155	151	0.97	2.8×10^9
5	<i>Cyathea Cooperi</i>	Rayol	167	69	12.9	147	390	311	0.78	2.1×10^9
6	<i>Buttia</i>	Éguilles	190	21.2	10.9	427	340	385	1.13	1.3×10^{10}
7	<i>Buttia</i>	Rayol	205	58.7	15.7	481	1303	1473	1.13	4.8×10^9
8	<i>Dicksonia Antartica</i>	Rayol	241	42.1	13.3	61	400	631	1.58	3.0×10^9
9	<i>Syagrus Romanzo ana</i>	Rayol	270	91.7	27.3	201	1717	1796	1.04	1.4×10^9
10	<i>Phoenix</i>	La Ciotat	300	73.8	19.7	310	840	946	1.12	6.4×10^8
11	<i>Rhopalostylis Sapida</i>	Rayol	378	100	31.2	310	2280	2434	1.07	5.1×10^8

Fig. 3. Characteristics of palms studied experimentally: species [column 1]; tree location [column 2]; length of the palm l [column 3]; mean width of the palm $2w$ [column 4]; diameter of the petiole at it base d [column 5]; mean thickness of the palm t [column 6]; volume of the palm V [column 7]; mass of the palm M [column 8]; density of the palm $\rho = M/V$ [column 9]; Young's modulus of the palm E [column 10].

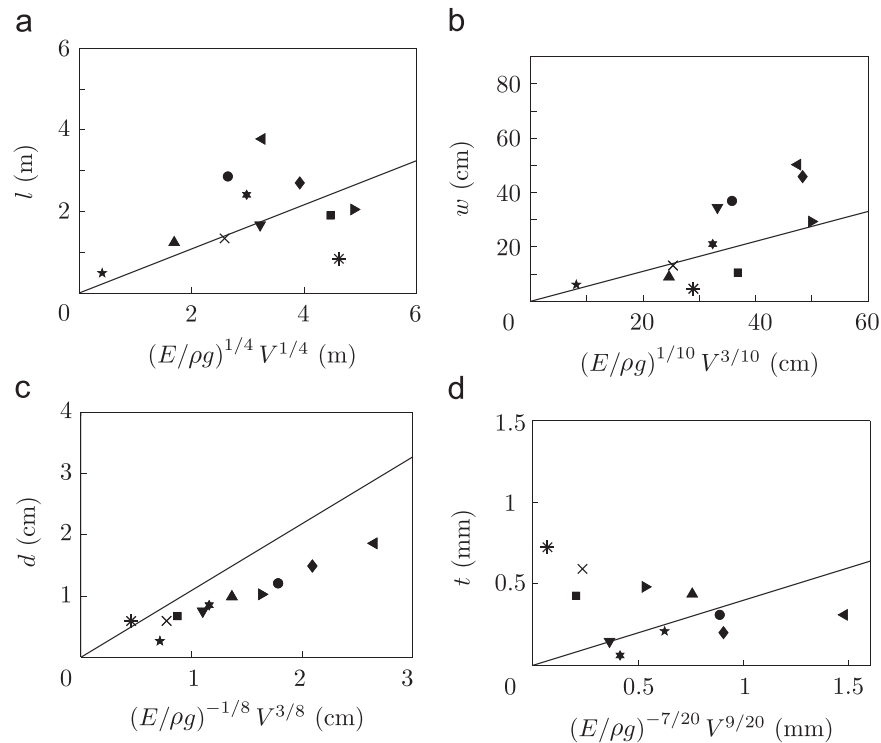


Fig. 5. (a) Palm length l of various species of palm tree as a function of $(E/\rho g)^{1/4} V^{1/4}$. ★ *Banana tree* (Exp. 1), * *Sica* (Exp. 2), ▲ *Phoenix canariensis* (Exp. 3), × *Macrozamia communis* (Exp. 4), ▼ *Cyathea cooperi* (Exp. 5), ■ *Buttia* (Exp. 6), ▴ *Buttia* (Exp. 7), * *Dicksonia antarctica* (Exp. 8), ◆ *Syagrus romanzoffiana* (Exp. 9), ● *Phoenix canariensis* (Exp. 10), ◀ *Rhopalostylis sapida* (Exp. 11). (b) Mean width w of various palms as a function of $(E/\rho g)^{1/10} V^{3/10}$. (c) Mean thickness t of palms of different species as a function of $(E/\rho g)^{-7/20} V^{9/20}$. (d) Diameter of the petiole d at the base of palms of different species as a function of $(E/\rho g)^{-1/8} V^{3/8}$. The black solid lines represent respectively Eqs. (5), (6), (7) and (8) for $\theta = 20^\circ$.

relation between the relative curvature $C - C_0$ at the base of a palm of a *Syagrus romanzoffiana* as a function of the torque $M_e g l_x$ applied to its free end. The correlation of the two quantities provides an estimation of the flexural rigidity EI of the palm along its principal direction. Assuming that in this situation, the second moment of area I is the one of a cylindrical beam of diameter d , ($I = \pi d^4/64$), we estimate the Young's modulus E of the palm. The same procedure has been done for every palm and the estimations of Young's modulus are reported in Fig. 3. For palms considered in this study, E is in a range from 10^7 to 10^{10} Pa consistently with Niklas (1994).

5. Comparison model and measurements and discussion

5.1. Comparison

In order to compare the experimental data with the model developed previously, we plot the palm length, mean width, mean thickness and the mean petiole diameter as a function of the scaling laws predicted by Eqs. (5), (6), (7) and (8), respectively, in Fig. 5.

First, for each parameter the model predicts the actual value within an order of magnitude. Second, the evolution of the data with the scaling laws are consistent. Third, the prefactors of the model yield an estimation of the parameter close to their absolute value.

One may notice that the *Sica* leaf does not follow the general trend for any of the parameters. Such a leaf is not optimally designed for self-support consistently with our observation; gravity does not impact the shape of a mature *Sica* leaf. As a general rule, when a leaf does not change in shape when gravity is

reversed (Mouli's test²), for instance, when its curvature does not change when flipped upside down, the leaf is too stiff and not optimally designed for self-support. To build such a leaf, too much biomass was used for self-support and one may think that other factors of selection prevail.

5.2. Discussion

The agreement between experiment and prediction supports our hypothesis that leaves are optimally designed for self-support. The discrepancy in terms of absolute value may come from the simplification of the geometry of the leaf made in the model (see Section 2.1). For all the leaves considered here, the midvein is not cylindrical but cone-shaped. Also, the non-uniform elasticity of the leaf can explain the difference between measurements and theory. Finally, the mechanical tricks make the leaf stiffer with the same amount of bio-material and thus must increase the length and the width of the leaf. We expected that our predictions of l and w would be lower than the measured values, which is the case. Note that palms are composed of separated leaflets which reduce the apparent density and increase w and l , see Eqs. (5) and (6).

In contrast, the model predicts the leaves' thickness for most of the leaves presented here but does not predict accurately the measurements for the smallest *Sica* leaf, which is too stiff compared to the optimal self-support prediction.

There is a minimal size of leaves for which the proposed model applies. If we consider that a leaf cannot be thinner than $100 \mu\text{m}$, we obtain a critical leaf length of 23 cm and a critical width of 4 cm (for $E = 10^8$ Pa, $\rho = 1.3 \times 10^3 \text{ kg m}^{-3}$ and $\theta = 20^\circ$). This means that

² From Bruno Mouli, INRA Clermont-Ferrand.

small leaves encountered in nature are too stiff and thus not optimized for self-support. Other selection parameters may influence on these leaves to determine their shape. Among these constraints, two examples are wind resistance and resistance to tearing. Also, Jensen and Zwieniecki (2013) discussed the maximal size of leaves resulting from the optimality of sap flow efficiency.

Our study shows that some large leaves are subjected to biomass allocation optimization. But, one may ask if it is preferable to concentrate much of the biomass of the tree in one leaf or if it is more efficient to create a large bunch of small leaves. According to our approach, there is no optimal position on this point. Indeed, the volume of the leaf dedicated to light interception is $V_{\text{light}} = 2 l w t$ that scales as $V_{\text{light}} \propto V^{1/4} V^{3/10} V^{9/20} \propto V$. In proportion, the amount of leaf useful for light interception is independent of the size of the leaf; having a bunch of small leaves or only one huge leaf of the same volume gives the same results in terms of light interception. In reality, the repartition between surface and thickness is different. A bunch of small leaves will have a larger surface to intercept light but a smaller thickness. This indicates that the transmission coefficient of the small leaves should be larger than the one of large leaves.

This particularity of leaf shape might lead to different light interception strategies: (i) in a strong light environment, a thick lamina allows a leaf to intercept a lot of light directly (light interception is exponential with lamina thickness), whereas a thin lamina will intercept less light and thus transmit more to under canopy leaves. One large leaf is favourable in this light environment. (ii) In a low light environment, a thin leaf already intercepts all of the available light. Being thicker is no longer efficient to intercept more light. However, with thin leaves the potential surface area of light interception is larger than with thick leaves. A bunch of small leaves is beneficial in such an environment.

Furthermore, areas with a strongly light environment are located close to the inter-tropical region, where the light is mainly direct light and thus the sun-leaf orientation matters. At higher latitudes, the available light is lower intensity and diffuse light that does not come from a preferred direction – leaf orientation does not matter. This reinforces our earlier point: in the inter-tropical region, leaves have to deal with strong direct light, so building one leaf properly oriented (*i.e.* horizontal) is favourable whereas in higher latitude regions, building a bunch of small leaves oriented in different directions would be more efficient in low and diffuse light environment. Because of these points, it may not be surprising that large palm trees are found in inter-tropical regions rather than close to the poles.

This hypothesis that leaf size is repartitioned with Earth's latitude needs to be verified in a statistical manner. Many selection factors may influence the leaf shape or change the optimal leaf size; see for instance the paper by Jensen and Zwieniecki (2013).

Our analysis has many biases on which we comment in the following: (i) there are many shapes of leaves in nature. We believe that for large leaves, arguments of optimal biomass usage would lead to similar results regardless the actual shape of the leaf. (ii) The leaf is created with a growth process that tends to change the mechanical properties of the leaf (pre-stresses, inhomogeneities, and natural curvatures). We believe that the arguments presented here apply regardless of the mechanical specificity of each leaf. This is because the dependence on global shape is much stronger than the dependence on any mechanical trick (see exponents in Eqs. (5)–(8), always smaller than 1/2). The leaves which measured dimensions are strongly different from predictions of the model would be leaves with a different strategy of orientation for light collection (such as diffuse light collection) or fulfilling other selection parameters.

6. Conclusion

This study shows that large leaves, as the palms studied here, are optimized to support themselves for direct light interception. However, most of the leaves of terrestrial plants are oversized for self-support and thus not optimal. Leaf over-sizing is easily observable: when you take a small leaf in your hand and flip it regarding the gravity, nothing happens and the leaf keeps its own curvature which results from growth (Mouli's test). This over-sizing supports the idea that different selection pressures are more relevant to explain the shape of the leaf rather than the one exposed in this paper. Among these selection pressures are storm survival, drought resistance, herbivory resistance, and enhancement of heat and gas exchange. The large number of selection pressures indicates that leaves are multi-functional organs for which the shape is the result of a complex trade-off. However, depending on leaf fitness, environment, and life history, one among those selection pressures may play a stronger role into leaf shaping than the others. The determination of which selective pressures shape the leaf in a given environment is a fascinating question that still requires further investigation. A better understanding of how competing selective pressure favours some leaves' shapes has strong agricultural implications, *e.g.* the artificial selection of plants to adapt to non-native habitats.

Another perspective of this work could be to relax some assumptions made in the model. Particularly, the simple geometry of palm leaves assumed in the model can be extended to other geometries using an evolutionary algorithm. We expect that such a procedure would predict the optimal geometry of a large leaf with more details (the midvein geometry, lamina shape and even the presence of secondary veins).

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Appendix A. Light interception

The goal of this appendix is to explain why a horizontal and flat position maximizes sunlight interception. Our approach considers a leaf as a thin plate, inclined by an angle θ with the local gravity, Fig. A1. A simple sun path across the sky is indicated by the dotted line. We assume that the sun provides a constant solar energy whatever its position on the sky (This is a simple assumption but

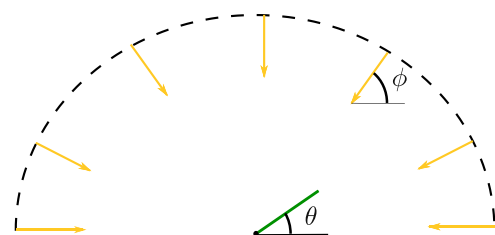


Fig. A1. Notation used to describe light interception by a leaf.

transmitted solar energy is larger when the sun is at its zenith. This effect strongly favours the horizontal position). Moreover we suppose that the sun shines from each position parallel sunbeams with an angle ϕ with respect to the horizontal. For the exact and detailed calculation of light interception, see Pisek et al. (2011) and Tadrast et al. (2014).

The total solar energy collected by the leaf on one day $\mathcal{E}$ is the sum of every sunbeam ($0 < \phi < \pi$)

$$\mathcal{E} \propto \left(\int_0^\pi \sin(\phi - \theta) d\phi + \int_0^{\pi+\theta} \sin(\phi - \theta) d\phi \right) \quad (\text{A.1})$$

The integration over ϕ yields

$$\mathcal{E} \propto (1 + \cos \theta) \quad (\text{A.2})$$

One observes that $\mathcal{E}$ is maximum when $\theta(s)=0$. The optimal orientation of a leaf for sunlight interception is a horizontal. Taking into account the variation of sunlight power during the day (i.e. with respect to solar inclination, ϕ) would reinforce the previous result.

Appendix B. Optimization of biomass allocation

We develop here the calculations to derive Eqs. (5)–(8) from the three first Eqs. (2)–(4) and surface optimization.

With Eq. (2), t writes

$$t = \sqrt{2} \left(\frac{E\theta}{\rho g} \right)^{-1/2} w^{3/2}. \quad (\text{B.1})$$

To simplify notations, in the following, the length $E\theta/\rho g$ is noted Θ . With this notation, t reads

$$t = \sqrt{2} \Theta^{-1/2} w^{3/2}. \quad (\text{B.2})$$

With Eq. (3), d^4 is

$$d^4 = \frac{2^6 \rho g}{3\pi E\theta} w t l^3. \quad (\text{B.3})$$

Injecting Eq. (B.1) in Eq. (B.3) allows to eliminate t ,

$$d^2 = \frac{2^{13/4}}{(3\pi)^{1/2}} \Theta^{-3/4} w^{5/4} l^{3/2}. \quad (\text{B.4})$$

Using those two intermediary equations, Eqs. (B.1) and (B.4), in the equation of volume, Eq. (4), the volume writes

$$V = 2^{3/2} \Theta^{-1/2} w^{5/2} l + \frac{2^{5/4} \pi^{1/2}}{3^{1/2}} \Theta^{-3/4} w^{5/4} l^{5/2}. \quad (\text{B.5})$$

Now to find the optimal leaf, we have to maximize the leaf surface $S=2lw$ for a given volume V . We use the Lagrange multipliers technique where λ is a dummy variable

$$S = 2Lw - \lambda \left(2^{3/2} \Theta^{-1/2} w^{5/2} l + \frac{2^{5/4} \pi^{1/2}}{3^{1/2}} \Theta^{-3/4} w^{5/4} l^{5/2} \right), \quad (\text{B.6})$$

that gives respectively, $\partial S/\partial L = 0$ and $\partial S/\partial w = 0$

$$2w = \lambda \left(\frac{2^{3/2}}{\sqrt{\Theta}} w^{5/2} + \frac{5 \cdot 2^{1/4} \pi^{1/2}}{3^{1/2}} \Theta^{-3/4} l^{3/2} w^{5/4} \right)$$

$$2l = \lambda \left(5 \sqrt{\frac{2}{\Theta}} w^{3/2} l + \frac{5 \pi^{1/2}}{2^{3/4} 3^{1/2}} \Theta^{-3/4} l^{5/2} w^{1/4} \right)$$

With these two lines, we eliminate λ and find the relationship between l and w

$$w^{5/4} = \left(\frac{2 \cdot 3^2}{11 \cdot 5 \pi} \right)^{1/4} \Theta^{1/4} l^{3/2}. \quad (\text{B.7})$$

Now, simple calculations with Eqs. (B.1), (B.4), (B.7) and (B.5) give

$$l = \left(\frac{2 \cdot 3^2 \theta}{5 \cdot 11 \pi} \right)^{1/4} \left(\frac{E}{\rho g} \right)^{1/4} V^{1/4}, \quad (\text{B.8})$$

$$w = \left(\frac{5^5 \pi \theta}{2^7 \cdot 3^3 \cdot 11^3} \right)^{1/10} \left(\frac{E}{\rho g} \right)^{1/10} V^{3/10}, \quad (\text{B.9})$$

$$d = \left(\frac{2^{11} \cdot 3 \cdot 5}{11^3 \pi^3 \theta} \right)^{1/8} \left(\frac{E}{\rho g} \right)^{-1/8} V^{3/8}, \quad (\text{B.10})$$

and finally

$$t = \left(\frac{\pi^3 \cdot 5^{15}}{3^9 \cdot 2^{11} \cdot 11^9 \theta^7} \right)^{1/20} \left(\frac{E}{\rho g} \right)^{-7/20} V^{9/20}. \quad (\text{B.11})$$

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Impact of a pressurised membrane: Contact time

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Contact time

ABSTRACT

Pressurised membranes are used in various situations involving humans (protections, sports, etc). In this context, impacts mechanics of membranes is critical since under or over inflated membranes may create injuries. The effect of inflation pressure on contact time is investigated on a model experiment using a spherical membrane. Both gas pressure and impact speed decrease contact time. Direct measurements of gas pressure and temperature variations indicate an adiabatic compression in strong interaction with the membrane. The contact time of an inflated membrane is described by three dimensionless numbers, (i) the relative inflation $\tilde{P}$, (ii) the presso-elastic number P_E that considers the gas-membrane interaction and (iii) the presso-inertial number I_P that scales membrane inertia to pressure forces. A model of pressurised membrane impact is provided and compared with experiments. It shows that contact time depends mainly on I_P with corrections on P_E and $\tilde{P}$.

1. Introduction

Inflatable structures are used for their capabilities: easy storage and transport, fast deployment, reduced weight and adjustable mechanical properties. Notably, pressurised membranes are used in various situations involving impacts with human beings. Pressurised membranes are widely used in sports (football, basketball, handball, etc.) but may generate injuries by a single or repetitive impacts [1,2]. Specifically in football, inflation pressure and impact speed have been shown to increase head accelerations during ball-to-head impacts [3]. In rare situations, gas expansion is used to create impacts, for instance with a pneumatic jackhammer [4]. However, pressurised membranes are most often used to milden impacts: running shoes equipped with heel air pockets or foam (ensemble of small air pockets) protect from running impacts [5], inflatable bike helmet [6] or car air-bag [7] protect from traumatic brain injuries (TBI). During an impact, the membrane reaction force depends on inflation pressure which is, for instance, a major concern for airbag-deployment and gas dosage as it may cause facial or eye injuries [8,9]. The inertial forces created by pressurised membranes are proportional to the typical acceleration U/t_c where U is the impact speed and t_c the contact time. Short contact time occurs for high pressure membranes and may be harmful. The relation between pressure, velocity and contact time is thus necessary to predict the danger of an

impact of a pressurised membrane.

Contact time has already been observed on various objects. Contact time of inelastic balls has been investigated to understand granular material dynamics [10]. Impacting droplets were studied in order to reduce contact time to avoid possible pinning and keep surfaces clean [11,12]. Contact time of soap bubbles has also been measured and modelled [13]. Pressurised membranes differ from those system by having their mass in the membrane and gas as a spring.

The static deformation of pressurised structures have been studied on spherical and ellipsoidal models [14,15], and is due to the competition between pressure and bending forces. An elastic shell deforms with an inverted cap instability [16], whereas the contact area of a pressurised membrane remains flat [17]. The dynamics of a membrane is mainly driven by gas pressure, membrane shape and bending rigidity [18–20]. During the impact, gas compression has been proposed to be adiabatic [18] or isothermal [21]. However, it has never been measured directly. Measurements of gas temperature, gas pressure and volume are performed simultaneously during the impact of a pressurised membrane.

In the following, we chose a spherical membrane (i.e. without bending rigidity) as a model experiment. The restoring force was only due to the combination of gas pressure and membrane shape. Differently from [21] where pressure elevation was not taken into account and from [18] where interaction between gas and membrane is not considered, we

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show that the main source of non-linearity arises from the membrane-gas interaction during the impact. Although impacts are usually characterised by contact time and coefficient of restitution, we mostly focused on the contact time of the impact.

In this paper, we first report measurements of the impact properties of pressurised membrane (contact time and coefficient of restitution) by systematically varying impact speed, inflation pressure and gas nature. We also describe the observed kinematics of the membrane during the impact. The second part of the paper is devoted to the gas dynamics during the impact and its strong interaction with the membrane elasticity. In a third part, we propose a model of the impact of a pressurised membrane. The impact time of a pressurised membrane is shown to be mainly dependent on two dimensionless numbers, the presso-inertial number and the presso-elastic number. A rationale is given for the dependency of the contact time with impact velocity arising from the non-linear equations of contact and gas dynamics of the pressurised membrane.

2. Impacts on pressurised membranes: experiments

An impact is characterised by the contact time t_c , and the coefficient of restitution $\eta = |U_{out}/U_0|$. $\eta = 1$ corresponds to a lossless impact where the outgoing speed is equal to the ingoing speed. We report here measurements of the impact properties of a pressurised membranes varying impact speed U_0 , inflation pressure $P - P_{atm}$ and gas nature.

2.1. Material and methods

Most studies of impacts on pressurized membranes have been carried out on sport balls [18,21,22] for application to sports but also for supply and cost reasons. Tests were carried out on a set of 4 identical beach-volley balls (Decathlon KIPSTA™, BV100 fun). Those spherical membranes were chosen to be a reference experimental model. They have a minimal composition, they are formed of a single layer of rubber equipped with a valve for inflation, and they are soft enough to expand when inflated at usual inflation pressures and thus to exhibit a strong coupling between gas pressure and membrane elasticity. The membrane weighted $m = 179$ g and had a thickness $e = 2.0 \pm 0.12$ mm. The Young modulus of the membrane was measured on a traction bench, $E = 4.22 \pm 0.1$ MPa. The deflated membrane radius was $R_0 = 8.2 \pm 0.1$ cm. All experiments were performed in the elastic range of the membrane (inflation between 5 and 35 kPa). The main membrane properties and notations are gathered in Table 1.

Membranes were inflated at four inflation pressures $P - P_{atm} = 0.1, 0.15, 0.25$ and 35 kPa and their pressure cross checked with a precise manometer (MPX4250A, NXP). Three different gases were used for inflation, (i) air (diatomic gas with Laplace coefficient $\gamma=7/5$), (ii) pure Helium and (iii) pure Argon which are both monoatomic gases ($\gamma = 5/3$). The pressurised membranes were released at heights (top of the membrane, $h + 2R$) $0.5, 0.75, 0.875, 1, 1.25, 1.35, 1.5, 1.75, 1.85$ and 2 m with no velocity. The membranes impacted on a massive and non deformable marble stone, Fig. 1. Impacts were recorded thanks to a high-speed camera (Photron Miro 110) at 5000 frames per second.

Impact velocities recorded in the experiment varied from 1 to 6 m/s. Positions were determined by tracking the circular shape on each frame using Matlab® 2015b algorithm. The algorithm works as follows, (i) first the scale in pixels per mm is defined and an approximate membrane radius is given after conversion in pixels. (ii) Second, the i th image is loaded. (iii) Third, the membrane location is found using *imfindcircle* Matlab® function based on the circular Hough transform. This function outputs the radius and center of the circular shape of the pressurised spherical membrane. (iv) Four, the ground position is detected automatically using the Hough transform for lines. Steps (ii) to (iv) are repeated for all images of the high speed film. With this algorithm, the center of the circle has been measured precisely even when the membrane was deformed, see Fig. 1b. The algorithm results, ground position,

the positions of the center of the circular shape of the membrane and its radius, were post treated. Contact occurred when the distance between the center of the circle and the marble is less than the membrane radius R . Consequently, contact time t_c was defined precisely by fitting linearly the circle center-ground distance vs time before and after impact. The absolute value of the slopes of those fits were used to determine the impact speed U_0 and the outgoing speed U_{out} . From these measurements, the coefficient of restitution was computed as $\eta = |U_{out}/U_0|$.

2.2. Evolution of contact time as a function of inflation pressure, impact speed and gas nature

The contact time t_c is reported in Fig. 2a as a function of inflation pressure $P - P_{atm}$ for different impact velocities. Impact velocities are colour-coded using the color scale of Fig. 2a. Contact time decreases with inflation pressure for a fixed impact speed. The spreading of impact times at a fixed inflation pressure is reduced when inflation pressure is increased.

For information, the coefficient of restitution $\eta = |U_{out}/U_0|$ is plotted in Fig. 2b as a function of inflation pressure $P - P_{atm}$ for different impact velocities. The coefficient of restitution increases slightly with inflation pressure and decreases with impact speed.

2.3. Kinematics of an impact of a spherical pressurised membrane

During an impact onto the marble considered as a target of infinite mass and stiffness compared to the membrane, the spherical pressurised membrane may be described by the position of its center of mass x_g or the indentation x , see Fig. 1b. The shape of the membrane during the impact is an indented spherical cap [18], which radius may slightly increase during the impact. The part of the membrane in contact with the ground is flattened and no ripples are observed. The volume, V , of a spherical cap and the flat area in contact with the ground, A , of a sphere of radius R indented by a length x reads,

$$V(x) = \frac{\pi}{3} (4R^3 + x^3 - 3Rx^2) \quad \text{and} \quad A(x) = 2\pi Rx \left(1 - \frac{x}{2R}\right). \quad (1)$$

The actual shape of the indented membrane differs from a pure sphere because of oval distortion, see zoom in Fig. 1b. This effect is taken into account in our model applying a correction to the actual surface in contact with the ground, $A_{contact} = \beta A$ with $\beta < 1$. The effect of oval distortion on the volume of the spherical cap remains negligible.

The deformation of the ball moves its center of mass which is no longer located at the center of the spherical shape, see Fig. 1b. When the ball gets in contact with the ground, $x = 0$ ($x_g = R$), the ball is not yet deformed and both the center of mass and the center of the sphere are at the same location. For larger and larger indentations the two centres separate one from another. Considering an homogeneous mass distribution of the membrane, the position of the center of mass during the contact reads

$$x_g = R \left(1 - \frac{x}{2R}\right)^2. \quad (2)$$

3. Gas-membrane interaction during an impact

3.1. Material and methods

The behaviour of the gas and its interaction with the membrane during an impact has been explored experimentally. In order to assess the hypothesis that the gas compression-expansion is adiabatic, one has to show that the time scale of thermal exchanges is larger than the impact time scale. To this aim, a first static compression experiment is performed to measure the characteristic time of thermal leaks. Second, the gas dynamical behaviour is analysed during the impact to understand how gas and membrane interact.

For both experiments, the membrane was punched with two holes of 5 mm of diameter. In each hole, a home-made hermetic feedthrough was inserted. The two feedthroughs allowed to place a micro-thermocouple and two pressure sensors inside the membrane simultaneously, see Fig. 3c. The total mass of the feedthroughs and sensors was 6 g, much lower than the membrane mass of 179 g (3.2% of the total mass).

The first pressure sensor, sensor A, was an absolute pressure sensor (Freescall semiconductor MPXAZ6115A, probing with an accuracy of 3.45 kPa), used to measure the initial pressure inside the membrane. The second sensor, sensor B, was a differential sensor (Freescall semiconductor MPXV7007) with accuracy of 500 Pa. This sensor was used to measure with accuracy the variations of pressure during impacts and static compression experiment. Differential pressure measurements were corrected by 1 ms, according to the sensor response time. Both sensors were linked to the same feedthrough. A manual valve allowed to set the reference of pressure for sensor B.

The indentation of the membrane was optically measured with a high-speed camera (Photron MIRO 110). The images were processed with a home-made Matlab® code similar to the one described in Section 2.1.

The temperature of the gas was recorded by a fast response type K thermocouple made by the welding of thin alumel and chromel wires (Fig. 3a,b). The welded hot junction is realised by a capacitance discharge device developed by the FEMTO-ST laboratory [23]. The thin wires were insulated from each other by a ceramic rod of diameter 0.8 mm. For the impact experiment, the use of extremely thin wires (7.6 μm in diameter) was required to get a sufficiently small thermal inertia to respond to sudden changes of temperature. The thermocouple of 7.6 μm wires had a time constant of ≈ 5 ms in air (natural convection) [23]. Conversely such a small thermocouple is mechanically fragile. To avoid breakage, a type K thermocouple with wires of 75 μm in diameter was used for the static compression experiment (time constant of ≈ 1 s). The home-made thermocouple was placed in the gas close to the membrane, Fig. 3b. Temperature signals were filtered for frequency content larger than 500 Hz and between 49.5 and 50.5 Hz.

Tension signals were converted in absolute temperature using the tabulated Seebeck coefficient [24]. Sensitivity of the type K thermocouple was $40\mu\text{V.K}^{-1}$ at 293 K leading to an uncertainty of ± 0.2 K in our experiments. The main source of error was the noise due to antenna effect of the thermocouples.

All sensors were linked to a NI instruments dacq for acquiring the signal through a Labview interface. Data were acquired at 10^4 Hz. During impact experiments, the high-speed camera was synchronised with the other measurements (pressures and temperature) with a trigger signal.

3.2. Experimental results

The membrane has been statically compressed between two plans. The inner pressure rose quickly, Fig. 4a, and was maintained for more than 80 s before being released. The rise of pressure induced an increase of the gas temperature, see Fig. 4b. The delay was due to the response time (≈ 1 s) of the 75 μm thermocouple. After reaching a maximum the temperature decreased following a damped exponential (straight line when plotted in lin-log scale, see inset). The characteristic time of decrease of temperature was $\tau_{\text{th}} = 13.5$ s.

As the gas cooling is not uniform within the membrane, the characteristic time may depend on the position of the thermocouple relative to the membrane. The measured time $\tau_{\text{th}} = 13.5$ s compares well with a characteristic diffusive time $\tau_{\text{th}} \sim l^2/D \approx 5$ s where $l = 1$ cm is the distance of the thermocouple to the membrane and $D = 20 \times 10^{-6} \text{ m}^2.\text{s}^{-1}$ is the thermal diffusion coefficient in air. To get rid off this effect, one can compute an upper bound for thermal transfer by conduction, considering that the gas cools down to the membrane temperature over a thickness $\sqrt{Dt_c}$

$$\frac{\delta \mathcal{E}_{\text{th}}}{\mathcal{E}_{\text{th}}} = \frac{3\sqrt{Dt_c}}{R} \quad (3)$$

Using typical values, $D = 20 \times 10^{-6} \text{ m}^2.\text{s}^{-1}$, $t_c = 10$ ms and $R = 10$ cm, one obtains $\delta \mathcal{E}_{\text{th}}/\mathcal{E}_{\text{th}} \approx 1.3\%$. The adiabatic hypothesis is well verified.

The temperature variations, pressure variations and indentation were measured simultaneously for impacts at different heights h . A typical measurement is shown in Fig. 5. The pressure and indentation evolved simultaneously due to the fast response of the sensors. Differently, the temperature only increased after a delay of ≈ 5 ms corresponding to the response time of the 7.6 μm thermocouple used in the experiment. The temperature increase shows that even for small indentation, gas compression is not isothermal and that gas temperature rises by few Kelvins.

3.3. Model of evolution of pressure during an impact

The pressurised membrane was released at different heights $0.5 \text{ m} < h < 1.85 \text{ m}$. The impact speed was much lower than sound velocity ($U_0 \ll 340 \text{ m/s}$), it thus remained under the hypothesis of quasi-static compression. As shown previously, the gas compression was also adiabatic, it can be modelled by the Laplace relation,

$$\frac{\Delta P}{\Delta V} = -\gamma \frac{P}{V}, \quad (4)$$

where γ is the Laplace coefficient. $\gamma = 5/3$ for monoatomic gases, $7/5$ for diatomic gases and $\gamma \rightarrow 1$ for polyatomic gases. During the impact, the membrane is an indented sphere which radius slightly increases because of pressure. The increase of radius of a thin elastic sphere reads [25],

$$R = R_0 + \frac{(P - P_{\text{atm}})R_0^2}{2eE} \quad (5)$$

The dependence between membrane radius and inflation pressure is shown in Fig. 6a and fitted with $E = 4.2$ MPa and $e = 2$ mm to determine $R_0 = 8.2$ cm. In the limit of small indentations $x \ll R$, one may linearise the membrane indented volume Eq (1), and the two above relations, Eqs. (4) and (5) to find the relation between gas pressure and indentation,

$$\frac{\Delta P}{P} = \frac{\gamma}{1 + (2 + 3\gamma)P_E + (1 + 3\gamma)P_E^2} \left(\frac{x}{R_0}\right)^2 \quad \text{with} \quad P_E = \frac{PR_0}{2eE} \quad (6)$$

P_E is the presso-elastic number which scales the inflation pressure to the elasticity of the membrane. For $P_E \ll 1$, the membrane is rigid and the increase of radius is negligible. In the contrary if $P_E \gg 1$, the membrane is flexible, the membrane radius increases and pressure variations are reduced. In the case of the membrane studied experimentally P_E is in the medium range with $P_E \approx 0.56$ (using values in Table 1). The relation between pressure and indentation is plotted in Fig. 5 for all impact experiments ($0.5 \text{ m} < h < 1.85 \text{ m}$), and compares well with the theory without fitting parameters. In the inset, the compression-expansion cycle is shown to occur without hysteresis (Fig. 6b).

4. Effect of gas nature and pressure on contact time

4.1. Conservation of momentum

In the chosen parametrisation, we compute the acceleration of the center of mass as a function of x by double differentiation of Eq. (2),

$$\ddot{x}_g = -\left(1 - \frac{x}{2R}\right)\ddot{x} + \frac{\dot{x}^2}{2R} \quad (7)$$

The right-hand-side of this equation is compounded of two terms: the first one corresponds to the variation of momentum due to the acceleration of the mass of the membrane that is not in contact with the

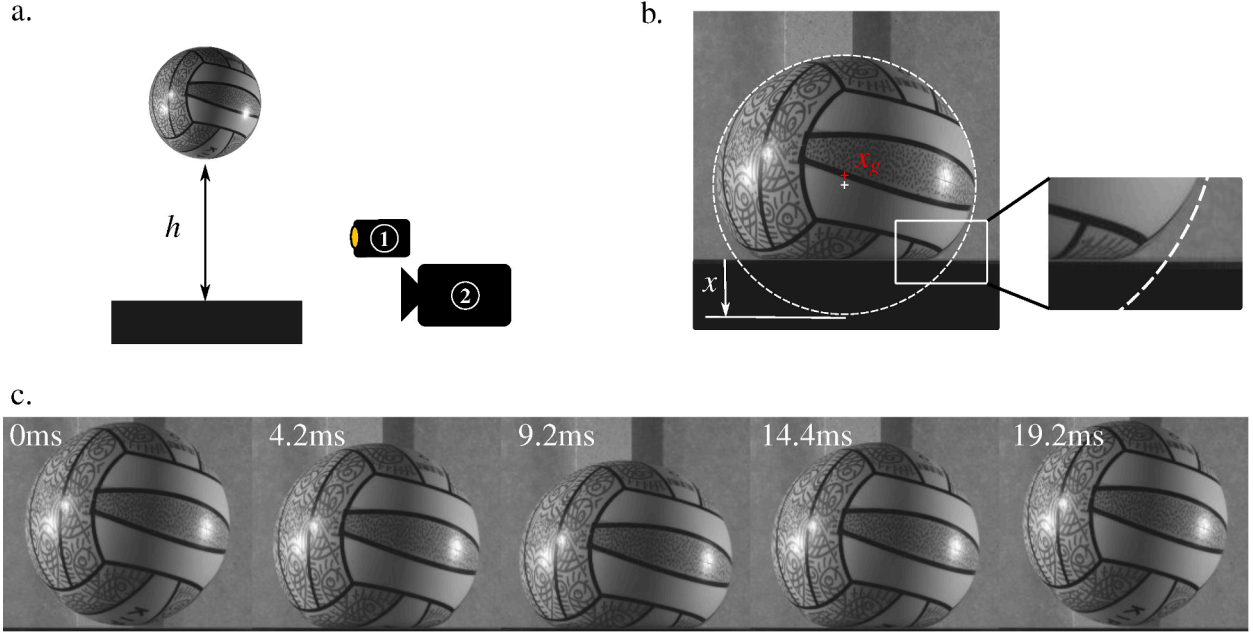


Fig. 1. a. View of the experiment. The pressurised membrane is released without velocity from a height h . ① Flicker-free lighting. ② high speed camera. b. Definition of indentation x and membrane center of mass g during impact. Zoom on membrane oval distortion. c. Chrono-photography of an impact event ($h = 1.85$ m and $P - P_{\text{atm}} = 15$ kPa).

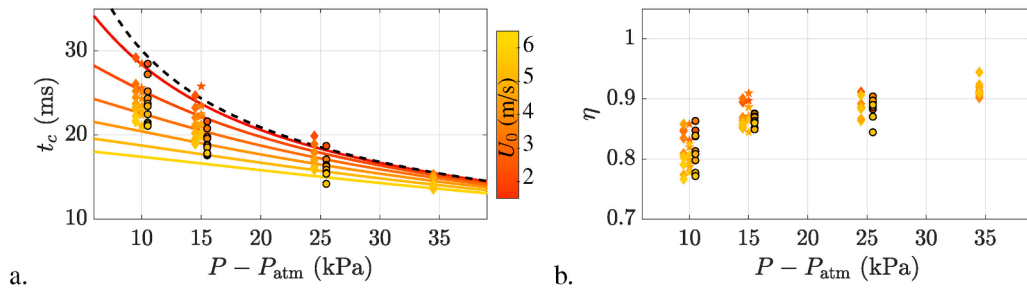


Fig. 2. a. Contact time, t_c as a function of inflation pressure $P - P_{\text{atm}}$. Coloured lines are the non-dissipative model prediction for contact time, Eq. (15), for different impact speeds. The speed color map will be conserved for all the figures. Dashed line correspond to the linear model independent of impact speed U_0 . b. Coefficient of restitution η as a function of inflation pressure $P - P_{\text{atm}}$. Inflation gas: $\blacklozenge$ Argon, $\star$ Helium and $\bullet$ Air.

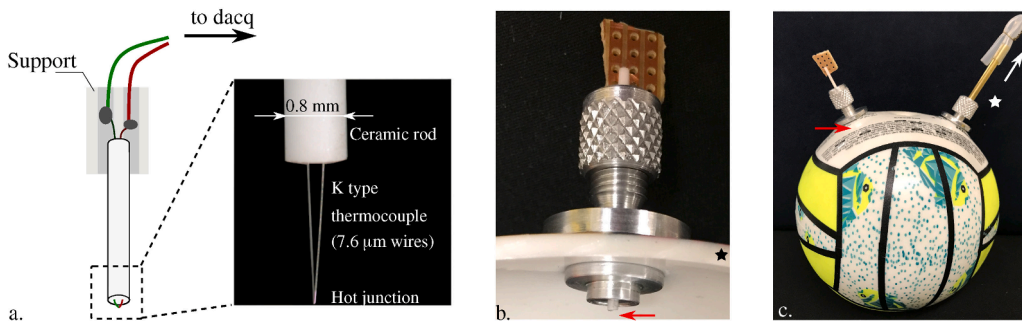


Fig. 3. Instrumented membrane for gas pressure and temperature measurement. a. Building of a fast K-type thermocouple. b. Cross section of the membrane showing the insertion of the thermocouple in the hermetic feed-through. Red arrow: thermocouple tip. Black star: membrane. c. Example of an instrumented mini-membrane (radius 57.5 mm) with two feedthroughs. One is devoted to temperature measurement (red arrow) with the micro-thermocouple tip inside the membrane and the other is devoted to the absolute and differential pressure measurements

(white star) with the white arrow indicating that the pressure sensors are outside the membrane. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

ground, $m(1 - x/2R)$. The second term corresponds to the variation of momentum associated to the membrane mass that actually reaches the ground during a given time interval, $m\dot{x}/2R$, which velocity drops from $\dot{x}$ to zero. When the membrane is not much squashed on the ground, $x \ll R$, the second term is much lower than the first one and the accel-

eration of the center of mass simply reduces to $\ddot{x}_g = -\ddot{x}(1 - x/2R)$.

The force exerted by the membrane onto the ground is the inner pressure, $P + \Delta P(x) - P_{\text{atm}}$ times the surface of contact, A . Conservation of momentum reads,

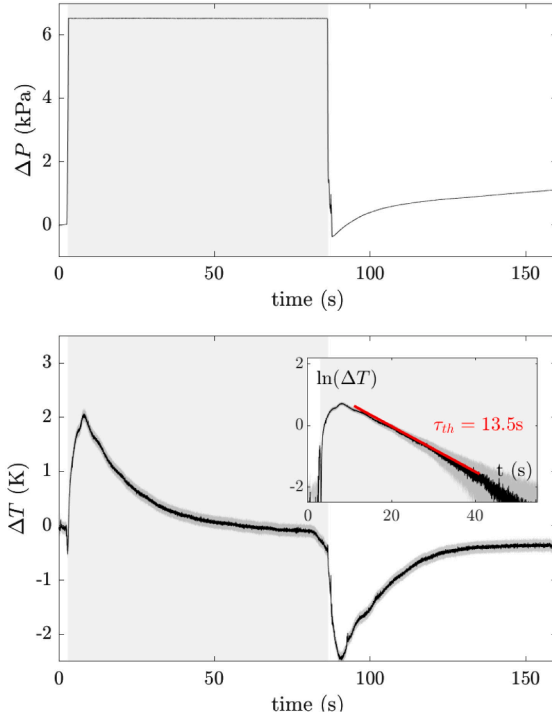


Fig. 4. Simultaneous evolution of a. gas pressure variations ΔP and b. temperature variations ΔT for a long (>80 s) compression-expansion cycle. (inset) lin-log graph of the gas temperature decrease showing the typical time $\tau_{th} = 13.5$ s.

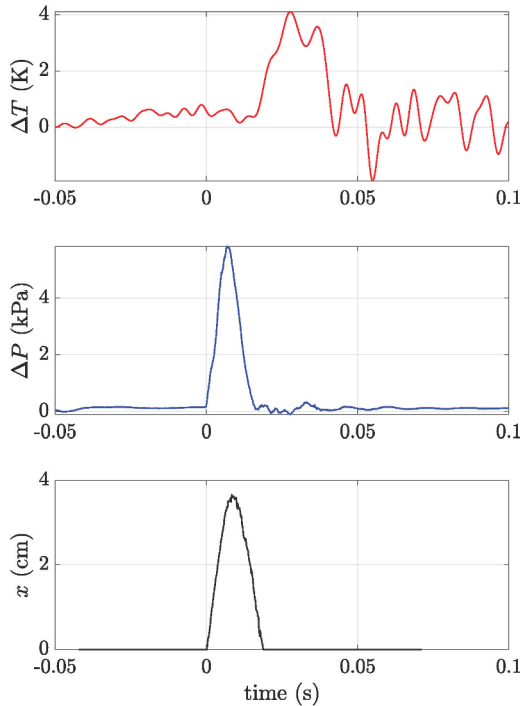


Fig. 5. Simultaneous measurements of temperature variations ΔT , pressure variations ΔP and indentation x (for impact conditions $P = 15.5$ kPa and $U_0 \approx 6$ m.s⁻¹). Temperature sensor responds to temperature variations with a delay and has a moderate noise level.

$$m\ddot{x}_g = \beta A(x) (P + \Delta P - P_{\text{atm}}), \quad (8)$$

where $\beta \approx 1/2$ is the fraction of the flattened membrane in contact with the marble, see Fig. 1b. Under the hypothesis of small indentation $\dot{x}^2/2R \ll \ddot{x}$, one obtains

$$-m\ddot{x} = 2\beta\pi R x (P - P_{\text{atm}}) \left(1 + \frac{\Delta P}{P - P_{\text{atm}}}\right). \quad (9)$$

The variation of radius due to the increase of pressure is negligible compared to the variation of inner pressure during the impact as $\Delta R/R = R_0 P_E \Delta P / P R < \Delta P / P \ll \Delta P / (P - P_{\text{atm}})$. The quantity $\Delta P / (P - P_{\text{atm}})$ which corresponds to the relative increase of pressure during the impact is thus the main source of force non-linearity in Eq. (9). In the following, we neglect variations of membrane radius R . At this step, we define the dimensionless variables $\tilde{x} = x/R$, $\tilde{t} = tU_0/R$ and make Eq. (9) dimensionless. We obtain the presso-inertial number $I_p = mU_0^2/2\beta\pi R^3(P - P_{\text{atm}})$ and the reduced pressure $\tilde{P} = (P - P_{\text{atm}})/P$. For convenience, we note $B = \gamma(1 + P_E^2)/(1 + (2 + 3\gamma)P_E + (1 + 3\gamma)P_E^2)$. Combining Eq. (9) and (6), one finds the dimensionless equation of evolution of the indentation,

$$\ddot{\tilde{x}} = -\frac{\tilde{x}}{I_p} - \frac{B\tilde{x}^3}{I_p\tilde{P}}. \quad (10)$$

With the initial condition at $\tilde{t} = 0$, $\tilde{x} = 0$ and $\dot{\tilde{x}} = 1$, the first integral of Eq. (9) reads

$$\dot{\tilde{x}} = \pm \sqrt{1 - \frac{1}{I_p} \left(\tilde{x}^2 + \frac{B}{2\tilde{P}} \tilde{x}^4 \right)}. \quad (11)$$

4.2. Pressure, temperature and indentation maxima

The maximal indentation occurs when $\dot{\tilde{x}} = 0$. There exists a single physical solution that reads

$$\frac{x_{\text{max}}}{R} = \sqrt{\frac{\tilde{P}}{B} \left(\sqrt{1 + \frac{2BI_p}{\tilde{P}}} - 1 \right)^{1/2}}. \quad (12)$$

Under the hypothesis of slow impact $I_p \ll 1$, one gets

$$\frac{x_{\text{max}}}{R} = \sqrt{I_p} \left(1 - \frac{BI_p}{4\tilde{P}} \right) + o(I_p^{3/2}). \quad (13)$$

Consequently, one finds the maximum of pressure and temperature elevations

$$\frac{\Delta P_{\text{max}}}{P} = B \left(\frac{x_{\text{max}}}{R} \right)^2 \quad \text{and} \quad \frac{\Delta T_{\text{max}}}{T} = \frac{(\gamma - 1)}{\gamma} \frac{\Delta P_{\text{max}}}{P}. \quad (14)$$

The comparison of the predicted maxima to the experimental values is shown in Fig. 7. The model predicts the right order of magnitude and tendency for indentation, pressure and temperature.

Since dissipation is not taken into account in this model, the contact time is twice the time needed to get to the maximal indentation,

$$t_c = \frac{2R}{U_0} \int_0^{\tilde{x}_{\text{max}}} \frac{d\tilde{x}}{\sqrt{1 - \frac{1}{I_p} \left(\tilde{x}^2 + \frac{B}{2\tilde{P}} \tilde{x}^4 \right)}} = \frac{2R}{U_0} \sqrt{\frac{2I_p}{1 + \frac{2BI_p}{\tilde{P}} + 1}} \mathbf{K} \left(-\frac{\sqrt{1 + \frac{2BI_p}{\tilde{P}} - 1}}{\sqrt{1 + \frac{2BI_p}{\tilde{P}} + 1}} \right). \quad (15)$$

where $\mathbf{K}$ is the first complete elliptic integral. We plot t_c for various inflation pressure and velocities in Fig. 2a. The model is in good agreement with the data trends. A discrepancy between model and data

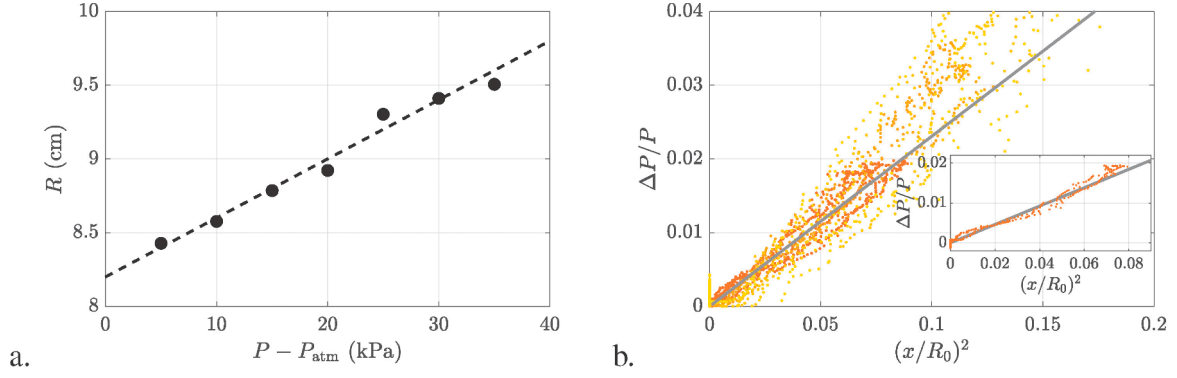


Fig. 6. a. Membrane radius vs Inflation pressure. Colours correspond to impact speed following colour map of Fig. 1. Black line corresponds to Eq. (5) with $E = 4.2$ MPa, $e = 2$ mm and $R_0 = 8.2$ cm. b. Example of normalised pressure vs. normalised indentation squared for an impact from height $h=0.5$ m. Black line corresponds to Eq. (5) with $P_E = 0.56$ and $\gamma = 7/5$ ($P = 114$ kPa, $E = 4.2$ MPa, $e = 2$ mm and $R_0 = 8.2$ cm).

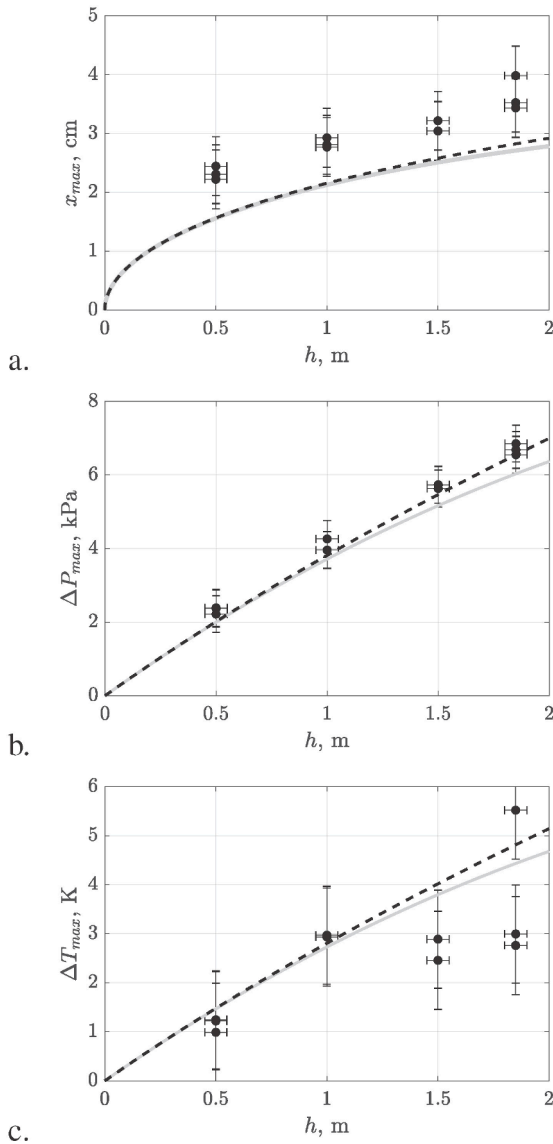


Fig. 7. Maximal values of a. indentation x , b. pressure variation ΔP and c. temperature variation ΔT , with release height of the membrane. Dashed lines correspond to the exact model prediction, Eq. (12) and (14) using $\gamma = 7/5$, $P = 114$ kPa, $T = 293.15$ K and membrane characteristics given in Table 1. Light grey lines correspond to the approximated predictions, Eq. (13) and (14) with the same set of parameters.

Table 1

Parameter and notations used in the paper. ($\dot{x}$) denotes the time derivative of x , index $_{\text{max}}$ denotes maximal value during impact.

PARAMETERS		
P	Absolute inner pressure	
P_{atm}	Atmospheric pressure	986 kPa
ΔP	Pressure variations during impact	
V	Gas volume	
T	Inner gas temperature	
R	Membrane radius	
h	Release height	
U_0	Impact speed	
g	Gravity acceleration	
t	Time	
x	Indentation	
x_g	Membrane center of mass	
t_c	Contact time	
MEMBRANE PARAMETERS		
R_0	Radius (deflated)	8.2 cm
m	Mass	179 g
E	Young modulus	4.2 MPa
e	Thickness	2 mm
DIMENSIONLESS NUMBERS		
γ	Laplace coefficient	-
$\tilde{p}$	Reduced pressure	
P_E	Presso-elastic number	
I_P	Presso-Inertial number	

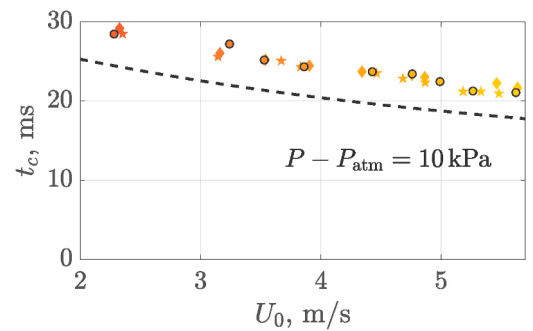


Fig. 8. Comparison of contact time prediction and measurement for trial at $P - P_{\text{atm}} = 10$ kPa. Black line: model prediction Eq. (15).

exists when comparing the magnitude of the contact time, figure 8. This discrepancy comes from the term in $\dot{x}^2$ in Eq 7 that increases the contact time and from ball oval distortion.

5. Discussion

The model developed for the dynamics of a pressurised membrane impacting a rigid surface relies on several assumptions which should be discussed. First, we assumed that no energy was dissipated during the impact (neither sound emission [22], vibrations nor visco-elasticity dissipation [21,26]), that leads to overestimate the maximal indentation $x_{\max}$ and to underestimate the contact time t_c . This point will be addressed in a future study on coefficient of restitution, however it is thought to have little effect on the contact time t_c . Indeed, using a linear damped oscillator model, one may estimate the lost energy as $2m\xi U_0^2$ where ξ is the reduced damping coefficient. This compares to the lost energy during the impact $(1 - \eta^2)mU_0^2/2$. One finds $\xi \simeq (1 - \eta^2)/4$. The reduced damping coefficient changes the contact time as $t_{c,\text{damp}} = t_c / \sqrt{1 - \xi^2}$. For $\eta = 0.9$, $(t_{c,\text{damp}} - t_c)/t_c \simeq 10^{-3}$.

Second, we made the hypothesis of small indentations of the membrane ($x/R \ll 1$) and consequently neglected the term $\dot{x}^2/2R$ in Eq. (7), this may induce an underestimation of the contact time. This term do not involve any loss of energy. Finally, we have accounted for oval distortion that arise because of membrane inertia through a simple corrective factor β . This is a raw assumption and taking into account the deformation of the ball due to inertia would be the next step of this theory. The quasi-static assumption is quite well verified in the limit of slow impact but may not be exact for impact at large velocities (approaching sound velocity or wave velocity in the membrane), possibly changing the physics of the impact.

Under these assumptions, we showed that the impact of a pressurised membrane is ruled by four dimensionless numbers. (i) $\tilde{P}$ is the normalised overpressure inside the membrane; (ii) the presso-elastic number P_E compares the inflation pressure with the membrane elasticity; (iii) I_P scales the initial kinetic energy with the work of the inner gas compression during the contact; finally, (iv) γ is the Laplace coefficient that depends on gas nature. The first three parameters have a major influence of the contact dynamics. In the contrary, γ is bounded between 5/3 for the adiabatic compression of a mono-atomic gas and 1 for an isothermal compression. In this range of variations (40%) it has only little influence on the contact time. The physical properties of the inflation gas (mono-atomic, polyatomic, etc) only change marginally the bouncing dynamics of the membrane. Differently, the increase of the inner pressure implies an increase of $\tilde{P}$ and P_E and provokes a reduction of the maximal indentation and of the contact time. When the impact speed is increased keeping the overpressure $\tilde{P}$ constant, I_P becomes larger and induces a larger maximal indentation and a shorter contact time.

In order to compare the model predictions with experiments, we characterised the impact dynamics of a model pressurised membrane onto a rigid substrate for different inflation pressures and impacting speeds. These experiments lied on the range of parameters where $P_E < 1$ and $I_P < 1$, corresponding to small membrane elasticity and slow impacts. The good agreement between these observations and model predictions justify the approximations discussed previously. One perspective of this work would be to change the properties of the membrane in order to explore the limit where the non-dimensional numbers P_E and I_P are larger than unity and verify model predictions in this limit.

This model of contact time of pressurised membranes allows to discuss the properties of inflated protections and sport balls. Biological damages are difficult to relate to a unique impact property as, for instance, traumatic brain injuries causes and criteria are numerous and complex [27]. Nevertheless, biological damages are somehow

dependent of the maximum force level. In the case of a free pressurised membrane the maximum force is related to the contact duration as $F_{\max} \sim mU/t_c$. Increasing contact time by decreasing inflation pressure or by choosing softer membrane may thus allow to reduce impact injuries. This should help to design safer sports balls [3].

In this paper, we considered an ideal target of infinite mass and stiffness. This may represent quite well the impact of a pressurised sport ball on a hard ground such concrete or wooden floor that may be encountered in basketball or handball. Differently, for outdoor sports such as football or rugby, the field is soft enough to be deformed elastically or plastically by the ball during the impact, changing the properties of impact [28,29]. This remark stands also for impact of pressurized balls or pressurised protections on humans which soft tissues (skin, muscles, etc.) may deform under impact. To take into account the mass of the target one has to consider the relative velocities in the frame of the center of mass. Compared to the case of infinite target mass $m_t \rightarrow \infty$, the amount of momentum exchanged during the impact is reduced by a factor $m_t/(m_i + m_t)$ where m_i is the mass of the impactor. Regarding the stiffness, the springs of the target k_t and impactor k_i are in series what decreases the total stiffness of the system $k_i k_t / (k_i + k_t)$ and increases contact time consequently. Thus taking into account a real target decreases the amount of momentum exchanged and increases contact time. The case of an impact on an ideal target represents thus an upper limit for inertial forces.

Regarding sports, it is unlikely that increasing contact time may increase precision since human response time is much larger than ball contact time. Differently, longer contact time may favour handling and control of the ball since it mostly relies on motion coordination rather than reflex. This explain why ball deflation has been used to cheat in American football where each team uses its own ball, [30]. Questions also arise for the level of trajectory modification following the impact of a spinning ball depending on its inflation pressure since it will increase indentation, contact surface and contact time.

6. Conclusion

We have shown that the contact dynamics of a pressurised membrane is made non-linear by the increase of the internal pressure resulting from the adiabatic compression of the gas. We have proposed a non-linear model that rationalizes the dependency of the contact time with the inflation pressure and the impacting speed. In addition, this model predicts a small effect of the gas nature, a fact that is observed in practice. Our model implies that the contact force F depends on the indentation x in a supra-linear way: $F \propto \tilde{x} + B\tilde{x}^3/\tilde{P}$. The bouncing dynamics of a pressurised membrane strongly differs from the Hertzian law $F \propto x^{3/2}$ which rules the impact of filled spheres made of elastic materials. It is also different from the linear law for shells before buckling $F \propto x$ if $x < e$ and the sub-linear law for shells after buckling $F \propto x^{1/2}$ for $x > e$ [31]. It would be interesting to study the implications of such differences in sports where these distinct types of balls are employed.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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B3. Robotique molle et bio-inspirée

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Controlled Locomotion of a Minimal Soft Structure by Stick-Slip Nonlinearity

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We present a locomotion mechanism that uses the stick-slip transition of a soft passive structure with an internal mechanical resonance. The structure is harmonically driven by a global vertical shaking and, because of its resonance dephasing and the threshold response of stick-slip transition, it can either move forward or backward. We establish a relation for the motion acceleration threshold that we experimentally validate. We identify a nontrivial regime close to the resonance with a velocity inversion for a constant excitation frequency and an increasing driving amplitude. We finally show that we can achieve a controlled multimodal motion by combining multiple internal resonances.

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The drift of an object placed on a vibrated table is a strategy that was used with a large variety of granular particles with head-tail properties in the bouncing regime. This is the case for boltlike particles [1], rods with a shifted center of mass [2], asymmetric chains [3], dimers with asymmetric internal dynamics [4], and asymmetric plant spikes [5]. Self-propulsion using asymmetric elements also exists with other power sources such as the Leidenfrost levitation and motion of uneven solids [6,7], the horizontal motion of asymmetric objects in vertical diffusion gradients [8], or Janus-like particles [9,10]. Rectified motion even exists with particles without head-tail asymmetry such as even dimers [11], trimers [12], or chiral dimers [13]. Rectified motion under vertical vibration also applies to the rotation of particles with chiral structure [14–17]. Other examples of simple self-propelled elements are polar disks [18], boltlike particles [19], walking [20–22] and spinning [23] robots, or asymmetric grains [24]. The vibration mechanism can also be embedded inside the granular particles as it is for the so-called “bristlebots” [25–27]. In the context of this Letter, we can also mention some interesting drifting mechanisms with vibrated fluids such as shaken droplets with nonvertical excitation [28,29], droplets on tilted supports [30,31], and walking droplets [32,33].

The above mechanisms for granular systems use elements in a bouncing regime mostly have a single mode of locomotion, either linear motion or/and steady rotation, and their structure is approximately rigid or, if not, no mechanical resonances are involved. In this Letter, we identify a new locomotion mechanism that combines stick-slip transition and mechanical resonance of a soft structure to obtain a controllable motile object with individually addressable forward and backward locomotion states.

Figure 1(a) represents the essential ingredients for our locomotion mechanism with a structure composed of a

rigid base represented by a rectangular block and a tilted flexible plate. The whole structure is placed on a vertically oscillating rigid table harmonically driven by a shaker at a frequency $\omega/2\pi$ and an acceleration amplitude a . The structure is in solid contact with the table and it can either stick or slide horizontally if the ratio of the tangential forces T to the normal forces N exceed the static friction coefficient.

Figure 1(b) represents the threshold acceleration a_c to set the structure in horizontal motion as a function of the frequency. The acceleration threshold is complementary to a resonancelike curve with an acceleration threshold decreasing as the driving frequency approaches the first mode resonance for the structure, which in this example is 14.44 Hz. The sign of the velocity depends on the driving frequency relatively to the structure internal resonance: for frequencies below the resonance frequency (i.e., between 10 and 14.5 Hz), the structure moves forward while above resonance (i.e., between 15 and 20 Hz), the structure moves backward. The threshold acceleration around the resonance is well below the gravity acceleration g , which indicates that the structure is in horizontal sliding motion without any jumps.

Figure 1(c) represents the structure velocity as a function of the shaker acceleration. Each connected set of data points are for a fixed frequency. For most of the frequencies, the velocity is linear with the acceleration and consistently with Fig. 1(b), the velocity is positive below the resonance and negative above the resonance. For the data points at $f = 14.5$ Hz close to the resonance, the threshold acceleration is the lowest at 4 m s^{-2} but surprisingly the velocity is nonmonotonous with the acceleration.

To explain the net motion of the structure under vibrations and notably the peculiar nonmonotonous regime close to resonance, we have to consider the coupling between solid friction and the linear resonance of the

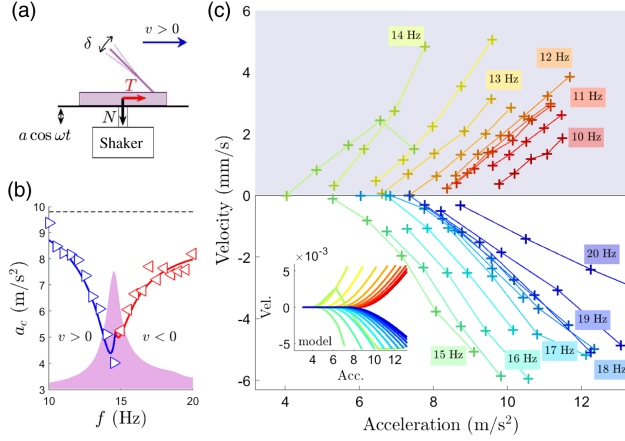


FIG. 1. (a) Schematics of the locomotion principle. The horizontal black line represents a rigid table set in vertical oscillatory motion by a shaker at an acceleration amplitude a and frequency $\omega/2\pi$. The soft structure is in frictional contact with the table and it is composed of a rigid base and a tilted soft plate. N is the normal contact force and T is the tangential contact force. δ is the amplitude of the cantilever first mode. (b) Acceleration threshold for steady horizontal motion as a function of the forcing frequency. The solid line is the model in Eq. (4) with $\omega_0/2\pi = 14.44$ Hz, $Q = 17.5$, $g = 9.8 \text{ m/s}^2$, $\alpha = 64^\circ$, $M = 0.08$, and $c_f = 0.25$. Blue, positive velocity; red, negative velocity. The resonance response of the soft plate first mode is represented in the background. (c) Velocity as a function of the shaker acceleration a . Each set of connected points is for a constant driving frequency. For frequencies from 10 up to 14.5 Hz, the structure moves forward (positive velocities) and for higher frequencies, it moves backward (negative velocities). The inserted plot is the result from the model presented in Fig. 2.

structure's first mode. The acceleration threshold for the directed motion is related to the threshold condition for sliding in the solid friction regime,

$$\frac{|T|}{|N|} = c_f, \quad (1)$$

in which T is the tangential force, N the normal force, and c_f the static solid friction coefficient [T and N are represented in Fig. 1(a)]. Without vibration, there is no tangential force $T = 0$ and the normal force is the total weight of the structure of mass m , $N = mg$. With the vibration, the net forces N and T have a modulation at the frequency of the shaker with

$$N = mg - ma \cos \omega t + m_1 \ddot{\delta} \cos \alpha \quad (2)$$

$$T = -m_1 \ddot{\delta} \sin \alpha, \quad (3)$$

in which m is the total mass, m_1 is the effective mass of the cantilever first mode, a is the acceleration amplitude of the shaker, and α is the angle between the flapping plate and

the horizontal. In Eq. (2), the term $-a \cos \omega t$ is the modulation of the vertical acceleration in the frame of the vibrating support. $\ddot{\delta}$ is the acceleration of the first mode of the soft plate, and it modulates the net forces N and T via the projection with angle α . The amplitude δ of the first mode is given by the linear mechanical response $\ddot{\delta} + (\omega_0/Q)\dot{\delta} + \omega_0^2\delta = a \cos \omega t \cos \alpha$, with $\omega_0/2\pi$ the resonant frequency of the mode and Q the quality factor. If the sliding condition in Eq. (1) is reached during the time evolution of N and T , the disk slides forward or backward, depending on the sign of T when the threshold is reached. By combining Eqs. (1), (2), and (3), we find two threshold accelerations for the horizontal motion,

$$a^*_{\pm} = \frac{g\sqrt{A^2 + B^2}}{A + \Omega_{\pm}^{-2}[(\omega_0^2 - \omega^2)A + (\frac{\omega_0\omega}{Q})B]} \quad (4)$$

with $A = 1 + (\omega_0^2 - \omega^2)/\Omega_{\pm}^2$ and $B = \omega_0\omega/(Q\Omega_{\pm}^2)$ in which $\Omega_{\pm}$ are two characteristic angular frequencies defined by $\Omega_{\pm}^{-2} = \omega^2 M \cos \alpha (\cos \alpha \pm \sin \alpha / c_f) / [(\omega_0^2 - \omega^2)^2 + (\omega_0\omega/Q)^2]$ with $M = m_1/m$ the mass ratio of the resonant mass to the total mass of the structure. The details for the derivation of $a^*_{\pm}$ is found in the Supplemental Material [34]. The positive solution is for $T = c_f N$ (positive velocity) and the negative solution is for $T = -c_f N$ (negative velocity). In Fig. 1(b), the blue solid line is the solution a^*_{+} and the red solid line is the solution a^*_{-} without free parameters (the resonant frequency and the quality factor were independently measured by small amplitude linear response and the friction coefficient was measured by a sliding angle method).

For a driving acceleration larger than that of the two threshold accelerations a^*_{+} or a^*_{-} , the disk has a net displacement during a cycle that we numerically compute by a double time integration of the horizontal net forces using friction laws. The overall sliding distance during an oscillation cycle defines the mean velocity via the period of oscillation $2\pi/\omega$. Figure 2 represents the velocity obtained from this numerical method as a function of the frequency and acceleration of the shaker with a resonant frequency $\omega_0/2\pi = 14.44$ Hz, $Q = 17.5$, $g = 9.8 \text{ m/s}^2$, $\alpha = 64^\circ$, $M = 0.08$, and $c_f = 0.25$, which corresponds to the experimental values for the structure used in Fig. 1.

Figure 2 also represents the time evolution of the normal force N and tangential force T/c_f , with c_f the static friction coefficient, in the static region $\mathcal{S}$ ($f = 13$ Hz, $a = 5 \text{ m/s}^2$), the positive region $\mathcal{P}$ ($f = 13$ Hz and $a = 8.5 \text{ m/s}^2$), the negative region $\mathcal{N}$ ($f = 16.5$ Hz and $a = 8.5 \text{ m/s}^2$) and the transition region $\mathcal{PN}$ ($f = 14.6$ Hz and $a = 7 \text{ m/s}^2$). The time evolution of N and T is given in Eqs. (2) and (3). In the $\mathcal{S}$ region, T/c_f remains smaller than N , which means that the structure is in stiction (no horizontal motion). In the positive region $\mathcal{P}$, T/c_f is larger than N during a portion of the cycle, which means that the sliding condition (1) is

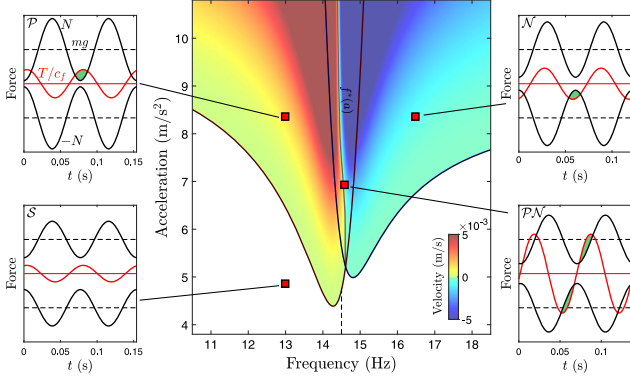


FIG. 2. Model for the velocity as a function of frequency and acceleration with four regimes (static S , positive P , negative N , and positive-negative PN). The two solid lines separating the four regimes are the solutions $a_{\pm}^*$ in Eq. (4). For each of the four regimes, we represent the tangential (T) and normal (N) forces as a function of time [Eqs. (2) and (3)]. The tangential forces are represented with the factor $1/c_f$ with c_f the static friction coefficient so that the intersection between T/c_f and N corresponds to the sliding threshold condition in Eq. (1). In region P and N , the sliding threshold is reached for positive and negative T , respectively. In the PN region, the sliding threshold is reached for positive and negative values during a cycle and the frontier $f^*(a)$ between positive and negative net motion is nonvertical.

reached for positive T and the structure slips in the positive direction during each cycle. In the negative region N , the phase shift between T and N is such that the sliding threshold is reached for $-T/c_f > N$.

In the region PN , the sliding threshold is reached for positive and negative values and we identify a nonlinear frontier $f^*(a)$ for the velocity cancellation. On this frontier, the structure slips positively and negatively during the cycle but the overall horizontal drift is zero. This frontier results from a nonlinear process: for a frequency such as $f = 14.5$ Hz, the horizontal velocity is positive if the acceleration amplitude is $a = 6 \text{ m/s}^2$ or negative if the acceleration amplitude is $a = 9 \text{ m/s}^2$. This is a surprising result because the mechanical resonance is in the linear regime, which means that the phase shift between N and T is independent of the amplitude a . To explain the net velocity sign change close to the resonance, we need to consider the threshold response of the stick-slip transition and how it affects the sliding dynamics of the structure. Close to the resonance, a negative slipping event is immediately followed by a positive slipping event. However, the cumulative sliding distance during one cycle is not a simple combination of the two isolated slipping phases because the initial state of the structure is not the same between the two phases: for the first negative slipping event, the structure is initially in stiction, while for the second positive slipping event, the structure has acquired a negative velocity and is initially sliding. We show in more detail in the Supplemental Material how this interaction between the

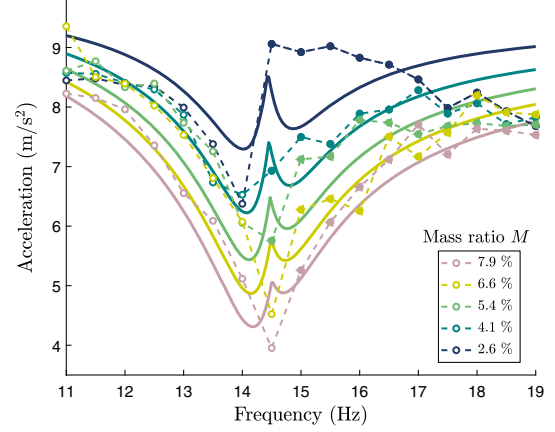


FIG. 3. Threshold accelerations of the structure with on board loads. The structure mass is $m_0 = 8.2$ g, and the added loads are $m_{\text{add}} = 0, 1.7, 3.8, 7.7$, and 16.6 g. The solid lines are the solutions $a^* = \min\{a_{\pm}^*\}$ of the model [Eq. (4)] for the different values of $M = m_1/m$ with m_1 the mass of the first mode and $m = m_0 + m_{\text{add}}$ the total mass.

two forcing phases depends on the driving amplitude and can modify the sign of the cumulative horizontal distance close to the resonance, even if the driving frequency is fixed.

The control of the horizontal velocity sign is closely related to the fact that the net forces N and T in Eqs. (2) and (3) oscillate with a different phase. The origin of this phase shift comes from the structure that can be decomposed in a nonresonant mass $m - m_1$ that moves in phase with the oscillating table, and a resonant mass m_1 that oscillates with a phase shift because of the harmonic response. In Fig. 3, we measure the influence of the mass ratio $M = m_1/m$ on the motion threshold. To vary M , we add items on the structure base that result in an increase of its nonresonant mass without modifying the resonant mass. We find that the threshold for motion is lowered with increasing mass ratio. For a mass ratio of 6.6%, the acceleration is typically half the gravity acceleration at the resonance with a quality factor independently measured at $Q = 17.5$. This is consistent with the relation $Qm_1 \sim m$ that compares the amplitude of the resonant mass oscillation to the amplitude of the nonresonant mass oscillation.

Another aspect of the velocity sign control is the behavior in the limit $M \sim 1$. This limit is when mostly all of the structure mass is resonant. This situation is, for example, the case for bristlebots forced by a vibrated table below the bouncing regime. In this case, the velocity inversion was predicted but experimental results were found inconclusive [35] around the resonant frequency of the structure. Our model shows that, in the limit $M \sim 1$, the region PN is much wider and the frontier $f^*(a)$ is even more complex than in the case presented in Fig. 2 for $M = 0.08$ (see Fig. 7 in Supplemental Material).

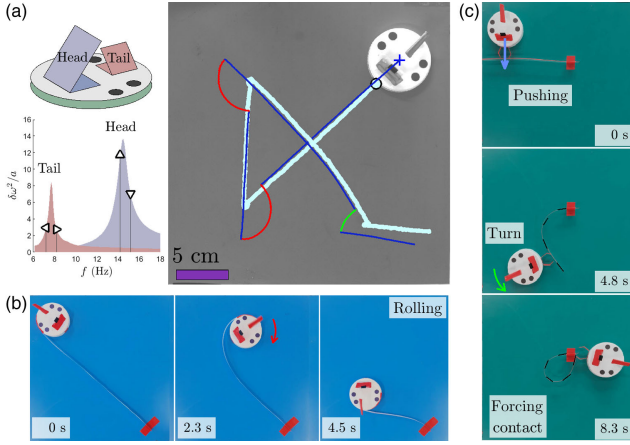


FIG. 4. (a) Schematics of a soft structure with two soft plates (head plate for forward and backward motion, tail plate for left and right turns). The trajectory is obtained by sequential excitation of forward motion and left or right turns (see Movie M1). The driving frequencies for forward (14.2 Hz), backward (15.2 Hz), left (7.2 Hz), and right turns (8.2 Hz) are indicated on the response function plot with triangular symbols pointing respectively up, down, left, and right. (b) Image sequence with the structure attached to a paper strip and rolling itself via a steady right turn (see Movie M5). (c) Image sequence showing the bending of a soft beam to a self-contact (see Movie M6).

In Fig. 4 we demonstrate that it is possible to combine the resonant oscillation of two cantilevers to obtain a fully controllable structure on a vibrated plate. In Fig. 4(a), we represent a structure composed of a rigid disk with two flexible plates (see Supplemental Material for the structure details). The head plate oscillates in the direction of the head-tail direction and produces a forward or a backward motion depending on whether the driving frequency is slightly before or after its resonant frequency (14.44 Hz). The second plate is placed at the tail and it oscillates perpendicularly to the head-tail direction. This plate produces a lateral drift force that rotates the structure either clockwise or counterclockwise depending on the excitation frequency relatively to its resonant frequency (8.5 Hz). The trajectory in Fig. 4(a) is obtained by vibration of the table following a frequency sequence that successively excites a combination of forward motion (14.2 Hz), right turn (7.2 Hz), and left turn (8.2 Hz) (see Movie M1). During the forward motion steps, the structure performs a random walk with a persistence length much larger than its size. By analyzing the velocity orientation during the forward motion steps, we find that the angular drift is typically of 0.1 rad for a traveled distance equal to the disk diameter (see Supplemental Material). In Fig. 4(b), we show an image sequence of the disk attached to a soft strip (paper strip with the other end perpendicularly fixed to the supporting table at the bottom right corner of the image). By a steady right turn, the structure rolls itself in the strip and ends up in a state in which 3/4 of the disk is covered by

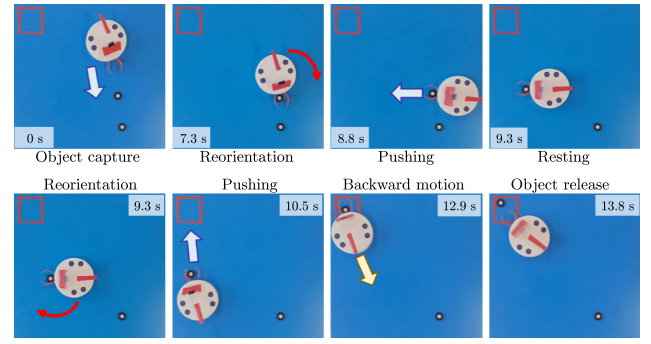


FIG. 5. Capture and delivery of two small cylinders (diameter 1.2 cm) into a target area. Movie M7 shows the full sequence.

the soft strip. The last image of this sequence is the final position reached with the net force from the tail plate balanced by the mechanical stiffness of the soft strip (see Movie M5). In Fig. 4(c), we show the bending a soft beam by a combination of forward motions and left turns in order to maintain a perpendicular bending force on the deforming beam. At the end of the sequence, the beam forms a closed loop. In the complete movie sequence (see Movie M6), the beam is then released to its initial configuration.

In the sequence of Fig. 5, we demonstrate how the structure can move small objects from some location on the supporting plate toward a target area identified by a red square (see Movie M7 for the complete sequence).

In this Letter, we identified a locomotion mechanism that uses simple and universal physical ingredients: solid friction and mechanical resonance. The coupling between the resonant structure and the stick-slip dynamics of a solid body subjected to vibrations gives rise to rectified motion in both the forward and backward directions. The use of multiple flaps further allows one to give rise to new degrees of freedom leading to moving and rotating structures that can be controllably steered. This easily applicable mechanism offers new possibilities for the investigation of active matter beyond directional motion or for the development of soft and minimal robotic structures [36–52].

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Pick up and release of micro-objects: a motion-free method to change the conformity of a capillary contact†

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We propose a new 3D-printed capillary gripper equipped with a textured surface for motion-free release. The gripper classically picks up micro-objects thanks to the capillary forces induced by a liquid bridge. Micro-objects are released by decreasing the volume of this bridge through evaporation. The latter can be either natural or speeded up by a heating source (an IR laser or the Joule effect). The volume reduction changes the conformity of the contact between the gripper and the object. We analyze the gripper performance and the capillary force generated, and then we rationalize the release mechanism by defining the concept of contact conformity in the context of capillary forces.

1 Introduction

Many strategies to handle objects are not suitable at the sub-millimeter scale. Vacuum grippers and friction grippers¹ are still predominant in pick-and-place industries. The constant miniaturization of objects makes the handling process more and more difficult, because of the associated size reduction of grippers and the difficulty to manage the positioning and handling of micro-objects without damaging them. These industries also have to comply with increasingly stringent standards on the reliability of the micro-objects they assemble.

Alternative handling mechanisms have been developed for small, fragile and soft objects. Several different physical strategies have been used to create the attractive or restoring forces necessary to handle micro-objects. Some of them allow one to avoid impacts or to fit to the shape of the handled object, such as smart actuated microgel grippers,² optical tweezers,³ and grippers based on a liquid–solid transition (catch-and-block strategy), the Bernoulli airflow principle or acoustic levitation.¹

Capillary gripping is another promising handling mechanism that consists in taking benefits from capillary forces, dominant over gravity at the sub-millimeter scale.⁴ These capillary forces are generated by a liquid meniscus that binds the object to the gripper. Their application in micro-assembly has been studied in ref. 4 and 5. Capillary forces result from a tension applied on the

gas–liquid–solid triple line interface and the Laplace pressure jump across the interface between the gas and liquid. This differential pressure is proportional to the curvature of the liquid meniscus. The current idea of using capillary forces as a gripping principle was first developed in 1998⁶ and 2002.⁷ Lambert *et al.*⁸ have demonstrated the feasibility of handling sub-millimeter balls of watch bearings. The wettability of the micro-object is preferably high to enhance the gripping ability. However Lambert⁸ showed that with the proper geometry of the gripper, even slightly hydrophobic components could be picked up. Higher surface tension of the handling liquid (*e.g.* water) leads to larger capillary forces. Other solvents like isopropanol, silicone oils or other organic liquids exhibit lower surface tension but better wettability. Volatile solvents may improve the gripping dynamics and also avoid residual traces of liquid after release.

Since those first studies, many solutions to the control of capillary forces for gripping applications have been developed. Force control represents a major challenge for capillary handling since it establishes the ability to pick up, to maintain and especially to release the handled objects. Both physical and geometrical parameters may be tuned to control capillary forces: electro-wetting^{9–11} or thermal stimulation¹² modify the wettability on the substrate, while liquid feeding^{13,14} or active change of the gripper geometry¹⁵ only modify the capillary bridge geometry.

Different release mechanisms have been considered in previous studies, see Table 1. The release mechanisms may be sorted into two main categories. The first involves applying an additional force to the handled part in order to break the liquid bridge. For example, meniscus shearing is studied in ref. 8, while ref. 16 explores release by means of the rotation of the handled micro-part induced by the compression under uneven micrometer-sized pillars. The additional

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Table 1 Censur of release mechanisms of capillary grippers

Ref.	Release mechanism	Release duration depending on	Minimal released weight	Size and weight of tested objects	Component possibly damaged by
8	Meniscus shearing	Mechanical limits of actuators*	—	$S: D = 0.5 \text{ mm}, W = 3.8 \times 10^{-3} \text{ mN}$	Mechanical loads*
16	Rotation of handled object	Mechanical limits of actuators*	—	$C: 1 \times 0.5 \times 0.5 \text{ mm}^3$	Mechanical loads*
17	Glue on substrate	Fluid motion*	—	—	Chemical contamination* Mechanical loads*
17	Inertia – vertical deceleration	Mechanical limits of actuators*	Depending on applied accelerations*	—	—
18 and 19	Inertia – vibrations	Mechanical limits of actuators*	Depending on amplitude and frequency*	$S: D = 0.04\text{--}0.20 \text{ mm}, W = 0.35 \times 10^{-6}\text{--}43.4 \times 10^{-6} \text{ mN},$ $C: 0.5 \times 0.5 \times 0.28 \text{ mm}^3, W = 1.63 \times 10^{-3} \text{ mN},$ $C: 1 \times 1 \times 0.52 \text{ mm}^3, W = 12.1 \times 10^{-3} \text{ mN}$	Mechanical loads*
6, 9, 10 and 20	Injection of gas Electrowetting	Fluid motion* Fluid motion*	— 77 mN	$C: 4.2 \times 4.2 \times 0.5 \text{ mm}^3, W = 0.2 \text{ mN}$ $W = 77 \times 10^{-3}\text{--}136 \times 10^{-3} \text{ mN}$	None* Electrical current*
15	Stretching of the gripper	Fluid motion*	—	$S: D = 0.5\text{--}2.85 \text{ mm}, W = 3\text{--}373 \text{ mN},$ Microscrews: $0.5\text{--}2.5 \text{ mm}, W = 28\text{--}310 \text{ mN}$	None*
22 and 23	Change in conformity	Fluid motion*	0.47 mN	$W = 0.295\text{--}2.18 \text{ mN}^*$	None*
12	Evaporation and capillary forces on substrate	Fluid motion*	No limit*	$S: D = 0.005\text{--}0.06 \text{ mm}$	None*

S: Sphere (diameter is given), C: cuboid (lengths are given), W: weight of object(s), *: own estimations.

force can also be developed by inertia, *e.g.* in ref. 17 where a large vertical deceleration is applied to the gripper, or in ref. 18 and 19 where vibrations are used. The rupture of the meniscus has also been achieved by generating a higher attraction force on the substrate on which the release is expected, *e.g.* through the addition of glue.¹⁷

The second category of solutions consists in releasing brittle micro-objects without applying additional loads on them. These solutions are based on the decrease of capillary forces. Bark⁶ suggests the injection of gas in order to change the internal pressure in the meniscus. Decreasing capillary forces can also be achieved by modifying the wettability: capillary forces are decreased thanks to electrowetting in ref. 9, 10 and 20. An elastic gripper is stretched in order to modify its wettability in ref. 15. Based on Israelachvili's model^{21–23} have investigated the change of curvature of their grippers, also known as conformity (see Section 4.1), in order to decrease capillary forces and therefore to release objects. Heating can also lead to the decrease of capillary forces by evaporation of the meniscus. This solution is studied in ref. 12 where this evaporation is coupled to the generation of capillary forces on the substrate.

A reliable, accurate, harmless and fast release mechanism is indeed necessary for industrial applications. An Ipulse M20 pick-and-place machine from Yamaha would for instance place one chip every 0.15 s with 4 picking heads. But contrary to inexpensive micro-objects which are mass-produced and which can be processed in fast assembly, the micro-manipulation of brittle objects must give priority to the integrity of the components.

In line with the second category of release mechanisms, this paper explores a motion-free release mechanism for capillary grippers based on a change of the conformity in the contact area. This change is allowed by surface texturing and bridge

volume reduction through evaporation. The surface of the gripper is textured with small cones called barbules. They avoid the persistence of strong capillary forces when the liquid bridge volume is reduced. Indeed the persistence of capillary forces at very low liquid volume associated with dry adhesion (van der Waals forces) may prevent release for non-textured grippers. The release ability of the gripper is thus enhanced by this surface texture.

This article first presents the set-up and methods to study this new release strategy. The experimental results are then compared to models. In particular, the concept of contact conformity is defined in the context of capillary gripping. Finally, conclusions and perspectives are discussed in Section 5.

2 Materials and methods

2.1 Gripper

The gripper used for all the experiments was a cone covered with spine-like microstructures called barbules (Fig. 1b). The gripper was 3D-printed with the two-photon-initiated polymerization-based stereolithography system Nanoscribe, with a 200 nm resolution. The gripper material was IP-L 780 UV-curable photoresin. The main cone had a height h of 700 μm and an aperture angle α of 140°. The barbules were also cones, of half aperture 12.5°, with a tilt angle respective to the cone surface of 50°. The distance from the surface of the cone to the head of the tip was 30 μm and the distance between two barbules was 50 μm . The barbule density was 50 barbules per mm^2 .

The barbules were designed following three principles: (i) the dimension of the surface texture is less than ten times smaller than the size of the gripper/micro-object, (ii) the barbules have a simple shape with a large aspect ratio and (iii) the barbule size is limited by the resolution of the 3D printer (400 nm).

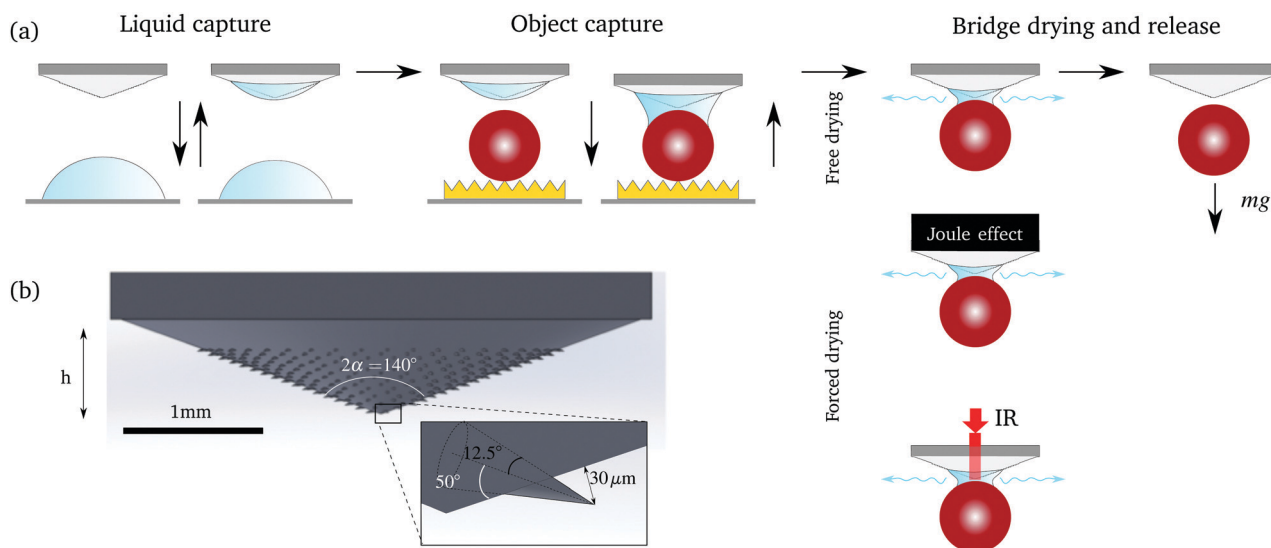


Fig. 1 (a) Schematics of the experimental procedure. First, the gripper was loaded by plunging it into a volatile liquid, here water. The gripper was then moved to create a capillary bridge between the gripper and the micro-object (here a sphere for clarity but idealized objects with a spherical cap and a cylindrical body were also considered in the experiment). Special care was taken to pick-up the micro-object from a textured surface to avoid any unwanted sticking on the surface. Finally, the object was released by drying the capillary bridge. The drying was either unforced or speeded up by heating the liquid by the Joule effect or with an IR-laser. (b) CAD geometry of the conical gripper. Close-up on a barbulement.

According to these criteria, we could print well-shaped conical barbules with dimensions of 30 μm , roughly 100 times the resolution of the 3D printer.

2.2 Experimental set-up

The experimental set-up is described in Fig. 1a. The gripper was mounted on a 3 degrees-of-freedom translation stage. High-precision mechanical stages were used for accurate positioning of the gripper in the (x,y) plane. The vertical motion of the gripper was controlled with a linear actuator (Z825B from Thorlabs) with a resolution of 0.5 μm .

A droplet of approximately 10 μL of purified water was deposited on a metallic substrate. To ensure reproducibility, the substrate was cleaned with ethanol and a new droplet was deposited before each experiment. The gripper was first vertically moved (z -axis) toward the droplet to form a liquid bridge. It was then moved upwards, which stretched and broke the liquid bridge. A liquid volume of $1.9 \pm 0.4 \mu\text{L}$ was left on the gripper. Reduced volumes of liquid ([0.7, 0.4, 0.1] μL) were obtained by drying the initial liquid bridge with an IR laser (1455 nm) at 491 mW (for [15, 20, 30] s respectively).

Idealized millimeter-sized objects, each consisting of a spherical cap on top of a vertical cylinder of radius 1 mm, were 3D-printed with a resin of density $\rho = 1.17 \times 10^{17} \text{ kg m}^{-3}$ (VeroWhitePlusRGD 835). Three cap radii were tested ($R_1 = 625 \mu\text{m}$, $R_2 = 950 \mu\text{m}$, and $R_3 = 1250 \mu\text{m}$) and the mass of the objects was varied from 1 mg to 30 mg by changing the length of the cylinder. The objects were initially placed on a micro-structured planar surface. The microstructures were again conical tips, whose purpose was to minimize solid–solid contact forces.

The gripper was moved downwards until a capillary bridge was formed with the object. The object then followed the gripper:

it could be lifted and moved horizontally at the same time as the gripper (see the video in the ESI†).

The water bridge volume was decreased, through either natural or forced evaporation. Two techniques were considered to force evaporation. The first was based on the Joule effect: an electrical current flowed in a ceramic heater plate and produced heat. The heat was transferred by conduction from the plate to the gripper and then to the water. The second technique involved an IR laser with a wavelength of 1455 nm. The gripper made of IP-L 780 UV-curable photoresin was indeed transparent to this wavelength. The IR light was efficiently absorbed by water with an absorption coefficient of $2.6 \times 10^3 \text{ m}^{-1}$. The absorbed energy was turned into heat, which induced water evaporation. The maximal power of our IR-laser was 0.5 W.

The experiment was imaged from the side with a camera (Canon EOS 5D Mark III) and a macro lens (Canon-MP-E 65 mm f/2.8 1-5x Macro). The resolution was 3.8 μm per pixel.

2.3 Force measurement set-up

The quasi-static capillary force was measured by the deflection of a calibrated steel cantilever. As depicted in Fig. 2a, the experimental set up comprised a clamped cantilever on which a glass sphere of 2 mm diameter was glued at a distance $L = 12.6 \text{ cm}$ from the clamping position. The width w and the thickness t of the cantilever were 1.27 cm and 100 μm respectively. The position of the cantilever x_p was initially zero. The deflection was measured with a non-contact displacement sensor (Keyence LC2440) with a resolution of about 0.2 μm . The gripper was mounted on a 3-axis motion system to easily set the gripper opposite to the sphere. The bridge axis was placed in the horizontal plane. The size of the bridge must also be significantly smaller than the capillary length in order to

preserve the axisymmetry of the bridge. The whole procedure was imaged from the side with the same equipment as for the gripping experiment.

The experimental procedure is drawn in Fig. 2b and c. A water droplet was deposited on the sphere ①. The gripper was moved towards the sphere until the liquid bridge formed ②, and consequently the cantilever was deflected. The gripper was pushed forward in the same direction until the zero position of the cantilever $x_p = 0$ was reached ③. Here the sphere was in contact with the gripper. The attractive capillary forces generated by the liquid bridge were balanced by the repulsive normal contact force exerted by the sphere on the gripper. The gripper was displaced in the x -direction with steps of 20 μm from the zero position. The cantilever consequently deflected until the gripper stopped touching the sphere ④. The recordings started during stage ③ but data were processed from ④ only, Fig. 2d.

We defined ξ as the gap distance between the tip of the gripper and the sphere. It was calculated as the difference between the motor position and the cantilever deflection. As the liquid bridge evaporated, its volume decreased, which led to reduced cantilever deflection ⑤. Rupture occurred abruptly when the configuration switched from ⑥ to ⑦.

The capillary force F was directly obtained through the deflection of the cantilever x_p when the gripper was not in contact with the sphere,

$$F = kx_p \quad \text{where} \quad k = \frac{3EI}{L^3} \quad (1)$$

was the stiffness of the cantilever, E the Young's modulus of the cantilever (steel) and $I = wt^3/12$ the second moment of area of the cross section. The spring constant has been directly measured using the classic pendant mass method. We obtained $k = 0.40 \pm 0.03 \text{ N m}^{-1}$.

3 Results

3.1 Gripping performance

The ability of this gripper to pick up and release millimeter-sized objects is first characterized. The relevant dimensional parameters are (i) the volume of water V used to pick up the object, (ii) the surface tension of water σ , (iii) the mass m of the object, (iv) the radius of curvature R of the spherical cap on top of the object, and (v) gravity g . An educated guess²⁴ would reveal that the density ρ of the liquid is not relevant, since the Laplace pressure induced by surface tension is much higher than the

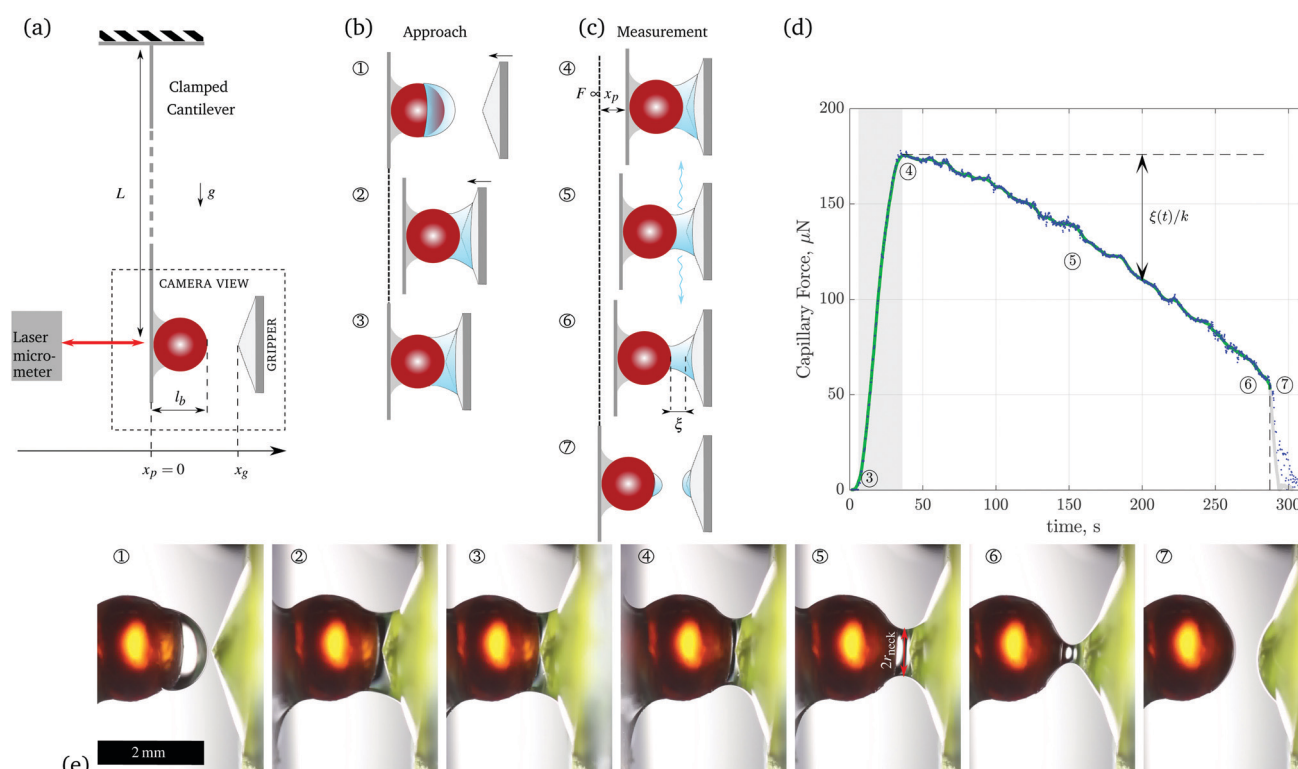


Fig. 2 Force measurement set-up and associated procedure. (a) Geometrical arrangement of the precision force measurement set-up. (b) Preparation of the set-up. ① A droplet was deposited on the bead. ② The gripper was moved forward in the x -direction to form the capillary bridge. As the capillary bridge was created, the cantilever was deflected by the capillary forces. ③ The gripper was again moved forward until the cantilever was set back to its undeflected position. (c) Recording and drying of the capillary bridge. ④ The experiment started by displacing the gripper backward. This motion deflected the cantilever by a distance x_p . ⑤ to ⑦ The capillary bridge dried and both the force F and the gap distance ξ were recorded until it broke-up. (d) Example of force and gap recording. The grey region corresponds to the end of the preparation to the measurement. The measurement was made in the blue region before the bridge broke-up between ⑥ and ⑦. (e) Images of the experiment illustrating configurations ① to ⑦.

hydrostatic pressure inside this sub-millimeter liquid bridge. From these relevant parameters, we can build two dimensionless numbers, namely the Bond number and the length ratio:

$$B_0 = \frac{mg}{\sigma V^{1/3}} \quad \text{and} \quad r = \frac{V^{1/3}}{R}. \quad (2)$$

The capability of the gripper as a function of (B_0, r) is summarized in the diagram of Fig. 3. Three zones are identified: (i) the gripper can pick up and release the object, (ii) the gripper can pick up the object but not release it, and (iii) the gripper cannot pick up the object. Variations of liquid volume for a given object (fixed mass m and curvature radius R) are represented as black lines in Fig. 3. The object is initially bound to the gripper with a large volume (top left). If the starting point is in the white region the solid line describing variations of volume only will remain in this region, meaning that the object will be picked-up but not released. In contrast, if the picking starts in the dark grey region, the dot-dashed line describing the volume variation remains in this region, meaning that it is impossible to pick-up the object. The dashed line starts in the light grey region, meaning that it is possible to pick-up and release the object. As the volume is decreased (towards the bottom right), the line enters the dark grey region and the object is released. The dotted line transits between the white and light grey regions, meaning that depending on the starting volume it is possible or impossible to release the object. This inconsistent behaviour is likely due to contact angle

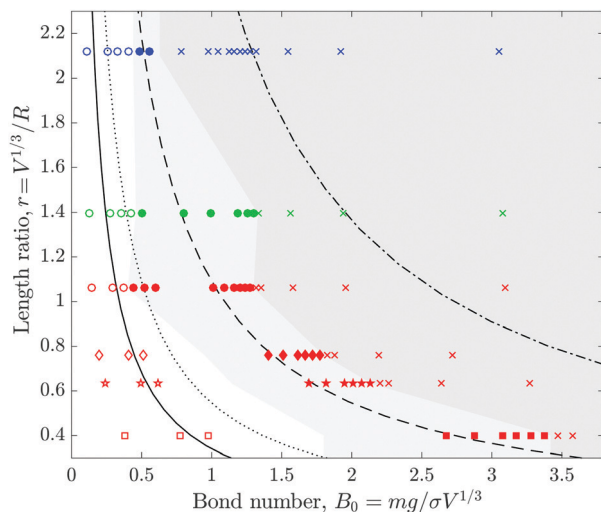


Fig. 3 Diagram of the gripper capability in terms of Bond number B_0 and length ratio r . Three radii of curvature of the object were considered, $R = 625 \mu\text{m}$ (blue), $R = 950 \mu\text{m}$ (green), and $R = 1250 \mu\text{m}$ (red). The symbols correspond to the liquid volume, $1.9 \mu\text{L}$ ($\circ$), $0.7 \mu\text{L}$ ($\diamond$), $0.4 \mu\text{L}$ ($\star$) and $0.1 \mu\text{L}$ ($\square$). The empty symbols and white background indicate the ability of the gripper to pick up the object but the impossibility to release it. The full symbols and light gray background indicate that the gripper can both pick up and release the object. Symbols $\times$ and the dark gray background indicate that the gripper could not pick up the object. The solid, dotted, dashed and dot-dashed lines represent variations of volume, every other parameter being kept constant, with $\sigma = 72 \text{ mN}$, $g = 9.81 \text{ m s}^{-2}$. Solid: $m = 20 \text{ mg}$, $R = 2 \text{ mm}$; dot-dashed: $m = 20 \text{ mg}$, $R = 2 \text{ mm}$; dotted: $m = 8 \text{ mg}$, $R = 2 \text{ mm}$; dashed: $m = 8 \text{ mg}$, $R = 1 \text{ mm}$.

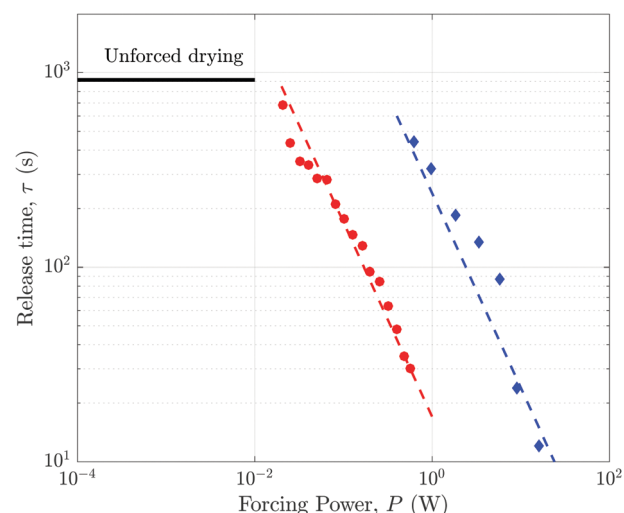


Fig. 4 Release time τ as a function of the input power P of the heating system (initial bridge volume $1.9 \mu\text{L}$). (red) IR laser. (blue) Joule effect. The dotted lines correspond to eqn (6) and (9), which confirm the scaling law $\tau \propto 1/P$. The unforced break-up time is reported as an asymptote (black line) in the log-log representation.

hysteresis and triple line pinning, which were not considered in this two-dimensional diagram.

The release performance of the evaporative gripper has also been quantified. The relation between the release time and the power used to warm up the liquid is presented in Fig. 4. For both the Joule effect and the laser heating, the release time τ is observed to be inversely proportional to the applied power P .

3.2 Gripping force

The gripping force F has been measured for the smooth cone and the textured cone as a function of temperature T , volume V , and gap distance ξ . The raw measurement of F as a function of V for the smooth cone is displayed in Fig. 5. The force increases with the volume of the liquid bridge.

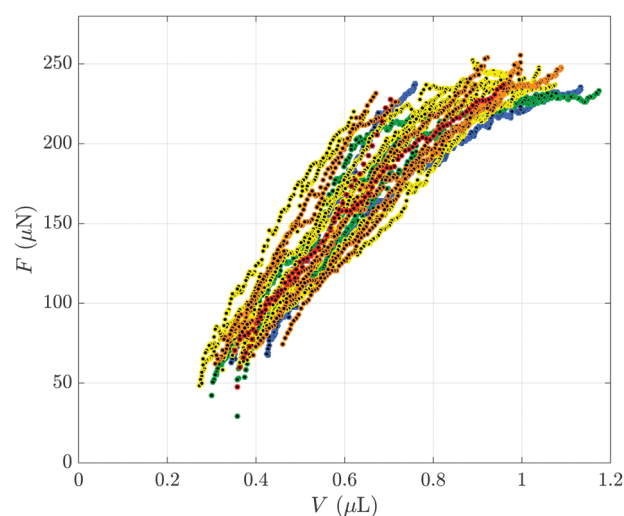


Fig. 5 Force F generated by the smooth gripper as a function of the bridge volume V . The colours indicate the temperature (blue: $T = 23 \text{ }^\circ\text{C}$; green: $T = 32 \text{ }^\circ\text{C}$; yellow: $T = 42 \text{ }^\circ\text{C}$; orange: $T = 47 \text{ }^\circ\text{C}$; and red: $T = 66 \text{ }^\circ\text{C}$).

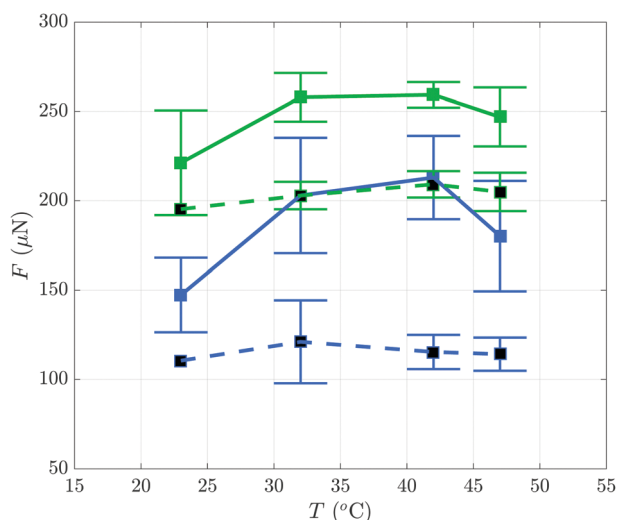


Fig. 6 Effect of temperature T and surface texture on the gripping force F . Coloured symbols and full line: barbule texture; black symbols and dashed line: smooth cone; blue (resp. green) corresponds to a volume of $V = 0.4$ μL (resp. 0.8 μL). Lines are a guide for the eyes where only temperature varies. Error bars correspond to one standard deviation.

The effects of temperature, volume and surface texture are reported in Fig. 6. We note that for a given volume and temperature the surface texture (in blue for the smooth cone and green for the textured cone in Fig. 6) increases the adhesion force generated by the gripper. We analysed these experimental data with ANOVA tests to tell if the effects of volume, temperature and surface texture variations on the force generated by the gripper are statistically relevant or not. Statistical tests were performed with quadratic models in volume and temperature,

$$F = a_0 + a_1V + a_2T + a_{11}V^2 + a_{12}VT + a_{22}T^2, \quad (3)$$

for results obtained with both smooth and textured grippers. The results are gathered in Table 2. As expected the volume has a strong effect on the generated force for both grippers, confirming the results in Fig. 5. However the surface texture has a non-trivial effect on the dependence of the force on the heating temperature. Temperature has no statistically relevant influence on the force in the case of the smooth cone (p -value > 0.1) but it has one for the textured cone. In this case the force is maximal between 30 and 40 °C.

4 Models

4.1 Model of the release time

The experimental results of Fig. 4 on the evaporation time of the bridge can be rationalized with theoretical arguments. In the absence of convection, natural evaporation is only driven by the diffusion of water vapour, as initially modelled by Langmuir.²⁵ As a crude approximation, the liquid bridge is here replaced by a hemispherical droplet of the same volume. If the fine geometry of the bridge is taken into account it will only change the prefactor but not the scaling as long as the diffusion occurs across sections whose area increases as the square of the distance to the droplet.

Table 2 ANOVA analysis dependence of force F on volume V and temperature T with a quadratic model, eqn (3). (a) Smooth gripper, and (b) textured gripper. Significant p -values ($p < 0.05$) are marked in bold face. SS: Sum of Squares, DoF: Degrees of Freedom, MS: Mean Square, F: F-test

(a)	SS ($\times 10^{-2}$)	DoF	MS ($\times 10^{-2}$)	F	p -value
Model, eqn (3)	1.02	5	0.20	6.89	<0.001
$V(a_1)$	0.52	1	0.52	17.39	<0.001
$T(a_2)$	<0.01	1	<0.01	0.007	0.931
$VT(a_{12})$	<0.01	1	<0.01	0.091	0.763
$V^2(a_{11})$	0.02	1	0.02	0.681	0.413
$T^2(a_{22})$	0.07	1	0.07	2.30	0.136
Residual	1.37	46	0.03		
Lack of fit	0.03	6	<0.01	0.147	0.989
Pure error	1.34	40	0.03		

(b)	SS ($\times 10^{-2}$)	DoF	MS ($\times 10^{-2}$)	F	p -value
Model, eqn (3)	7.27	5	1.45	13.7	<0.001
$V(a_1)$	0.75	1	0.75	7.09	0.009
$T(a_2)$	0.73	1	0.73	6.89	0.010
$VT(a_{12})$	0.24	1	0.24	2.28	0.133
$V^2(a_{11})$	0.05	1	0.05	0.446	0.505
$T^2(a_{22})$	4.38	1	4.38	41.3	<0.001
Residual	13.35	126	0.11		
Lack of fit	0.28	6	0.05	0.426	0.8604
Pure error	13.07	120	0.11		

The diameter δ of this hemisphere is then $\delta(t) = (12V(t)/\pi)^{1/3}$. The relative humidity ϕ in the air surrounding the bridge is by definition the ratio of the partial vapour pressure P_v and the vapour pressure at saturation P_v^0 . It satisfies the Laplace equation for pseudo-steady-state molecular diffusion in spherical coordinates, namely $\nabla^2\phi = 0$. If we assume that the gradients of ϕ are only radial, the vapour concentration profile is $\phi(r) = \phi_\infty + (1 - \phi_\infty)\delta(t)/2r$, where r is the radial coordinate and ϕ_∞ the relative humidity of the room during the experiment. The diffusive mass flux J is then calculated as $J = DM_w P_v^0 \nabla\phi/(RT)$, which leads to $J = DM_w P_v^0 (1 - \phi_\infty)\delta/(2r^2 RT)$ where D is the diffusion coefficient of water vapour in air, M_w the molar mass of water, R the ideal gas constant and T room temperature. The constants are gathered in Table 3. A mass balance of water between the liquid and gas phases then yields:

$$\rho \frac{dV}{dt} = -2\pi r^2 J \quad (4)$$

The volume of the liquid bridge evolves from an initial value V_0 as

$$V(t) = V_0 \left(1 - \frac{t}{\tau}\right)^{3/2} \quad \text{with } \tau = \frac{(3V_0/2\pi)^{2/3} \rho RT}{2DM_w P_v^0 (1 - \phi_\infty)} \quad (5)$$

Complete drying occurs at time τ , which varies as $V_0^{2/3}$. This time has been measured experimentally $\tau_{\text{exp}} = 917 \pm 44$ s and is reported in Fig. 4. The model evaluated with the parameter values from Table 3 predicts $\tau = 2150$ s, which is larger than in experiments but still of the same order of magnitude. The discrepancy can be attributed to the replacement of the bridge geometry by a hemisphere, the assumption of purely diffusive evaporation, and the fact that some liquid may remain at break-up.

When heat is brought to the bridge by the laser or Joule effect, the evaporation is not driven by vapour diffusion anymore. For the heating strategy based on the Joule effect, the

Table 3 Parameters and constants used in the models

Variables & coordinates			
V	Bridge volume	h_1	Bridge height
r, z	Coordinates	ξ	Gap
t	Time	F	Capillary force
R_s	Sphere radius	R_c	Cone contact radius
α	Cone opening angle	λ	Contact conformity
θ_1, θ_2	Contact angles	δ	Equivalent diameter
P_v	Water vapour pressure	φ	Relative humidity
Experimental parameters		Value	
φ_∞	Ambient relative humidity	0.55	
V_0	Initial bridge volume	$0.5 \text{ to } 21 \times 10^{-9} \text{ m}^3 (\mu\text{L})$	
T	Room temperature	293.15 K	
P	Forcing power	0.1 to 10 W	
m	Initial mass of liquid	$0.5 \text{ to } 21 \times 10^{-6} \text{ kg}$	
η	Yield of evaporation process	—	
Constants		Value	
σ	Water surface tension	72 mN	
P_v^0	Saturated water vapor pressure	$2.34 \times 10^3 \text{ Pa}$	
D	Diffusion coefficient of water vapor	$2.82 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$	
M_w	Water molar mass	$18 \times 10^{-3} \text{ kg mol}^{-1}$	
R	Ideal gas constant	$8.314 \text{ J mol}^{-1} \text{ K}^{-1}$	
ρ	Water density	$1.0 \times 10^3 \text{ kg m}^{-3}$	
c_p	Water specific heat capacity	$4.21 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$	
h	Water specific latent heat	$2.26 \times 10^6 \text{ J kg}^{-1}$	
κ	Attenuation coefficient at 1455 nm	$2.6 \times 10^3 \text{ m}^{-1}$	

supplied power P yields an increase of thermal energy ηPt after time t , where the factor $\eta < 1$ is the fraction of produced heat that reaches the bridge. This thermal energy is used to first warm up the liquid to 100 °C (heat capacity c_p) and then to evaporate it (latent heat h). As the former requires about 7 times less energy than the latter, it can be neglected in the first approximation. The energy balance then yields:

$$\eta Pt = h\rho[V_0 - V(t)], \quad (6)$$

where $h = 2.26 \times 10^6 \text{ J (kg K)}^{-1}$ is the specific latent heat of water. Therefore, the volume decreases linearly with time:

$$V(t) = V_0 \left(1 - \frac{t}{\tau}\right) \quad \text{where } \tau = \frac{\rho h V_0}{\eta P} \quad (7)$$

is the time of complete evaporation of the bridge. The yield η in eqn (7) is estimated as 1.8% by fitting the experimental data (Fig. 4).

When the bridge is evaporated by laser-induced heating, the amount of absorbed energy depends on the liquid thickness through the Beer–Lambert law. Therefore, the yield now depends on the thickness of the bridge, which scales as $\beta V^{1/3}$, where β is an unknown geometrical coefficient. For thin films, the absorbance is $\alpha \beta V^{1/3}$, where α is the attenuation coefficient of the electromagnetic wave sent by the laser (here 1455 nm) in water. The energy balance then yields

$$(\eta \kappa \beta V^{1/3}) P = -\rho h \frac{dV}{dt}. \quad (8)$$

The temporal evolution of the liquid volume is then,

$$V(t) = V_0 \left(1 - \frac{t}{\tau}\right)^{3/2} \quad \text{where now } \tau = \frac{3\rho h V_0^{2/3}}{2\eta \kappa \beta P}. \quad (9)$$

Again, a comparison of this theoretical prediction to the experimental data in Fig. 4 indicates that the factor $\eta \beta$ is about 11.8% (6 times more efficient than the Joule effect).

Both models of forced evaporation suggest that the release time is inversely proportional to the input power P . However, its dependence on the initial liquid volume V_0 differs: it is proportional in the case of the Joule-induced evaporation, and less than proportional in the case of laser-induced evaporation. We expect that the use of a larger forcing power might keep reducing the releasing time, especially for the laser method, which is not limited by the thermal conduction in the gripper. For instance, we obtained a release time of $\simeq 10$ s with 0.5 W with the IR laser, and we thus expect that using a more powerful laser of 50 W we would achieve the release in $\simeq 0.1$ s, which would be close to industrial standards. The miniaturization of the components would also go with a reduction of the initial liquid bridge, and therefore with a substantial decrease of the evaporation time.

We showed that heating the bridge enhances the evaporation process and decreases the time needed to release the micro-object. However heating the bridge does not change the gripping ability, especially to release micro-objects. The ability to pick up and release an object mainly depends on the geometry of the gripper.

4.2 Gripping force

The capillary force results from both the Laplace pressure and the tension at the contact line. In the absence of other effects, it is given by

$$F = 2\pi\sigma r_{\text{neck}} - \pi\sigma r_{\text{neck}}^2 \left(\mathcal{C} + \frac{1}{r_{\text{neck}}} \right) \quad (10)$$

where r_{neck} is the neck radius, see Fig. 2e, and $\mathcal{C}$ is the curvature at the neck along the x direction. Experimentally, $-\mathcal{C}r_{\text{neck}} \simeq 0.6\text{--}1.2$. In those conditions the capillary force simply reads,

$$F = K\sigma r_{\text{neck}} \quad (11)$$

where $K = 1 - \mathcal{C}r_{\text{neck}} \simeq 1.6\text{--}2.2$. The experimental data are fitted by the above relation with $K = 1.5$, see Fig. 8a, which is close to the range [1.6, 2.2].

We now relate r_{neck} to the controllable parameters of the experiment, namely the volume V and the gap ξ . The exact relation between these parameters may be deduced by solving numerically the Young Laplace differential equation in an axisymmetric frame. We here look for an empirical approximation to this relation. Conveniently, we define the equivalent radius r of a straight cylinder of the same volume placed between the cone and the sphere with a gap ξ with the notations defined in Fig. 7. The equivalent radius is given by

$$\begin{aligned} \frac{V}{\pi} = & r^2 \left(\xi + R_s \left(1 - \sqrt{1 - (r/R_s)^2} \right) + r \cot \alpha \right) - \frac{r^3}{3} \cot \alpha \\ & - R_s^3 \left(1 - \sqrt{1 - (r/R_s)^2} \right)^2 + \frac{R_s^3}{3} \left(1 - \sqrt{1 - (r/R_s)^2} \right)^3. \end{aligned} \quad (12)$$

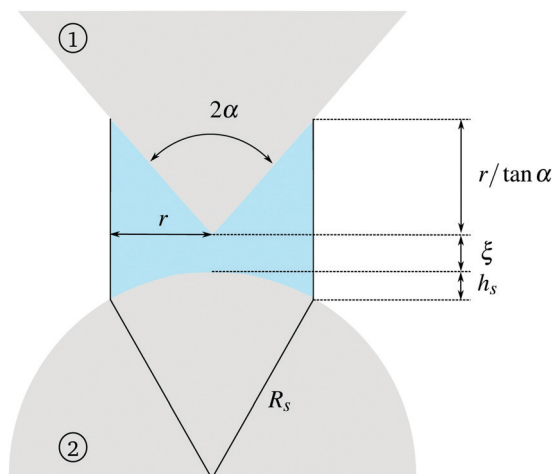


Fig. 7 Notations used to compute the equivalent radius of a volume of liquid between a cone and a sphere. $h_s = R_s(1 - \sqrt{1 - (r/R_s)^2})$.

The experimental data of the equivalent radius r , computed from eqn (12), are plotted in Fig. 8b as a function of the neck radius. The experimental points collapse on a master line which can be approximated by an affine law as

$$\frac{r}{R_s} = 0.15 + \frac{r_{\text{neck}}}{R_s} \quad (13)$$

This empirical relation depends in a non trivial manner on the contact angles and on the pinning of the contact line. Indeed, we expect that small contact angles create a more curved bridge with a smaller neck radius r_{neck} than large contact angles would create. With eqn (11) and (13), we deduce the relation between the force and the equivalent radius of the bridge,

$$F = K\pi\sigma(r - 0.15R_s) \quad (14)$$

5 Discussion

The contact conformity²⁶ refers to the shape of the contact, as defined in solid mechanics. There is conformal contact when two solids fit one another similarly to a ball in a spherical cup of identical diameter. For a capillary gripper, the contact is said to be conformal when it maximizes the capillary force, which is somehow related to the geometrical conformity borrowed from contact mechanics but not only.⁵ The capillary force is maximized when two solids have a large contact area and a large curvature where the Laplace pressure jump occurs. One knows by intuition that the capillary force between a sharp cone and a sphere (small conformity) is lower than the one between a wide open cone and a sphere (large conformity). This may however be shown directly using the capillary force model derived hereinabove. If we take into account the full force, eqn (11), along with the equivalent radius, eqn (13), and its definition related to the gap and volume, eqn (12), we observe that the force increases as the cone widens and then gets reversed (*i.e.* $\alpha > 90^\circ$). An example of the force as a function of the cone aperture angle α is plotted in Fig. 9 using the following set of parameters: $V = 0.5 \mu\text{L}$, $\sigma = 72 \text{ mN}$, $\xi = 0.3 \text{ mm}$ and $R_s = 1 \text{ mm}$.

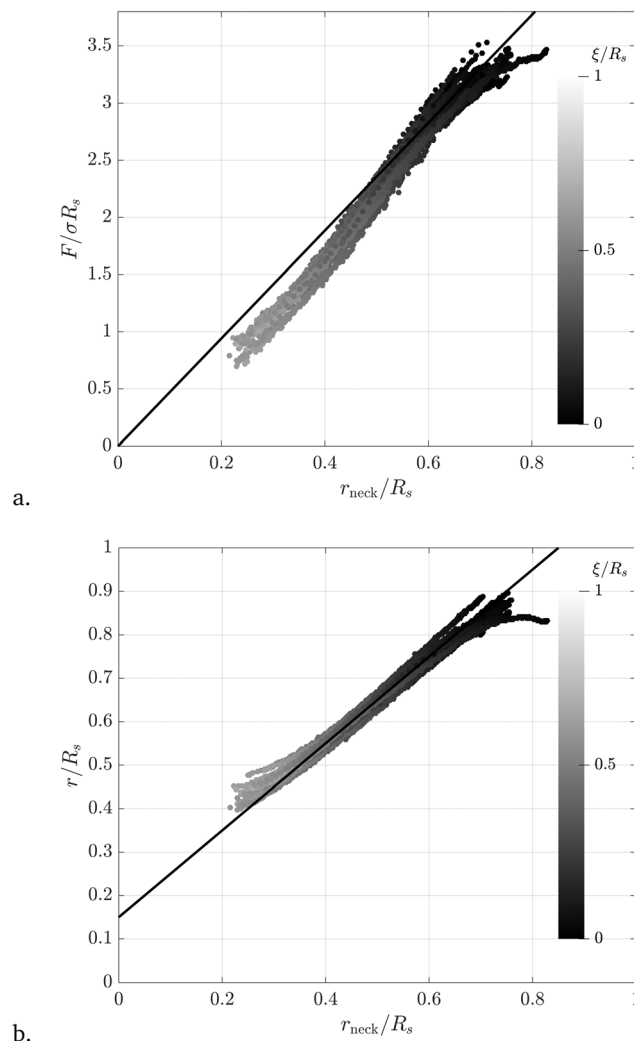


Fig. 8 (a) Normalized force of the gripper $F/\sigma R_s$ as a function of the normalised neck radius r_{neck}/R_s . The solid line corresponds to the fit of the experimental data with $F = K\pi\sigma r_{\text{neck}}$ and $K = 1.5$. (b) Normalized equivalent cylinder radius r/R_s computed from eqn (12). The black line corresponds to the fit with eqn (13). The grey scale indicates the gap value ξ/R_s .

To generalize the discussion to any capillary contact, we consider two solids of radii of curvature R_1 and R_2 respectively at the contact location. We also assume that the liquid makes a contact angle θ_1 on the first solid and a contact angle θ_2 on the second one. The capillary force F between two rounded solids reads,²⁷

$$F = 2\pi\sigma \left| \frac{R_1 R_2}{R_1 + R_2} \right| (\cos \theta_1 + \cos \theta_2) \quad (15)$$

At this step, we define the contact conformity as

$$\lambda = \frac{R_1 R_2}{\min(|R_1|, |R_2|)(R_1 + R_2)} \quad (16)$$

When the contact conformity is perfect, *i.e.* when the two objects fit perfectly, $R_1 = -R_2$, the conformity is infinite. Such a configuration would maximize the attractive capillary forces. Conversely, if the first object has a much smaller radius of

curvature than the second one, $|R_1| \ll |R_2|$ for instance, the conformity is $\lambda \simeq 1$, which leads to a capillary force proportional to the small radius of curvature R_1 .

The concept of contact conformity may now be used to understand how our gripper works. The object has a radius of curvature R_s . To grab it, we use a wet cone for which the typical radius of curvature r^* depends on the liquid volume: during the pick-up phase, the conformity should be maximized, which suggests to use a large liquid volume (*i.e.* $r^* \gg R_s$). The picking conformity is then only driven by the object radius $\lambda \simeq R_s$. When the bridge dries, r^* decreases and so does λ . In some cases this might be sufficient to detach the object. However we noticed during preliminary tests that the object usually sticks on the side of a smooth cone, where the corresponding curvature is finite. On the textured gripper, the barbules finally come into contact with the object as the liquid dries up. If the barbule radius R_b (radius of the cone at the contact position) is smaller than the one of the object $R_b \ll R_s$, the conformity is strongly reduced, which leads to a force of the order of the barbule radius R_b . With the textured gripper, the conformity at picking up ($\lambda \sim R_s$) is much larger than the conformity at releasing ($\lambda \sim R_b$), making detachment possible. Consequently, the object may detach from the gripper as soon as the decrease of conformity leads to a capillary force smaller than the object weight.

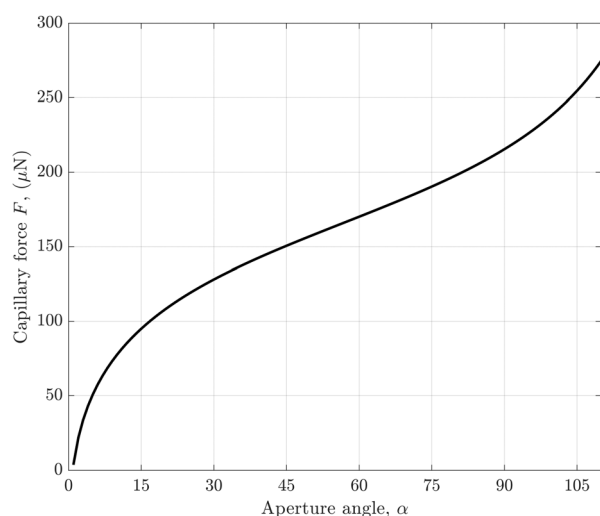


Fig. 9 Force of the gripper as a function of the aperture angle α . $\alpha > 90^\circ$ corresponds to a reversed cone. This figure was plotted from eqn (11) and (12) using $V = 0.5 \mu\text{L}$, $\sigma = 72 \text{ mN}$, $\xi = 0.3 \text{ mm}$ and $R_s = 1 \text{ mm}$.

This capillary gripper releases objects without applying additional loads. We can therefore imagine using this technique on brittle objects in a pick-and-place context. In the case of the Joule effect stimulation, the thermal diffusion that occurs through the gripper would fix a lower limit to the release time even if the power is increased. The release time would be rather long compared to industry standards. However, we expect the release time to be strongly reduced, down to a fraction of a second, if the IR-laser power is increased.

Even if millimeter-sized objects were considered as a first step, the precision of the stereolithography printing technique, *i.e.* 200 nm, allows one to consider smaller objects. The barbules that we added to the gripper were also a first attempt and their shape can certainly be optimized. Any other shape giving a small conformity to the contact (*i.e.* having a large radius of curvature, such as spines or hairs) may be efficient. Some spine-like textured surfaces, inspired by cicada wings, have already been shown to be anti-mist and anti-biofouling²⁸ and might be of inspiration for the design of future grippers. Further optimization of the conformity at both the gripper and the texture scale would be interesting. For instance, a gripper with a radius of curvature matching that of the picked object and textures with a very small radius of curvature compared to the object size might make a very efficient gripper.

An example of a conformal gripper producing a large initial capillary force is displayed in Fig. 10a. This gripper fits the picked object when the liquid volume is large but as the liquid dries the volume is reduced to obtain several non conformal contacts, Fig. 10b and c. At this step the capillary force is strongly reduced and the picked object should be released. However, in some cases, the object may remain attached to the gripper even after drying. This unwanted sticking would be due to some small capillary bridges of the order of the Kelvin radius (about 5 nm at 20 °C with 90% relative humidity) that would remain between the gripper and the object, Fig. 10d. Those bridges have already been shown to create significant capillary forces.²⁹ Moreover they arise at a scale where van der Waals forces also play a role in the adhesion between the object and the gripper.

The wettability of the gripper has not been varied in this study but is known to play an important role in the gripping of micro-objects. The IR-heating has also not been completely studied. It has recently been shown that the hot-spot location has an effect on the evaporation rate,³⁰ which may be optimised to reduce the release time. The size of the smallest object that

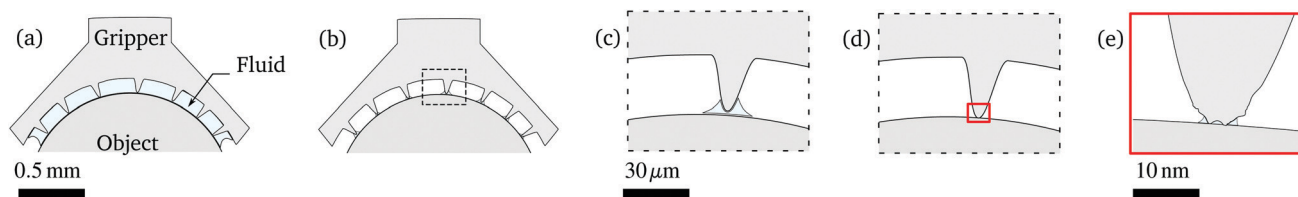


Fig. 10 Change of contact conformity during the drying process. (a and b) As the bridge dries, the contact is changed from one conformal macro-contact to several non-conformal micro-contacts. (c) Close-up of a non conformal contact. The object is released when its weight is larger than the sum of the capillary forces of each bridge. (d) and (e) The release of the object may be limited by condensation of liquid at the scale of the kelvin radius ($\sim 5 \text{ nm}$).

our gripper could seize and release may be estimated with the vertical asymptote of Fig. 3, with a bond number of $B_0 \simeq 0.5$. If we consider a metallic object of density $\rho = 7000 \text{ kg m}^{-3}$ bound with a volume $V = 0.1 \text{ }\mu\text{L}$ of water $\sigma = 72 \text{ mN m}^{-1}$, the minimal dimension is $L \sim 620 \text{ }\mu\text{m}$. Now, if the gripper and barbule shape are optimized and inertial loads are used, one would expect to release much smaller objects.

In summary, we designed a 3D-printed capillary gripper based on a change of conformity of the contact for pick-up and release. The contact conformity is changed by evaporation of the liquid bridge. It switches scale, from the large cone to the small barbules. The evaporation may be forced by the Joule effect or laser absorption to reduce the release time. The capillary force generated by the gripper depends directly on the contact conformity, *i.e.* the fit between the gripper and the object. We use the capillary conformity as an efficient proxy to explain the pick up and release mechanism. We hope that future capillary grippers will be designed using the contact conformity concept.

Conflicts of interest

There are no conflicts to declare.

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